1. Let $f(x) = \frac{1}{x}$, then $f^{(50)}(-1) =$

(a) $-50\,!$ (correct)
(b) $49\,!$
(c) $51\,!$
(d) undefined
(e) $50\,!$

$f = x^{-1}$

$f^{(1)} = -x^{-2}$

$f^{(2)} = (-1)(-2)\,x^{-3}$

$f^{(50)} = (-1)(-2)(-3)\cdots(-50)\,x^{-51}$

$f^{(50)}(-1) = 50!\,(-1)^{-51} = -50!$

2. The point(s) on the curve $y = x^3 + 3x^2 - 3x + 3$ where the tangent is parallel to the line $3x + y = 15$ is (are):

(a) $(0,3)$ and $(-2,13)$ (correct)
(b) $(0,3)$ only
(c) $(-2,13)$ only
(d) $(0,15)$ and $(-2,21)$
(e) $(0,15)$ only

Similar to #51/Sec 3.1

$y = -3x + 15$

$\Rightarrow m = -3$

$y' = 3x^2 + 6x - 3 = -3$

$\Rightarrow 3x^2 + 6x = 0$

$3x(x+2) = 0$

$x = 0$ or $x = -2$

$(0,3)$ $(-2,13)$

3. Let $P(x) = \dfrac{F(x)}{G(x)}$, where F and G are the function whose graphs are shown then $P'(2) =$

#50/sec#3.2

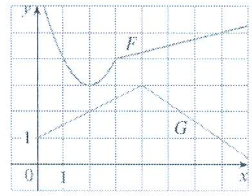
(a) $-\dfrac{3}{8}$

(b) 0

(c) undefined

(d) $-\dfrac{3}{4}$

(e) $\dfrac{3}{2}$



(correct)

$$P'(x) = \frac{F' \cdot G - G' \cdot F}{G^2}$$

$$P'(2) = \frac{F'(2)G(2) - G'(2)F(2)}{G^2(2)}$$

4. Which one of the following statements is always **True**?

(a) $\dfrac{d}{dx}(\tan^2 x) = \dfrac{d}{dx}(\sec^2 x)$

(b) $\dfrac{d}{dx}(10^x) = x\,10^{x-1}$

(c) If f is differentiable, then $\dfrac{d}{dx} f(\sqrt{x}) = \dfrac{f'(x)}{2\sqrt{x}}$

(d) $\lim\limits_{x\to 0} (1+x)^{1/x} = 1$

(e) $\dfrac{d}{dx}|x^2| = 2|x|$

(correct)

$$\frac{0 - \left(\frac{1}{2}\right)(3)}{4} = \boxed{-3/8}$$

True, False Quiz
end of chapter #3

5. The equation of the tangent line to $f(x) = 2x\sin x$ at $\left(\frac{\pi}{2}, \pi\right)$ is

(a) $y = 2x$ (correct)

(b) $y = 2x - \pi$

(c) $y = \pi\left(x - \frac{\pi}{2}\right)$

(d) $y = \pi x$

(e) $y = \pi$

#25/sec3.3

$f'(x) = 2\sin x + 2x\cos x$

$f'\left(\frac{\pi}{2}\right) = 2 + 0 = 2$

$y - \pi = 2\left(x - \frac{\pi}{2}\right)$

$\boxed{y = 2x}$

6. $\lim_{x\to 0} \dfrac{\sin x}{x + \tan x} =$

(a) $\frac{1}{2}$ (correct)

(b) 0

(c) ∞

(d) 1

(e) $-\infty$

$\lim_{x\to 0} \dfrac{\frac{\sin x}{x}}{\frac{x}{x} + \frac{\tan x}{x}}$

#45/sec3.3

$\lim_{x\to 0} \dfrac{1}{1+1} = \boxed{1/2}$

7. If $f(x) = \sqrt{\dfrac{x^2+1}{x^2+4}}$, then $f'(1)$ is

$$f'(x) = \frac{1}{2}\left(\frac{x^2+1}{x^2+4}\right)^{-1/2} \cdot \left(\frac{x^2+1}{x^2+4}\right)'$$

(a) $\dfrac{3}{5\sqrt{10}}$ (correct)

(b) $\sqrt{\dfrac{2}{5}}$

(c) $\sqrt{5}$

(d) 1

(e) $\dfrac{1}{2}$

$$\left(\frac{x^2+1}{x^2+4}\right)' = \frac{(x^2+4)(2x)-(x^2+1)(2x)}{(x^2+4)^2}$$

$$f'(x) = \frac{1}{2}\left(\frac{x^2+4}{x^2+1}\right)^{1/2} \frac{6x}{(x^2+4)^2}$$

#22 Sec 3.4

$$f'(1) = \frac{1}{2}\sqrt{\left(\frac{5}{2}\right)}\ \frac{6}{25} = \frac{3}{5\sqrt{10}}$$

8. If $g(x)$ is a twice differentiable function and $f(x) = x\,g(x^2)$, then, $f''(x) =$

(a) $6xg'(x^2) + 4x^3g''(x^2)$ (correct)

(b) $4xg'(x^2) + 2x^2g''(x^2)$

(c) $g'(x^2) + x^2g''(x^2)$

(d) $2x^2g(x^2) + x^3g''(x^2)$

(e) $2g''(x^2)$

#72/sec#3.4

$$f'(x) = g(x^2) + x g'(x^2)(2x)$$

$$= g(x^2) + 2x^2 g'(x^2)$$

$$f''(x) = 2x g'(x^2) + 4x g'(x^2) + 2x^2(2x) g''(x^2)$$

$$= 2x g'(x^2) + 4x g'(x^2) + 4x^3 g''(x^2)$$

$$= 6x g'(x^2) + 4x^3 g''(x^2)$$

9. If $\sin y + \cos x = 1$, and $-\frac{\pi}{2} < y < \frac{\pi}{2}$, then $y''(0) =$

(a) 1 (correct)

(b) 0

(c) -1

(d) $\frac{\sqrt{2}}{2}$

(e) $-\frac{\sqrt{2}}{2}$

$$y' \cos y - \sin x = 0$$

$$y' = \frac{\sin x}{\cos y}$$

$$y'' = \frac{\cos x \cdot \cos y + y' \sin y \sin x}{\cos^2 y}$$

$$\left.\begin{array}{l} x = 0 \Rightarrow \sin y + 1 = 1 \\ \Rightarrow \sin y = 0 \Rightarrow y = 0 \end{array}\right\} \quad y''(0) = \frac{(1)(1) + 0}{1^2} = 1$$

10. If $y = (\tan^{-1}(bx))^2$ and $y'\left(\frac{1}{b}\right) = \pi$, then $b =$

(a) 4 (correct)

(b) 3

(c) 2

(d) 1

(e) 0

$$y' = 2(\tan^{-1}(bx)) \cdot \frac{b}{1 + b^2x^2}$$

$$y'\left(\frac{1}{b}\right) = 2\tan^{-1} 1 \cdot \frac{b}{1+1} = \pi$$

$$\tan^{-1}(1)\, b = \pi$$

$$\frac{\pi}{4} b = \pi \Rightarrow \boxed{b = 4}$$

11. If $y = (1+\sqrt{x})^x$, then $y'(1) =$

(a) $\frac{1}{2} + \ln 4$ (correct)

(b) $\frac{1}{4} + \ln 2$

(c) $\frac{1}{4} + 2\ln 2$

(d) 2

(e) $\ln 2$

$$\ln y = x\ln(1+\sqrt{x})$$

$$\frac{y'}{y} = \ln(1+\sqrt{x}) + x\,\frac{\frac{1}{2\sqrt{x}}}{1+\sqrt{x}}$$

$$y' = \left[\ln(1+\sqrt{x}) + \frac{\sqrt{x}}{2(1+\sqrt{x})}\right]y$$

$$y'(1) = 2\left[\ln(2) + \frac{1}{4}\right] = 2\ln 2 + \frac{1}{2} = \ln 4 + \frac{1}{2}$$

12. If $P = (2, b)$ is a point on the curve $x^2 + y^2 + 2x - xy + 6y - 13 = 0$, with $b \neq 1$, then, the slope of the tangent line to the curve at P is:

(a) $\frac{11}{6}$ (correct)

(b) $\frac{13}{7}$

(c) $\frac{11}{3}$

(d) $\frac{11}{7}$

(e) $\frac{13}{3}$

$P = (2, b)$

$4 + b^2 + 4 - 2b + 6b - 13 = 0$

$b^2 + 4b - 5 = 0$

$(b+5)(b-1) = 0, \quad b \neq 1$

$b = -5 \Rightarrow (2, -5)$

Now $2x + 2yy' + 2 - y - xy' + 6y' = 0$

$$y' = -\frac{2x+2-y}{2y-x+6}$$

$$y'\Big|_{(2,-5)} = -\frac{4+2+5}{-10-2+6} = \boxed{11/6}$$

13. An object is moving along a straight line with position function $s(t) = \frac{t^3}{3} - \frac{19t^2}{2} - 5t\,(t \geq 0)$.($s$ measured in meters, and t in seconds). Then the object reaches the velocity $15m/s$ after

(a) $20\,s$
(b) $15\,s$
(c) $25\,s$
(d) $30\,s$
(e) $10\,s$

$s'(t) = t^2 - 19t - 5 = 15$ (correct)

$t^2 - 19t - 20 = 0$

$(t-20)(t+1) = 0$

$t = 20$

14. A particle is moving along a hyperbola $xy = 8$. As it reaches the point $(4, 2)$, the y-coordinate is decreasing at a rate of $3\,cm/s$. How fast is the x-coordinate of the point changing at that instant:

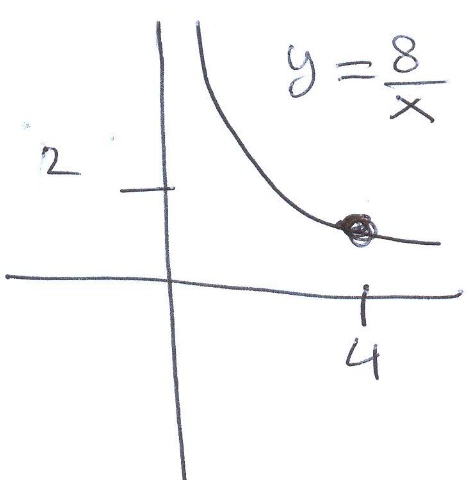
(a) $6\,cm/s$
(b) $8\,cm/s$
(c) $-6\,cm/s$
(d) $12\,cm/s$
(e) $-8\,cm/s$

$xy = 8$ (correct)

$y\frac{dx}{dt} + x\frac{dy}{dt} = 0$

$2\frac{dx}{dt} + 4(-3) = 0$

$\frac{dx}{dt} = 6$



#10/sec 3.9

15. For $y = 2x - x^2$, $x = 2$ and $\triangle x = -0.4$. The value of $dy + \triangle y$ is equal to

(a) 1.44 (correct)

(b) 0.16

(c) −0.16

(d) −0.44

(e) −1.44

$X = 2$

$X + \Delta X = 1.6$

$\Delta y = y(1.6) - y(2)$

$= (3.2 - 2.56) - 0$

$= 0.64$

$dy = f'(2)\,dX$

$= (2 - 2(2))(-0.4)$

$= (-2)(-0.4)$

$= 0.8$

$0.64 + 0.8$

$= 1.44$

#19/sec 3.10