

Syllabus of the Comprehensive Exam

MATH 533 Complex Analysis

Topics

- **Complex functions:** limits, continuity, analyticity, rational functions, exponential functions, trigonometric functions, branches of logarithm.
- **Analytic functions:** harmonic functions, harmonic conjugates, Cauchy-Riemann equations, power series representation.
- **Conformal mappings:** mapping properties of elementary functions, linear (fractional) transformations.
- **Complex integration:** line integral, index or winding number, Cauchy's theorem, Cauchy's integral formula, Cauchy's estimate, Morera's theorem, Goursat's theorem.
- **Zeros of analytic functions:** entire functions, Liouville's theorem, fundamental theorem of algebra, open mapping theorem.
- **The maximum modulus theorem:** The maximum modulus principle, Schwarz's, and Schwarz-Pick lemma.
- **Singularities:** removable and essential singularities, poles, Laurent series.
- **Residues:** residue theorem, calculus of residues and evaluation of real integrals, meromorphic functions,
- **Argument principle, Rouché's theorem.**

References

- Ahlfors, *Complex Analysis*, 2nd ed., McGraw-Hill, 1979.
- Conway, *Functions of One Complex Variable*, 2nd ed., Springer-Verlag, 1978