Midterm-Version2

November 4, 2023

Midterm Exam Version 2- Recitation MATH 102 - PYTHON

1 Question 1

The estimate of the area under the graph of $f(x) = \frac{37}{23x^2+1}$ from $x = 0$ to $x = 3$, using $n = 15$ approximating rectangles with left end-points is:

[1]:
```
import sympy as smp   #Here 'as' means that we type 'smp' whenever we want to␣
 ↪recall this library
import numpy as np    #Here 'as' means that we type 'np' whenever we want to␣
 ↪recall this library

# We now define the variables and the function
x=smp.symbols('x', real=True)
f = smp.lambdify(x, 37/(1 + 23*x**2), "numpy")
# The lambdify keyword is used to allow the function to take values over
# an array of values, namely, xvalues as you are going to see below.
# This is like the substitution but more straightforward
a=0; b=3; n=15

# You can now display them
display(f(x))
print("a=",a)
print("b=",b)
print("n=",n)
```

$$\frac{37}{23x^2 + 1}$$

```
a= 0
b= 3
n= 15
```

[3]:
```
# We now define the line-space for x and then the line space for the values of␣
 ↪the function f at each x.
xvalues = np.linspace(a, b, n+1)
yvalues = f(xvalues) #You can check the sample points by using display(xvalues)

# Now we evaluate the width of subinterval, midpoints and then the Riemann sums.
```

```
deltax = (b-a)/n # The equal width of the each subinterval
#midpoints= (xvalues[1:]+xvalues[:-1])/2 # Array of midpoints
#display(midpoints) # To display the midpoints
#yvalues_M = f(midpoints) # The values of the function on midpoints
Re_L = (np.sum(yvalues[:-1])) * deltax # Reimann Sum when the samples are the␣
↪left-end points
#Re_R = (np.sum(yvalues[1:])) * deltax # Reimann Sum when the samples are the␣
↪right-end points
#Re_M = (np.sum(yvalues_M)) * deltax # Reimann Sum when the samples are the␣
↪midpoints
print("Approximating area using left-end points is ", Re_L)
#print("Approximating area using right-end points is ", Re_R)
#print("Approximating area using midpoints is ", Re_M)
```

```
Approximating area using left-end points is  15.299882255778611
```

2 Question 2

If the acceleration of a moving particle is

$$a(t) = 19t^2 - 33t + 21,$$

with initial velocity $v(0) = 8$, the TOTAL distance traveled by the particle when $0 \leq t \leq 11$ is:

[7]:
```
# (a) First we find the velocity as the antiderivative of the acceleration␣
↪function
t = smp.Symbol('t', real=True)
c = smp.Symbol('c') # We need to define the constant of integration
a=smp.Function('a')(t)
v =smp.Function('v')(t)
a= 19*t**2 -33*t+21
display(a)
v =smp.integrate(a)+c # Notice we added the constant of integration c
display(v)
```

$19t^2 - 33t + 21$

$c + \frac{19t^3}{3} - \frac{33t^2}{2} + 21t$

[8]:
```
v.subs(t,0)  # Use the command sub() to get the velocity at t=0
c1 =smp.solve(v.subs(t,0)-8,c)[0] # We write the equation as v(0)-6=0 and solve␣
↪with respect to c.
# IMPORTANT: [0] means we take only the entry instead of a the whole list of␣
↪solutions.
display(c1)
v = v.subs(c,c1)
display(v)
```

8

$$\frac{19t^3}{3} - \frac{33t^2}{2} + 21t + 8$$

[9]:

```
d=smp.integrate(smp.Abs(v), (t,0,11))
display(d)
display(float(d))
```

$$\frac{206635}{12}$$

17219.583333333332

3 Question 3

Considering the function $f(x) = 13x^2 - 7x - 6$ on the interval $[-10, 40]$, the value(s) of c such that $f_{ave} = f(c)$ is (are):

[10]:

```
# PART 1
import numpy as np   #Here 'as' means that we type 'smp' whenever we want to␣
 ↪recall this library
import sympy as smp   #Here 'as' means that we type 'smp' whenever we want to␣
 ↪recall this library
from sympy import Symbol, Function, lambdify, sqrt, exp , integrate # we import␣
 ↪these functions from smp

x = smp.Symbol('x', real=True) # Set x as a real variable
f = smp.Function('f')(x) # define f as a function of x
f = 13*x**2-7*x-6 # The expression of f
display(f) # To check whether is the correct expression of f
a=-10 # Lower limit of the integral
b=40 # Upper limit of the integral

# value takes the results of the integration of f over the given interval [a,b]

value=smp.integrate(f,(x,a,b))
display(value)
```

$$13x^2 - 7x - 6$$

$$\frac{828350}{3}$$

[11]:

```
# We need to compute the average value of f
# NOTE: Make sure that b > a here

delta=0 # Initialize the variable delta

if (b <=a ):
    print("Error: value of b <= a")
else:
```

```
    delta=b-a
    ave=value/delta
    display("Average value of f = ", ave)
```

'Average value of f = '

$$\frac{16567}{3}$$

[12]:
```
# Part 2
# Define c as the real solution to f(c)=ave
c = smp.Symbol('c', real=True)

# Define the solist of solutions that solve f(c)=ave
solist =smp.solve(f.subs(x,c)-ave,c)

# Print the items in solist
display(solist)
```

[7/26 - sqrt(2587701)/78, 7/26 + sqrt(2587701)/78]

Note there are two elements in solist.

[13]:
```
# We can autmatize this and display the value of c only if it meets our␣
↪conditions

for i in range(len(solist)):
    if ((solist[i] >=a) & (solist[i]<=b)):
        display("Possible value of c = ", solist[i])
```

'Possible value of c = '

$$\frac{7}{26} + \frac{\sqrt{2587701}}{78}$$

4 Question 4

If we use an appropriate substitution first, then evaluate the integral:

$$\int_0^3 \frac{111x dx}{\sqrt{243x^2+1}},$$

we get:

[18]:

```python
# The command transform from Sympy will substitute an expression in the␣
↪integral, with
# the corresponding limits, by a variable.
i= smp.Integral(111*x / smp.sqrt(1 + 243*x**2),(x,0,3)) # We use the command␣
↪Integral to produce an unevaluated integral
display(i)

# Define u
u = smp.Symbol('u', real=True)

k=i.transform(243*x**2+1,u) # This will substitute 3*x+5 by u and changes the␣
↪limits of integration accordingly
display(k)
```

$$\int_0^3 \frac{111x}{\sqrt{243x^2+1}}\,dx$$

$$\int_1^{2188} \frac{37}{162\sqrt{u}}\,du$$

[19]:
```python
# The integral value can now be found

smp.simplify(k)
display(float(k))
```

20.910053128418788

[41]:
```python
val=float(k)+2*2188-1
val
```

[41]: 4395.910053128418

[42]:
```python
valb=float(k)+3*2188-1
valb
```

[42]: 6583.910053128418

[44]:
```python
valc=float(k)+4*2188-1
valc
```

[44]: 8771.910053128418

[45]:
```python
vald=float(k)+2188-1
vald
```

[45]: 2207.910053128419

[46]:
```python
vale=float(k)+5*2188-1
vale
```

[46]: 10959.910053128418

5 Question 5

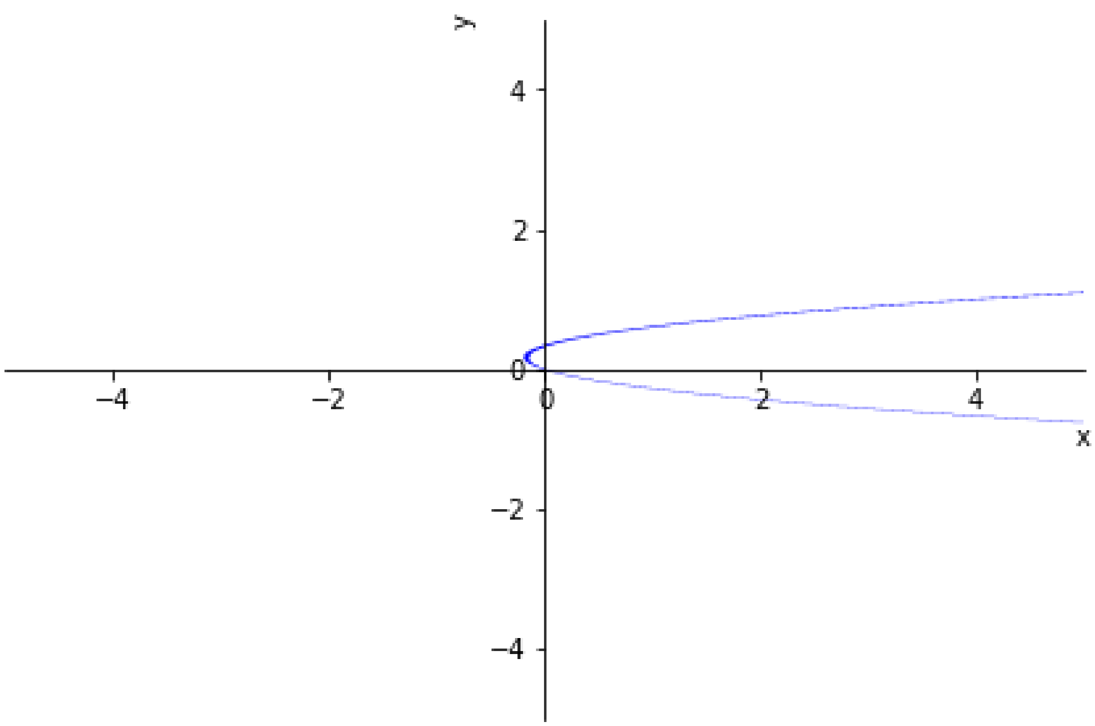
The area of the region enclosed between the curves:

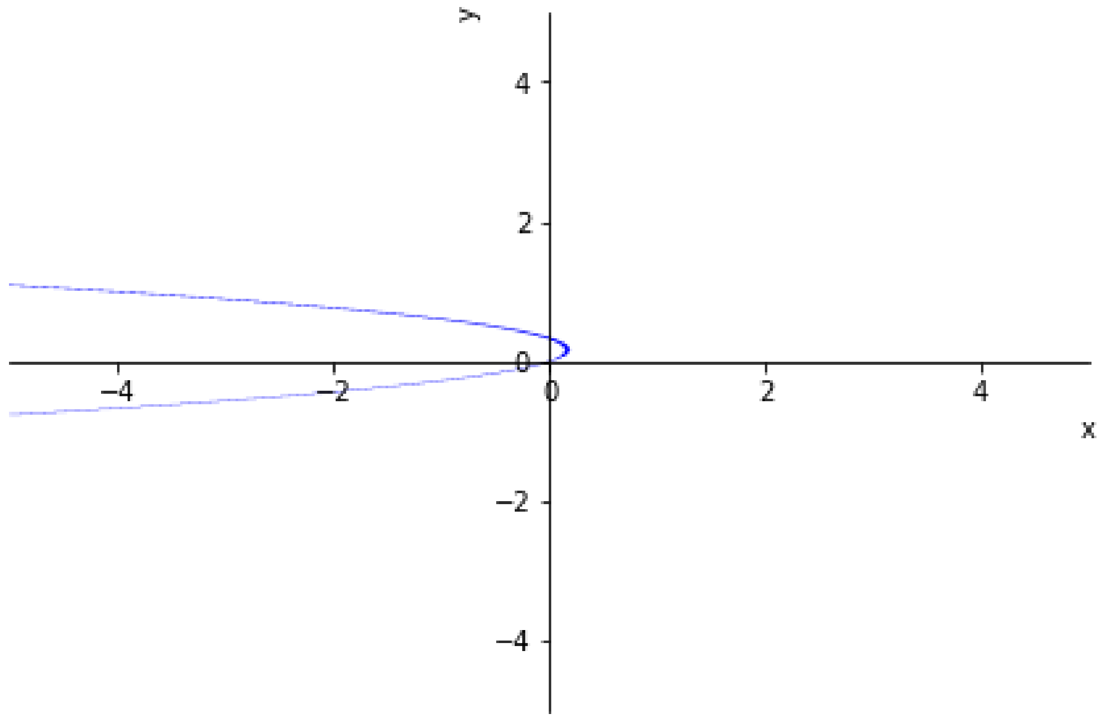
$$x = 6y^2 - 2y \quad \text{and} \quad x = 2y - 6y^2$$

is equal to:

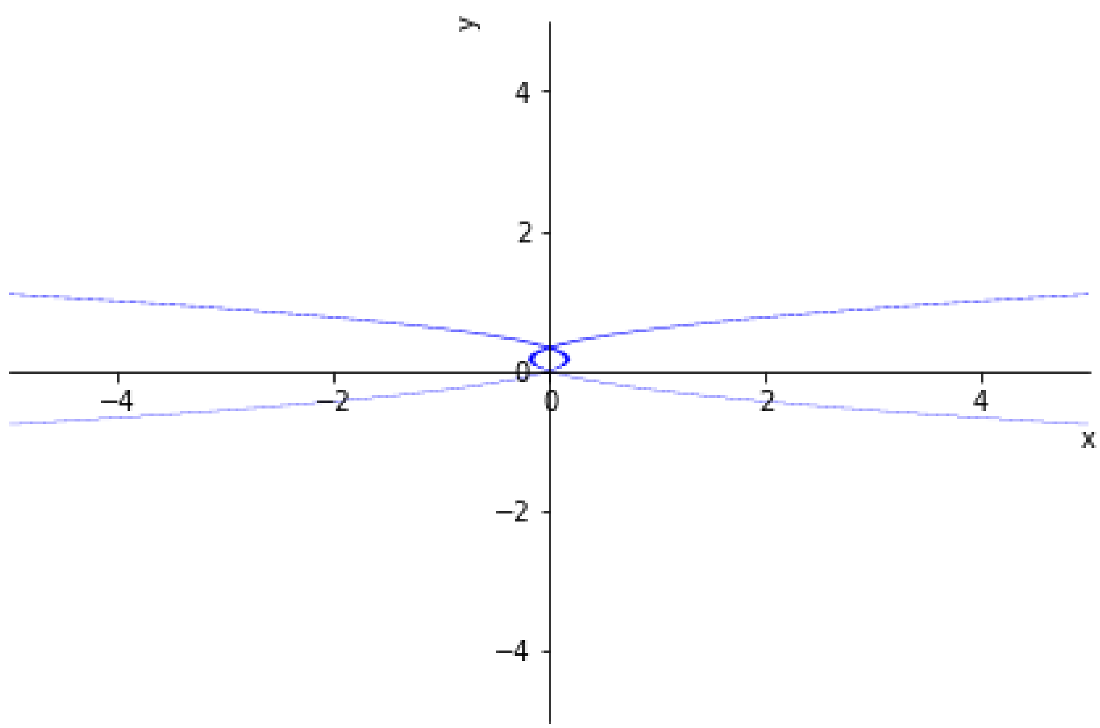
[1]:
```python
from sympy import *

x, y = symbols("x y", real=True)
# store the intersection points in t
sol = solve((x-6*y**2+2*y, x-2*y+6*y**2), x, y)
display(sol)
p1 = plot_implicit(x-6*y**2+2*y)
p2 = plot_implicit(x-2*y+6*y**2)
```

[(0, 0), (0, 1/3)]

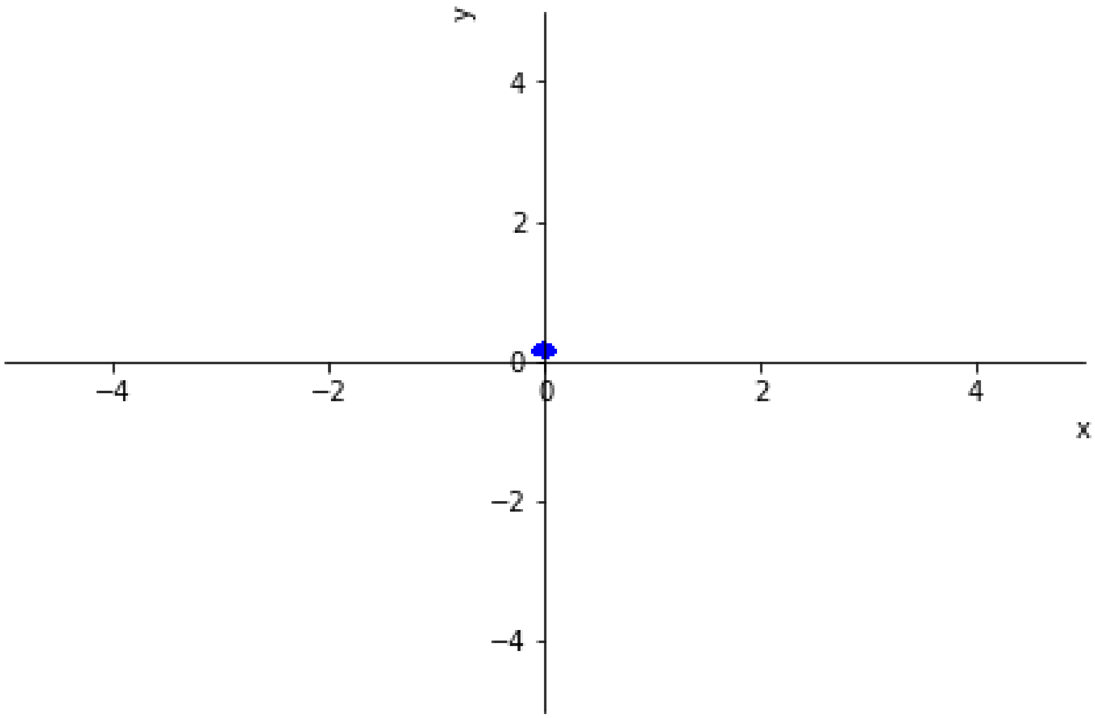




[2]:
```
p1.extend(p2)
p1.show()
```



[3]:
```
plot_implicit(And(x<2*y-6*y**2, x>6*y**2-2*y))
```



[3]: <sympy.plotting.plot.Plot at 0x287ddc3ddf0>

[4]:
```
integrate((2*y-6*y**2)-(6*y**2-2*y),(y,0,1/3))
```

[4]:

$$0.0740740740740741$$

6 Question 6

If $f(x) = 5239\ cosh^{-1}(4x)$, then $\int_{\frac{1}{3}}^{\frac{1}{2}} f(x)dx =$

[28]:
```
f=5239*smp.acosh(4*x)
i= smp.Integral(f,(x,1/3,1/2)) # We use the command Integral to produce an␣
↪unevaluated integral
display(i)
simplify(i)
```

$$\int\limits_{0.333333333333333}^{0.5} 5239 \operatorname{acosh}(4x)\, dx$$

[28]:
947.335375219489

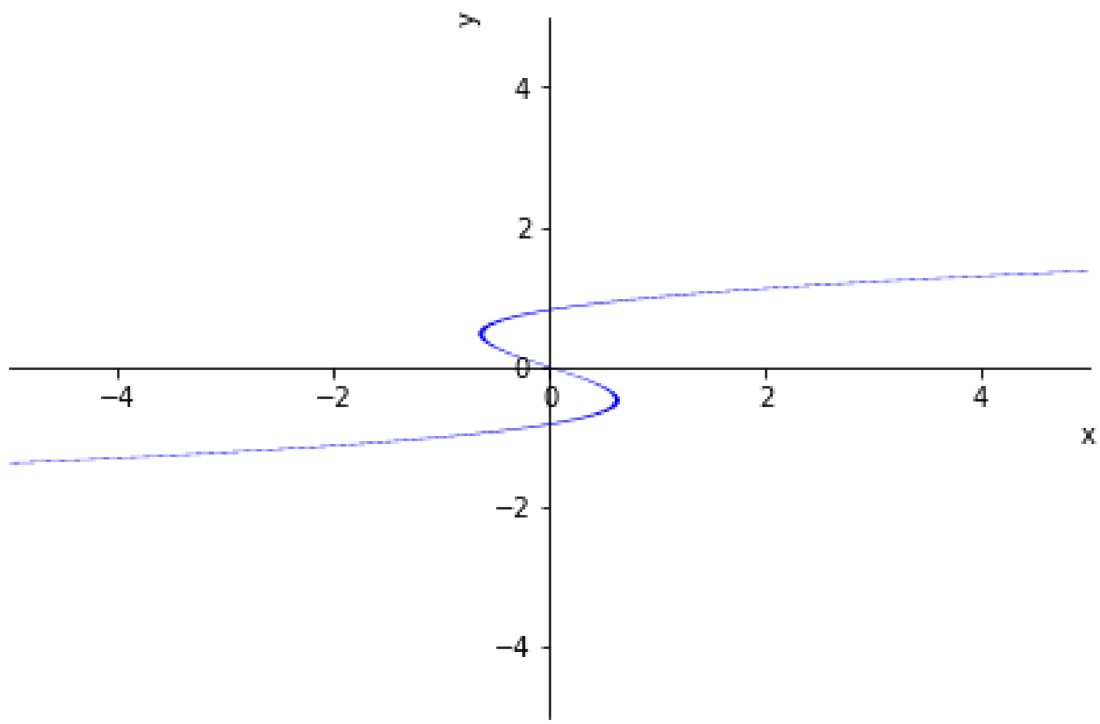
7 Question 7

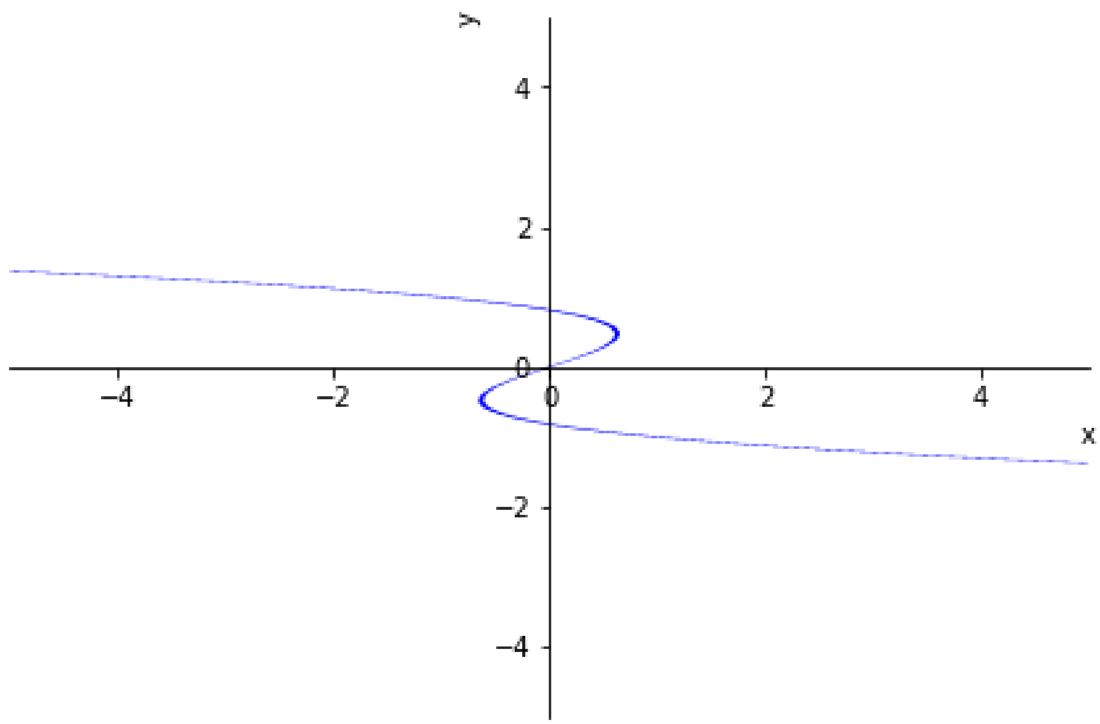
The area of the region enclosed between the curves $x = 3y^3 - 2y$ and $x = 2y - 3y^3$ is equal to:

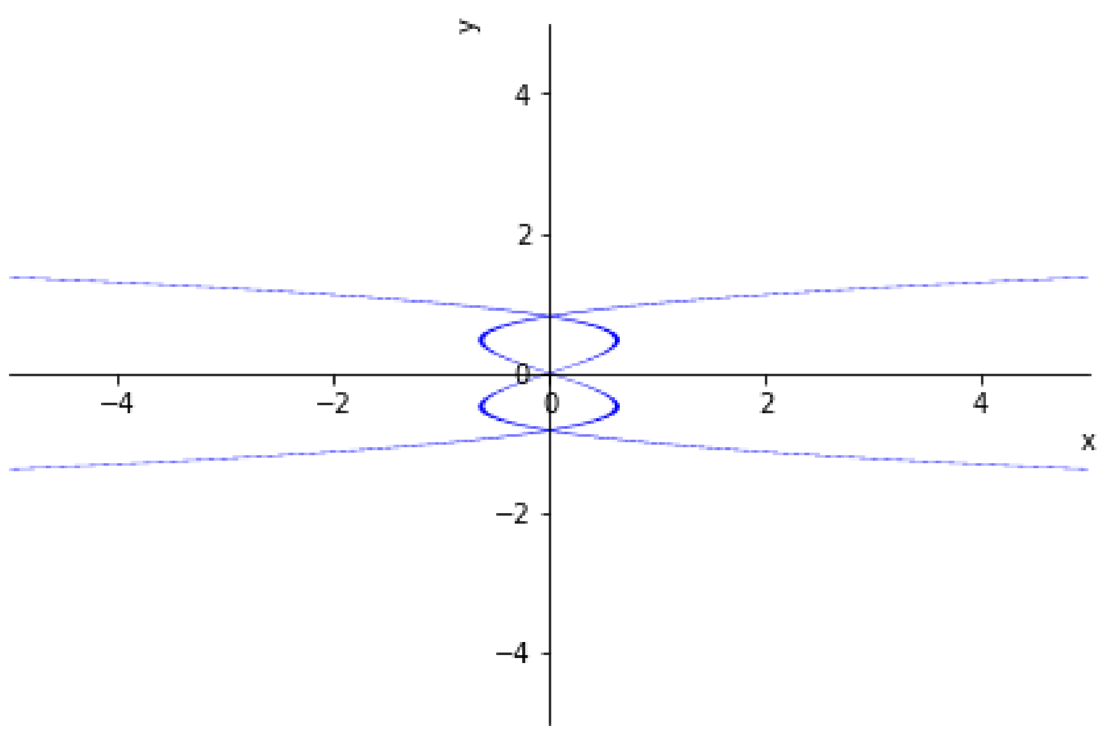
[32]:
```
x, y = symbols("x y", real=True)
# store the intersection points in sol
sol = solve((x-(3*y**3-2*y), x-(2*y-3*y**3)), x, y)
display(sol)
```

```
[(0, 0), (0, -sqrt(6)/3), (0, sqrt(6)/3)]
```

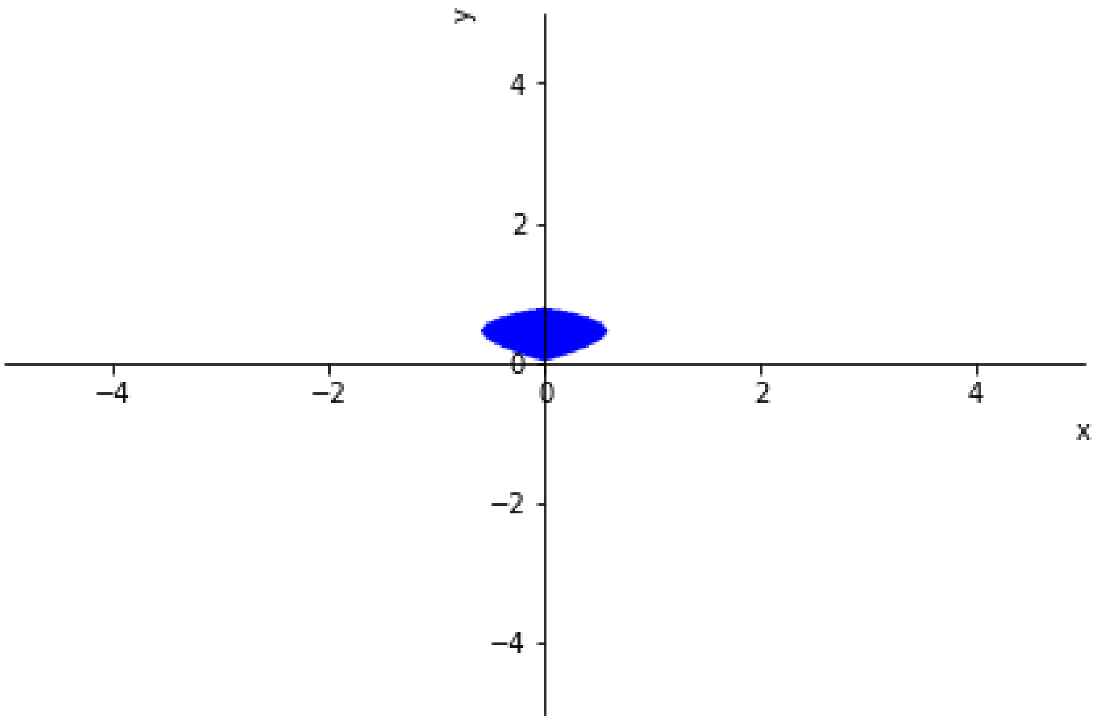
[33]:
```
p1 = plot_implicit(x-(3*y**3-2*y))
p2 = plot_implicit(x-(2*y-3*y**3))
p1.extend(p2)
p1.show()
```




y
4
2
0
−4
−2
0
2
4
x
−2
−4


y
4
2
0
−4
−2
0
2
4
x
−2
−4

[34]:
```
plot_implicit(And(x>3*y**3-2*y, x<2*y-3*y**3, y>0))
```



[34]:
```
<sympy.plotting.plot.Plot at 0x169571a4550>
```

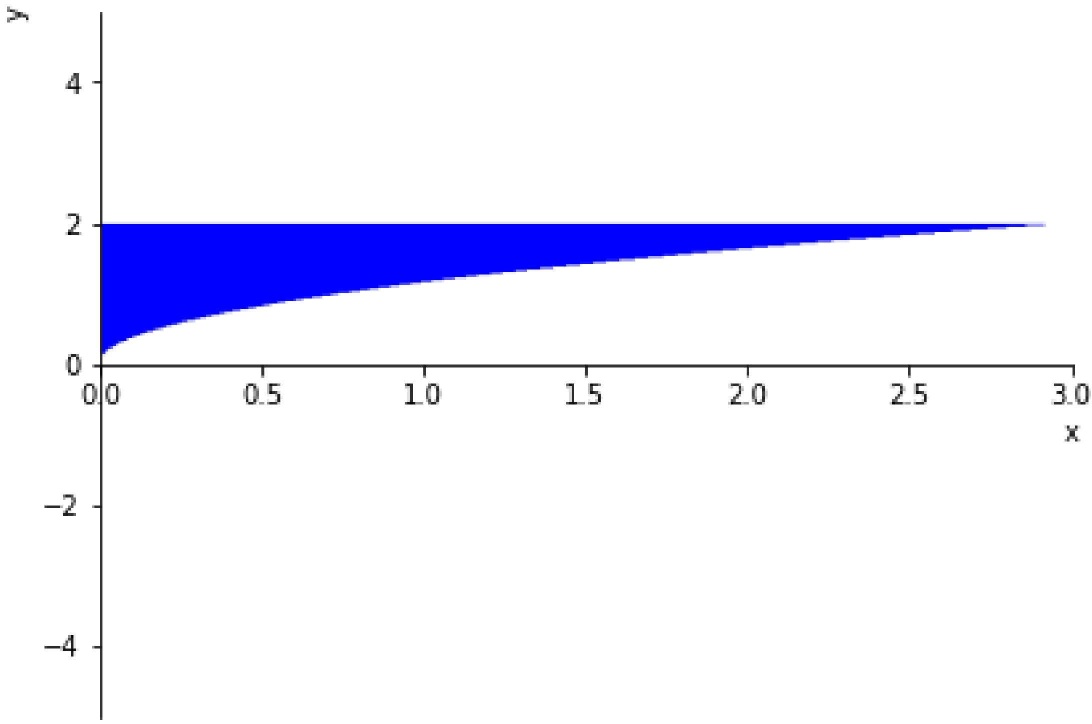
[35]:
```
A=2*integrate(4*y - 6*y**3,(y,0,sqrt(6)/3))
display(A)
```

$$\frac{4}{3}$$

8 Question 8

The volume of the solid obtained by rotating the region bounded by the curves $4x = 3y^2$, $x = 0$ and $y = 2$ about the y-axis is given by:

[36]:
```
p3 = plot_implicit(And(y < 2 , 3*y**2 - 4*x > 0, y> 0, x>0), (x,0,3))
```



[39]:
```
r = ((Rational(3,4))*y**2)
vol = pi*integrate(r**2, (y,0,2))
display(vol)
```

$$\frac{18\pi}{5}$$

[ ]: