

Syllabus of the Comprehensive Exam

MATH 551 Abstract Algebra

Topics

- **Rings:** Associative rings; Homomorphisms of rings; Ideals
- **Modules:** Morphisms of modules; Exact sequences; Direct sums; Direct products.
- **Functors:** The Hom functor; The Tensor functor; Natural isomorphisms; Adjointness of the Tensor and the Hom functors.
- **Special Classes of Modules:** Free modules; Vector spaces; Projective modules; Injective modules; Flat modules.
- **Chain Conditions:** Noetherian modules; Artinian modules; Modules of finite length; Noetherian rings; Artinian rings.
- **Prime Ideals:** Maximal ideals; Prime and primary ideals; Primary decomposition.
- **Structure of Modules:** Simple modules; Semisimple modules.
- **Radicals of Rings:** The Jacobson radical; The prime radical; The nil radical.
- **Structure of Rings:** Semisimple rings; Structure theorems of rings.

References

- P. Grillet, Abstract Algebra, 2nd edition, Graduate Text in Mathematics 242, Springer (2007).
<https://link.springer.com/book/10.1007/978-0-387-71568-1>
- T. Hungerford, Algebra, Graduate Text in Mathematics 73, Springer (1974).
<https://link.springer.com/book/10.1007/978-1-4612-6101-8>