King Fahd University of Petroleum & Minerals

Department of Mathematics

Syllabus of the Comprehensive Exam

MATH 565 Advanced Ordinary Differential Equations I

Topics

- **Existence, Uniqueness, and Continuity of Solutions**: Picard–Lindelöf (Cauchy-Lipschitz) theorem; Continuation and maximal interval of existence (Extension of solutions; blow-up in finite time); Dependence on initial conditions and parameters; Gronwall inequalities and applications
- **Linear Systems**: General theory (Structure of the vector space of solutions for homogeneous systems); Systems with constant coefficients (Matrix exponential, fundamental solutions, classification); Systems with periodic coefficients (Floquet theory, monodromy matrices, stability analysis); Nonhomogeneous problems.
- **Stability Theory**: Stability of equilibrium and periodic solutions for linear systems (Lyapunov, asymptotic, and exponential stability; instability); Stability under nonlinear perturbations (Principle of linearized stability, Lyapunov's indirect method); Lyapunov direct (second) method; Stability of linear equations with coefficients approaching a limit (Uniform convergence and stability implications).
- **Qualitative Theory and Hyperbolicity:** Phase space analysis and orbits (Phase portraits, types of critical points, classification: nodes, saddles, foci, centers); Hyperbolic equilibria and stability; Topological conjugacy (Linearization near hyperbolic fixed points, Hartman-Grossman theorem); Invariant manifolds.
- **Planar Systems and Index Theory:** Two-dimensional nonlinear systems (Classification of planar phase portraits; behavior near singularities); Index theory; Bendixson's criterion (Nonexistence of periodic orbits in planar systems; divergence conditions).
- **Poincaré–Bendixson Theory**: Positive and negative orbits, alpha/omega limit sets (Invariance under flow, structure of limit sets); Invariant and minimal sets; Poincaré–Bendixson theorem; Applications (Locating and classifying periodic orbits and limit cycles in physical systems).
- **Bifurcations and Center Manifolds:** Local bifurcation analysis (Saddle-node, transcritical, pitchfork, and Hopf bifurcations); Existence and properties of center manifolds.
- **Hamiltonian Systems and Conserved Quantities**: Hamiltonian structure (Symplectic geometry, conservation theorems, physical interpretations); First integrals and integrability (Existence, calculation, Poisson brackets); Examples (Classical mechanical systems, phase portrait analysis, special solutions).

References

- F. Verhulst *Nonlinear Differential Equations and Dynamical Systems* (Second Edition, 1996. Revised 2006)
- F. Brauer and J. A. Nohel *The qualitative theory of ordinary differential equations: an Introduction.* Dover Publications, Inc. NY (1969)