

King Fahd University of Petroleum & Minerals

Department of Mathematics

Syllabus of the Comprehensive Exam

MATH 556 Algebraic Geometry

Topics

- **Affine algebraic varieties:** Algebraic sets in $\mathbb{A}^n$; ideals and varieties; irreducibility and prime ideals; coordinate rings $k[V]$ and their basic properties; Hilbert's Nullstellensatz (weak and strong) and the bijection between radical ideals and affine varieties.
- **Zariski topology:** Closed sets, closure and irreducible components.
- **Morphisms of varieties & embeddings:** Regular maps and isomorphisms; morphisms given by polynomials; Veronese and Segre maps.
- **Projective varieties:** Projective space $\mathbb{P}^n$; homogeneous coordinates and homogeneous ideals; projective Nullstellensatz; homogeneous coordinate ring; regular and rational maps between projective varieties.
- **Quasi-projective varieties:** Open subvarieties of projective varieties; quasi-affine vs. quasi-projective; morphisms of quasi-projective varieties.
- **Regular and rational functions:** Regular functions on affine and projective varieties; function fields $k(V)$; dominance and rational maps.
- **Hilbert polynomial, dimension, and degree:** Krull dimension for affine/projective varieties; Hilbert function and Hilbert polynomial; degree of a projective variety and basic consequences.
- **Tangent space and smoothness:** Zariski tangent space via Jacobian; singular vs. nonsingular points; dimension and smoothness.
- **Birational geometry and blow-ups:** Birational maps and equivalence; blow-up of a point and linear subspace in $\mathbb{A}^n$ and $\mathbb{P}^n$.

References

- K. E. Smith, L. Kahanp, P. Keklinen, and W. Traves, *An Invitation to Algebraic Geometry*, Springer, 2000.