

King Fahd University of Petroleum & Minerals

Department of Mathematics

Syllabus of the Comprehensive Exam

MATH 645 Combinatorics

Topics

- **Basic Counting:** The sum and product rules for sets, permutations and words, combinations and subsets, set partitions, permutations by cycle structure, integer partitions, compositions, lattice paths, pattern avoidance.
- **Counting with Signs:** The principle of inclusion–exclusion, sign-reversing involutions, the Reflection Principle.
- **Counting with Ordinary Generating Functions:** The algebra of formal power series, the sum and product rules for ogfs, revisiting integer partitions, recurrence relations and generating functions, rational generating functions and linear recursions.
- **Counting with posets:** Partially ordered sets, chains, antichains, and operations on posets, lattices, the Möbius function of a poset, the Möbius inversion theorem, quotients of posets, computing the Möbius function, binomial posets.
- **Counting with Group Actions:** Groups acting on sets, Burnside’s lemma, the cycle index, Redfield–Pólya theorem.

References

- B. E. Sagan, *Combinatorics: the art of counting*, Graduate Studies in Mathematics 210. Providence, RI: American Mathematical Society (AMS), 2020.