

Syllabus of the Comprehensive Exam

MATH 531 Real Analysis

Topics

- **Lebesgue measure:** The Lebesgue outer measure; The sigma-algebra of Lebesgue measurable sets; Outer and inner approximations of Lebesgue measurable sets; Properties of the Lebesgue measure; The Borel-Cantelli lemma.
- **Lebesgue measurable functions:** Sums, products, and compositions; Simple approximation; Littlewood's three principles; Egoroff's theorem; Lusin's theorem.
- **Integration Theory:** The Riemann integral; The Lebesgue integral (any type of functions); Properties of the Lebesgue integral; Convergence theorems (Fatou's lemma, the monotone convergence theorem, the Lebesgue's dominated convergence theorem); Uniform integrability and the Vitali convergence theorem; Convergence in measure; Characterizations of Riemann and Lebesgue integrability.
- **Differentiation and integration:** Continuity and differentiability of monotone functions; Functions of bounded variations; Absolutely continuous functions; Integration derivative; Convex functions.
- **The Classical L_p Spaces:** Inequalities in L_p (Hölder's and Minkowski's inequalities); Completeness of L_p and the Riesz-Fisher theorem; Convergence in L_p ; The Riesz representation for the dual of L_p ; Weak sequential convergence in L_p .
- **General measure spaces:** Measures and measurable sets; Signed measures (the Hahn and Jordan decompositions); Measurable functions; Integration of general measurable functions; the Carathéodory measure induced by an outer measure; the construction of an outer measure.

References

- H. L. Royden & P. M. Fitzpatrick, *Real Analysis*, 4th ed.