

Syllabus of the Comprehensive Exam

MATH 535 Functional Analysis

Topics

- **Normed & Banach Spaces:** Norms and equivalent norms; completeness; Cauchy sequences; subspaces, closures, quotients; Baire category.
- **Bounded Linear Operators:** Operator norm, continuity, boundedness; invertible operators;
- **Convex Sets & Separation:** Convexity; extreme points (basic); Minkowski functional; separation/support via Hahn–Banach
- **Linear Functionals & Duality:** Dual spaces; norm of a functional; annihilators; adjoint between duals;
- **Reflexive Spaces:** Unit ball weak compactness; reflexivity of ℓ^p ; basic non-reflexive examples.
- **Weak & Weak-* Topologies:** Weak vs norm convergence; weak-* on X^* ; Banach–Alaoglu;
- **Banach Contraction Principle:** Fixed points in complete metrics; error bounds; Picard/integral-equation applications.
- **Hahn–Banach Theorem:** Norm-preserving extension; geometric separation; dual-norm computations.
- **Uniform Boundedness (Banach–Steinhaus):** Standard consequences.
- **Open Mapping & Closed Graph Theorems**
- **Hilbert Spaces:** Inner product, norm; orthogonality; projection theorem; orthonormal bases; adjoint operators
- **Riesz Representation (Hilbert):** Identify $H^* \cong H$; functionals as inner products; L^2 applications.

References

- E. Suhubi , *Functional Analysis*, (Kluwer, 2003)