King Fahd University of Petroleum & Minerals

Department of Mathematics

Syllabus of the Comprehensive Exam

MATH 568 Advanced Partial Differential Equations I

Topics

- **First-Order Equations**: Linear first-order partial differential equations (Method of characteristics: derivation, geometric interpretation, Canonical forms and integrating factor methods); Nonlinear first-order equations (Quasilinear and fully nonlinear cases, Breakdown of solutions, formation of shocks, rarefactions, and singularities, Existence and uniqueness; Burgers' equation, Hamilton-Jacobi equations).
- **Classification of Second-Order Equations:** General second-order PDEs in two (or more) variables (Standard and canonical form derivation, Definitions and criteria for elliptic, parabolic, and hyperbolic types (discriminant analysis), Laplace's equation, heat equation, wave equation).
- **Classical Equations and Models:** Wave equation (Cauchy and boundary value problems in one and multiple dimensions, d'Alembert's formula, method of characteristics, energy and uniqueness principles, Propagation of waves, finite speed of propagation); Heat equation (Initial and boundary value problems, Maximum principle and uniqueness of solutions, Fundamental solution, Weak solutions, smoothing effects). Laplace's equation (Harmonic functions and their properties, Mean value property, uniqueness by maximum principle).
- **Solution Methods**: Green's functions (Definition and construction for ODEs and PDEs; Fundamental solution, Application to Poisson, wave, and heat equations, Representation formulas, physical interpretation, uniqueness; Conformal mapping (Riemann mapping theorem and basic examples, Applications to boundary value problems, transformation of domains, Explicit conformal maps; Separation of variables, Eigenfunction expansions, Fourier series, and boundary value problems).
- **Sturm-Liouville Theory:** General theory (Linear self-adjoint differential operator, Regular and singular Sturm-Liouville problems); Properties of eigenvalues and eigenfunctions (Orthogonality and completeness, Spectral decomposition for physical systems); Applications (Expansion of solutions to PDEs using Sturm-Liouville eigenfunctions, Examples from Fourier series, Legendre, Bessel equations).
- **Energy methods for nonlinear IBVPs**

References

- P. O'Neil, *Beginning Partial Differential Equations,* Second Edition, John Wiley, 2008
- Y. Qing Han; *A basic course in Partial differential equations*, First Edition, AMS, 2011