

King Fahd University of Petroleum & Minerals

Department of Mathematics

Syllabus of the Comprehensive Exam

MATH 527 Differential Geometry

Topics

- **Curves in Euclidean space:** Arc length; tangent, normal, and binormal vectors (Frenet frame); curvature and torsion; FrenetSerret formulas; isoperimetric inequality for plane curves.
- **Local theory of surfaces in $\mathbb{R}^3$:** Regular parametrized surfaces and coordinate patches; first and second fundamental forms; normal vector field; local isometries and examples.
- **Geodesics:** Length and energy; EulerLagrange/geodesic equations; existence and uniqueness; Clairauts relation; geodesic curvature.
- **Curvature of surfaces:** Principal curvatures, lines of curvature; mean and Gaussian curvature; Gauss's *Theorema Egregium*; minimal surfaces.
- **Gauss–Bonnet theorem:** Local and global forms; relation to Euler characteristic; applications and examples.
- **Manifolds and differential forms:** Smooth manifolds; tangent and cotangent spaces; differential forms, exterior derivative, orientation; Stokes theorem.
- **Introduction to Riemannian geometry:** Riemannian metrics; Levi–Civita connection; geodesics and exponential map; curvature tensors; model spaces of constant curvature.

References

- W. Khnel, *Differential Geometry: Curves, Surfaces, Manifolds*, 3rd ed., AMS (Student Mathematical Library), **2015**.
- T. F. Banchoff & S. T. Lovett, *Differential Geometry of Curves and Surfaces*, 3rd ed., CRC Press, **2022**.