**KING FAHD UNIVERSITY OF PETROLEUM &MINERALS**

Department of Mathematics

MATH 551 (Abstract Algebra)

Semester 232 (Spring 2024)

Prof. Jawad Abuihlail

**Description:** Basic definitions of rings and modules. Homomorphisms. Sums and products. Exactness. Hom and tensor. Adjoint isomorphism. Free, projective and injective modules. Chain conditions. Primary decomposition. Noetherian rings and modules. Artinian rings. Structure theorems.

**Prerequisite:** Graduate Standing.

**Textbook:** P. Grillet, *Abstract Algebra,* 2nd edition, Springer (2007).

https://link-springer-com.kfupm.idm.oclc.org/book/10.1007/978-0-387-71568-1

**References:**
1) T. Hungerford, *Algebra,* Springer-Verlag, New York-Berlin (1980).
2) S. Lang, *Algebra*, Revised 3rd edition, Springer (2005).
3) R. Wisbauer, Foundations of Module and Ring Theory, Routledge, 1st edition (2018).

**Grading:**

| First Major | Second Major | Homework | Projects/Presentations | Final Exam |
|:---:|:---:|:---:|:---:|:---:|
| 20% | 20% | 10% | 15% | 35% |

**Exams:**

| | 1st major | 2nd major | Final |
|:---:|:---:|:---:|:---:|
| **Date** | 29.2.2024 | 25.4.2024 | **TBA** |

**Attendance:** Students are expected to attend all lecture classes.

➢ If a student misses a class, he/she is responsible for any announcement made in that class.

➢ A DN grade will be awarded to any student who accumulates more than 20% unexcused absences or 33% excused and unexcused absences

**Objectives:**

(1) To identify ring-theoretic and module-theoretic properties and identify these properties in familiar rings and modules.
(2) To provide proofs to simple assertions of ring- and module-theoretic principles.

**Learning Outcomes:**

Upon successful completion of this course, the student should be able to

| Code | CLO |
|---|---|
| **1** | **Knowledge and Understanding** |
| 1.1 | Demonstrate rigorous understanding of the foundations of Rings and Modules. |
| 1.2 | Demonstrate rigorous understanding of the Basics of Category Theory (Categories, Functors, Limits, Colimits, Adjointness). |
| **2** | **Skills** |
| 2.1 | Prove and apply results on Noetherian modules, associated primes, primary decomposition, and Hilbert's Basis Theorem. |
| 2.2 | Prove and apply results on Artinian and indecomposable modules, including Krull - Schmidt Theorem and various semisimplicity results. |
| 2.3 | Prove and apply basic results on free, projective, and flat modules. |
| 2.4 | Prove and apply results on injective modules, including Baer criterion and divisibility. |
| 2.5 | Prove and apply results on semisimple rings, including structure results. |
| **3** | **Values** |
| | Manage complex ethical and professional issues and make informed judgements on ethical codes and practices. |

## Detailed Syllabus

| Week(s) | Section(s) | Title |
|---|---|---|
| 1 – 2 | **Chapter III. Rings** | |
| | III.1-III.2 | Rings, Subrings and Ideals |
| | III.3 | Homomorphisms |
| | III.4 | Domains and Fields |
| | III.11 | Noetherian Rings |
| 3 | **Chapter VII. Commutative Rings** | |
| | VII.1. | Primary Decomposition |
| 4 – 5 | **Chapter XVI: Categories** | |
| | XVI.1 | Definitions |
| | XVI.2 | Functors |
| | XVI.3 | Limits and Colimits |
| | XVI.4 | Completeness |
| | XVI.6 | Adjoint Functors |
| 6 – 7 | **Chapter VIII. Modules** | |
| | VIII.1 | Definition |
| | VIII.2 | The Adjoint Functor Theorem |
| | VIII.3 | Direct Sums and Products |
| | VIII.4 | Free Modules |
| | VIII.5 | Vector Spaces |
| | VIII.8 | Chain Conditions (Modules of Finite Length) |
| 8 – 10 | **Chapter IX. Semisimple Rings and Modules** | |
| | IX.1 | Simple Rings and Modules |
| | IX.2 | Semisimple Modules |
| | IX.3 | The Artin Wedderburn Theorem |
| | IX.5 | The Jacobson Radical |
| | IX.6 | Artinian Rings |
| 11 – 13 | **Chapter X. Projectives and Injectives** | |
| | X.1 | Exact Sequences |
| | X.3 | Projective Modules |
| | X.4 | Injective Modules |
| 14 – 15 | **Chapter XI. Construction** | |
| | XI.1 | Groups of Homomorphisms |
| | XI.2 | Properties of Hom |
| | XI.5 & XI.6 | Tensor Products and their Properties |
| | XI.8 | Flat Modules |

## Projects:

| TEAM I | TEAM II | Due |
|---|---|---|
| UFDs | PIDs | 30.1.2024 |
| Pushouts | Pullbacks | 20.2.2025 |
| Additive Categories | Subgenerators | 24.3.2024 |
| The Projective Cover | The Injective Hull | 30.4.2024 |
| Primitive Rings | Quasi-Frobenius Rings | 15.5.2024 |

**Homework Problems**

**Note:** Problems with (*) should be solved.

| Section | Due | Problems | # of required questions |
|---------|-----|----------|-------------------------|
| 3.1 | 10.2.2024 | 4, 8*, 12, 13 | 2 |
| 3.2 | | 6*, 7, 9 | 2 |
| 3.3 | | 4*, 8, 9, | 2 |
| 3.4 | | 1, 4*, 5 | 2 |
| 3.11 | | 1, 2, 5* | 2 |
| 7.1 | | 3, 15*, 17, 18, 19, 20, 21* | 5 |
| 16.1 | 26.3.2024 | 2, 8*, 10 | 2 |
| 16.2 | | 3, 4, 8* | 2 |
| 16.3 | | 2, 5*, 8, 11, 12, 14, 15* | 4 |
| 16.4 | | 3, 5*, 7*, 11, 12 | 4 |
| 16.6 | | 2, 5*, 6, 9 | 2 |
| 8.1 | 19.3.2024 | 5, 6, 8*, 9, 20* | 3 |
| 8.2 | | 3*, 6*, 8, 9, 10 | 3 |
| 8.3 | | 2*, 5, 7*, 10, 12*, 14 | 4 |
| 8.4 | | 1*, 2*, 9, 10*, 11 | 4 |
| 8.5 | | 2, 4, 5*, 6*, 9* | 4 |
| 8.8 | | 4, 5, 6*, 8*, 10, 11*, 14, 15 | 4 |
| 9.1 | 23.4.2024 | 3, 4, 5* | 2 |
| 9.2 | | 1, 4*, 6 | 2 |
| 9.3 | | 2, 3*, 6, 7 | 3 |
| 9.5 | | 1, 3, 5, 7*, 8, 16* | 3 |
| 9.6 | | 1, 3, 4, 8* | 3 |

| 10.1 | 20.5.2024 | 4, 5, 7*, 8, 9*, 10 | 4 |
|------|-----------|---------------------|---|
| 10.3 | | 2, 3, 4*, 5, 7*, 8, 9 | 4 |
| 10.4 | | 2 (correction: **direct product**), 5, 7, 8, 9, 10* | 3 |
| 11.1 | 20.5.2024 | 4, 6, 7* | 2 |
| 11.2 | | 2, 5, 6, 10, 11* | 3 |
| 11.5 | | 3, 4*, 11, 12, 13* | 3 |
| 11.6 | | 6, 10*, 12, 14, 17* | 3 |
| 11.7 | | 2, 6, 7*, 8, 9, 10* | 3 |