King Fahd University of Petroleum and Minerals
Department of Mathematics and Statistics
Math 642
Comprehensive exam- Linear control theory- 2021-2022
Net Time Allowed: 150 minutes

Name:——————————————————————

ID:——————————Section:——————Serial:——————

Q#	Marks	Maximum Marks
1		20
2		20
3		20
4		20
5		20
Total		100

1. Write clearly.
2. Show all your steps.
3. No credit will be given to wrong steps.
4. Do not do messy work.
5. Calculators and mobile phones are NOT allowed in this exam.
6. Turn off your mobile.

1. Show that if the state equation

$$x' = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} x + \begin{bmatrix} B_1 \\ 0 \end{bmatrix} u$$

is controllable then the pair (A_{22}, A_{21}) is controllable.

2. A continuous linear time invariant dynamical system is described by

$$x'(t) = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} u(t), \quad x(t_0) = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}, \quad t \geq t_0$$

$$y(t) = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} x(t)$$

(a) Show that the system is not controllable, and obtain the controllable Kalman decomposition.

(b) Is the system stabilizable? Justify your answer.

(c) Is the system detectable? Justify your answer

(d) What is the system response at time $t \geq t_0$, when $u(t) \neq 0$?

3. Consider the continuous time state equation

$$x'(t) = \begin{bmatrix} 1 & 1 & -2 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 2 & 0 & 0 \end{bmatrix} x(t)$$

Let $u = pr - kx$. Find the feedforward gain p and state feedback gain k so that the resulting system has eigenvalues -2 and $(-1 \pm j1)$ and will track asymptotically any step reference input.

4. Design a full state estimator for the system described by

$$x' = \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 2 \end{bmatrix} u$$
$$y = \begin{bmatrix} 0 & 2 \end{bmatrix} x.$$

Select the following eigenvalues $\{0.5 + j0.5, 0.5 - j0.5\}$ for the estimator.

5. Consider the continuous time system

$$x'(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$
$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t).$$

Use LQR with $Q = \begin{bmatrix} 1 & 0 \\ 0 & \mu \end{bmatrix}$, $(\mu \geq 0)$ and $R = 1$ to find the optimal state feedback gain K.

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