

Syllabus of the Comprehensive Exam

MATH 525 Graph Theory

Topics

- **Graphs:** Simple graphs, multigraphs, degree sequence and Handshaking Lemma; isomorphism; complements; common families (K_n , C_n , P_n , $K_{m,n}$); distance, eccentricity, radius, diameter.
- **Subgraphs:** Induced and non-induced subgraphs; spanning subgraphs; edge/vertex deletions; contractions and subdivisions.
- **Connected Graphs:** Components; cut vertices and bridges; blocks (2-connected components); distance metrics and metric properties.
- **Trees:** Definitions and equivalent characterizations; leaves and centers; Cayley's formula and Prüfer codes; spanning trees; minimum spanning trees (Kruskal, Prim).
- **Connectivity:** Vertex/edge connectivity $\kappa(G)$, $\lambda(G)$; cuts and separators; Menger's theorem; edge/vertex-disjoint paths; ear decompositions; global vs. local connectivity.
- **Coloring:** Vertex colourings and chromatic number $\chi(G)$; greedy and Brooks' theorem (overview); edge colourings and Vizing's theorem; list colouring (intro); planarity and the Four-Colour Problem.
- **Matchings:** Matchings, maximal vs. maximum, perfect matchings; augmenting paths (Berge's lemma); bipartite matchings and Hall's theorem; König's theorem; Tutte's 1-factor theorem (overview); Edmonds' blossom idea.
- **Traversability:** Eulerian trails/circuits (Euler's theorem; Hierholzer's algorithm); Hamiltonian paths/cycles; sufficient conditions (Dirac, Ore), Bondy–Chvátal closure; Hamiltonicity in special classes.

References

- J. A. Bondy U. S. R. Murty, *Graph Theory*, Grad. Texts in Math., 244 Springer, New York, 2008,