1. $\displaystyle\lim_{x\to -3} \frac{x^2-9}{2x^2+7x+3} =$

Similar to #15/2.3

(a) $\frac{6}{5}$ (correct)

(b) $-\infty$

(c) 0

(d) $\frac{1}{2}$

(e) -3

$$\lim_{x\to -3} \frac{(x-3)\cancel{(x+3)}}{(2x+1)\cancel{(x+3)}} = \frac{-6}{-5} = \boxed{6/5}$$

2. $\displaystyle\lim_{x\to 0^+} \left(\frac{1}{x} - \ln x\right) =$

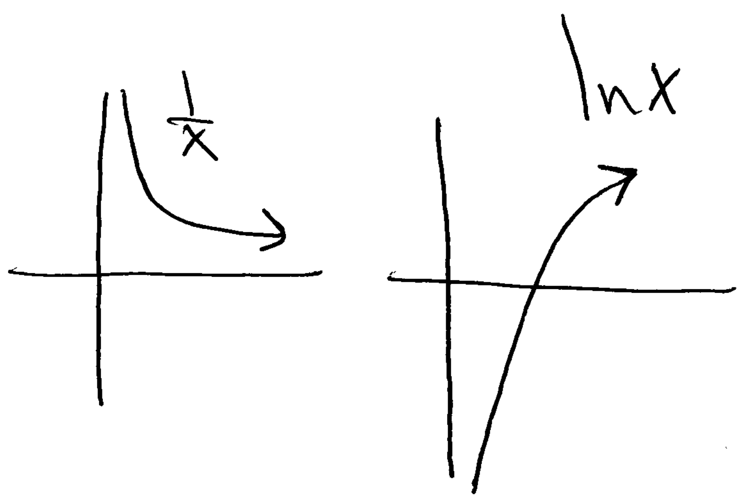
(a) ∞ (correct)

(b) 0

(c) $-\infty$

(d) 1

(e) -1



$$\lim_{x\to 0^+} \left(\frac{1}{x} - \ln x\right) = \infty - (-\infty) = +\infty$$

3. The graph of the function $f(x) = \dfrac{x^3 - 3x^2 + 2x}{x^2 + x}$ has:

(a) one vertical asymptote
(b) two vertical asymptotes
(c) three vertical asymptotes
(d) no vertical asymptote
(e) has vertical asymptotes at $x = 0$

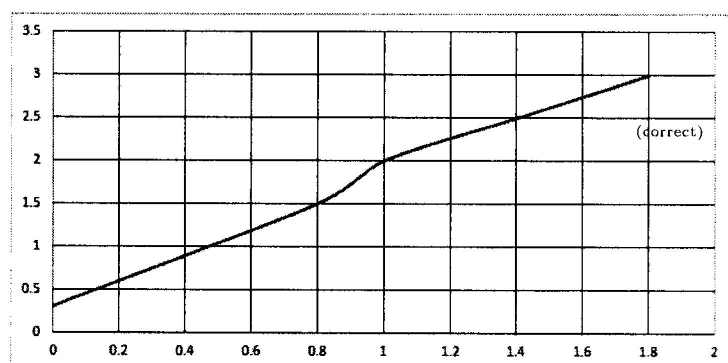
$\dfrac{\cancel{x}(x^2-3x+2)}{\cancel{x}(x+1)}$ (correct)

$\lim_{x \to -1} \dfrac{x^2-3x+2}{x+1} = \infty$

$\boxed{x = -1}$ $\lim_{x \to -1} \dfrac{(x-1)(x+2)}{(x+1)}$

4. Use the graph to find δ such that if $0 < |x-1| < \delta$, then $|f(x) - 2| < 0.5$

(a) 0.2
(b) 0.4
(c) 0.3
(d) 1
(e) 2



$|f(x) - 2| < 0.5$ Similar to #2/2.4

$-0.5 < f(x) - 2 < 0.5$

$1.5 < f(x) < 2.5 \Rightarrow$ From graph

$0.8 \leq x < 1.4$

So $\delta = \min\{1 - 0.8, 1.4 - 1\} = \boxed{0.2}$

5. $\lim_{x \to \frac{1}{2}^-} \dfrac{2x-1}{|2x^3 - x^2|} =$

Similar to #43/2.3

(a) -4
(b) 2
(c) -2
(d) 4
(e) $-\frac{1}{2}$

(correct)

$$\lim_{x \to \frac{1}{2}^-} \frac{2x-1}{|x^2||2x-1|}, \quad |x^2| = x^2$$

$$2x - 1 = 0 \Rightarrow x = \tfrac{1}{2}$$

$$- \quad \tfrac{1}{2} \quad +$$

$$\lim_{x \to \frac{1}{2}^-} \frac{(2x-1)}{-x^2(2x-1)} = -1/x^2 = \boxed{-4}$$

6. $\lim_{x \to 2} \dfrac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1} =$

$$\frac{2-2}{1-1} = \frac{0}{0}$$

(a) $\frac{1}{2}$
(b) $-\infty$
(c) 0
(d) $\sqrt{2}$
(e) $-\sqrt{2}$

(correct)

$$\lim_{x \to 2} \frac{\sqrt{6-x} - 2}{\sqrt{3-x} - 1} \cdot \frac{\sqrt{6-x} + 2}{\sqrt{6-x} + 2} \cdot \frac{\sqrt{3-x} + 1}{\sqrt{3-x} + 1}$$

$$\lim_{x \to 2} \frac{(\sqrt{6-x})^2 - (2)^2}{(\sqrt{3-x})^2 - (1)^2} \cdot \frac{\sqrt{3-x} + 1}{\sqrt{6-x} + 2}$$

Similar to #62 Sec#2.3

$$\lim_{x \to 2} \frac{6-x-4}{3-x-1} \cdot \frac{\sqrt{3-x} + 1}{\sqrt{6-x} + 2}$$

$$\lim_{x \to 2} \frac{2-x}{2-x} \cdot \frac{\sqrt{3-x} + 1}{\sqrt{6-x} + 2} = \boxed{\frac{1}{2}}$$

7. The horizontal asymptote(s) of $f(x) = \dfrac{x^2 + x^4}{x^2 - x^4}$ is (are):

(a) $y = -1$
(b) $y = 1$
(c) $y = -1$ and $y = 1$
(d) $x = 1$
(e) $x = -1$

$y = \dfrac{+1}{-1} = \boxed{-1}$ (correct)

Similar to #45/2.6

8. $\lim\limits_{x \to -\infty} \dfrac{\sqrt{9x^6 - x}}{x^3 + 1} =$

(a) -3
(b) 2
(c) 1
(d) -1
(e) 3

$$\frac{\sqrt{x^6\left(9 - \frac{1}{x^5}\right)}}{x^3\left(1 + \frac{1}{x^3}\right)}$$

(correct)

$$= \frac{-\cancel{x^3}\sqrt{9 - \frac{1}{x^5}}}{\cancel{x^3}\left(1 + \frac{1}{x^3}\right)}$$

as $x \to \infty$, $\dfrac{-3}{1} = \boxed{-3}$

Similar to # 23/2.6

9. If $f(x) = \begin{cases} cx^2 + 2x & \text{if} \quad x < 2 \\ x^3 - cx & \text{if} \quad x \geq 2 \end{cases}$ is continuous everywhere, then $c =$

(a) $\frac{2}{3}$ (correct)
(b) 2
(c) -1
(d) $\frac{3}{5}$
(e) 1

$\lim_{x \to 2^-} (cx^2 + 2x) = 4c + 4$

$\lim_{x \to 2^+} (x^3 - cx) = 8 - 2c$

$4c + 4 = 8 - 2c$

$\Rightarrow \boxed{c = 2/3}$

Similar to #45/2.5

10. Which of the following functions f has a removable discontinuity at a

(I) $f(x) = \dfrac{x^4 - 1}{x - 1}$, $a = 1$

(II) $f(x) = [\![\sin x]\!]$, $a = \pi$

(III) $f(x) = \dfrac{1 - x}{1 - |x|}$, $a = 1$

(a) (I) and (III) only (correct)
(b) (I) and (II) only
(c) (I) only
(d) (I), (II) and (III)
(e) (II) only

I): $\lim_{x \to 1} \dfrac{\cancel{(x-1)}(x+1)(x^2+1)}{\cancel{(x-1)}} = 4$ ✓

II) $\lim_{x \to \pi^+} [\sin x] = -1$, $\lim_{x \to \pi^-} [\sin x] = 0$ X

III) $\lim_{x \to 1^+} \dfrac{1-x}{1-|x|} = \dfrac{1-x}{1-x} = 1 = \lim_{x \to 1^-}$ ✓

11. Let f be a differentiable function such that $f'(a) = 4$, then $\lim_{h\to 0} \frac{f(a+5h) - f(a)}{2h} =$

(a) 10 (correct)
(b) 5
(c) 2
(d) 11
(e) $\frac{5}{2}$

Re-write

$$\frac{f(a+5h)-f(a)}{5h}\left(\frac{5}{2}\right)$$

let $t=5h \Rightarrow$

$$\lim_{h\to 0}\frac{f(a+5h)-f(a)}{5h} = \frac{5}{2}\lim_{t\to 0}\frac{f(a+t)-f(a)}{t} = \frac{5}{2}(4) = \boxed{10}$$

12. Let $f(x) = ax^2 + bx + 1$. Assume that $f(1) = 2$ and the tangent line to the graph $y = f(x)$ of f at the point $(1, 2)$ has the following equation:

$y = 3x + 2$, then $7a + 11b =$

(a) 3 (correct)
(b) 18
(c) 77
(d) 0
(e) 5

$$f(1) = \boxed{a+b+1=2}$$

$$f'(1) = \lim_{x\to 1}\frac{f(x)-f(1)}{x-1}$$

$$= \lim_{x\to 1}\frac{(ax^2+bx+1)-(a+b+1)}{x-1}$$

$$= \lim_{x\to 1}\frac{(ax^2-a)+(bx-b)}{x-1}$$

$$= \lim_{x\to 1}\frac{a(x^2-1)+b(x-1)}{x-1}$$

$$\lim_{x\to 1} a(x+1)+b = \boxed{2a+b=3}$$

Now $2a+b=3$
$a+b+1=2$

$\Rightarrow a=2$
$b=-1$

So $7(2)+11(-1)$
$=14-11$
$=\boxed{3}$

13. Let f be the function defined by

$$\begin{cases} 2x+b, & \text{if} \quad x \le -2 \\ x^2+ax+3b, & \text{if} \quad x > -2 \end{cases}$$

If f is differentiable at $x=-2$, then $4a+5b =$ $4(6)+5(2)$ $=34$

(a) 34 (correct)
(b) 45
(c) 54
(d) 23
(e) 20

$f(x)$ is cont. at $x=-2$

$f(-2) = \lim_{x\to -2^-} f(x) = \lim_{x\to -2^+} f(x)$

$-4+b = 4-2a+3b$

$\Rightarrow \boxed{a-b=4}$

also, f is Diff. at $x=-2$

$f'_-(-2) = f'_+(-2) \Rightarrow LHD = RHD \Rightarrow 2 = 2(-2)+a$

$\boxed{a=6}$

$\boxed{b=2}$

14. Which of the following statements is always **True**?

(a) If $\lim_{x\to a} f(x)$ exists but $\lim_{x\to a} g(x)$ does not exists, then $\lim_{x\to a}[f(x)+g(x)]$ does not exist (correct)

(b) If $\lim_{x\to 5} f(x) = 2$ and $\lim_{x\to 5} g(x) = 0$, then $\lim_{x\to 5} \dfrac{f(x)}{g(x)}$ may exists

(c) If neither $\lim_{x\to a} f(x)$ nor $\lim_{x\to a} g(x)$ exists, then $\lim_{x\to a}[f(x)+g(x)]$ does not exist

(d) If $\lim_{x\to 3}(f(x)g(x))$ exists, then the limit must be $f(3)\,g(3)$

(e) If $\lim_{x\to 0} f(x) = \lim_{x\to 0} g(x) = \infty$, then $\lim_{x\to 0}(f(x)-g(x)) = 0$

From T/F Quiz at End of chapter 2.

15. Let $y = f(x)$ be a function such that

(i) $f(0) = f'(0) = 1$

(ii) $f(2) = f'(2) = 0$

(iii) $f'(1) = -1$

Which of the following graphs could be the graph of $y = f(x)$?

(a) graph (a)

(b) graph (b)

(c) graph (c)

(d) graph (d)

(e) graph (e)