~ #38 §2.3

1. If $1 + \sqrt{x+3} \le g(x) \le x^2 - 5x - 2$, for all x, then $\lim_{x\to 6} g(x) =$

(a) 4 (correct)

(b) does not exist

(c) 3

(d) 5

(e) 6

By the Squeeze Theorem

$\lim_{x\to 6} (1+\sqrt{x+3}) = 1+\sqrt{9} = 1+3 = 4$

$\lim_{x\to 6} (x^2-5x-2) = 36-30-2 = 4$

$=$

So $\lim_{x\to 6} g(x) = 4$

#32 §2.6

2. $\lim_{x\to\infty} (2\cos(3x)) =$

(a) does not exist (correct)

(b) 0

(c) ∞

(d) 2

(e) -2

Since $f(x) = 2\cos(3x)$ oscillates at infinity then the limit does not exist

~ #47
§ 2.5

3. If f and g are **continuous** functions such that $g(2) = 4$ and $\lim_{x\to 2}[3f(x) - 2f(x)g(x)] = 20$, then $f(2) =$

(a) -4
(b) -5
(c) -3
(d) -2
(e) -1

$3 f(2) - 2f(2)g(2) = 20$

$3f(2) - 2f(2)\cdot 4 = 20$

$-5f(2) = 20$

$f(2) = -4$

f & g are continuum

$\Rightarrow \lim_{x\to 2} f(x) = f(2)$

(correct)

& $\lim_{x\to 2} g(x) = g(2)$

~ #25
§ 2.3

4. $\lim_{t\to 0} \dfrac{\sqrt{1-t^2} - \sqrt{1+t^2}}{t^2} =$

(a) -1
(b) -2
(c) 2
(d) 0
(e) $\frac{1}{2}$

$\lim_{t\to 0} \dfrac{\sqrt{1-t^2} - \sqrt{1+t^2}}{t^2} \cdot \dfrac{\sqrt{1-t^2} + \sqrt{1+t^2}}{\sqrt{1-t^2} + \sqrt{1+t^2}}$ (correct)

$\lim_{t\to 0} \dfrac{(1-t^2) - (1+t^2)}{t^2\left(\sqrt{1-t^2} + \sqrt{1+t^2}\right)}$

$\lim_{t\to 0} \dfrac{-2t^2}{t^2\left(\sqrt{1-t^2} + \sqrt{1+t^2}\right)}$

$\lim_{t\to 0} \dfrac{-2}{\sqrt{1-t^2} + \sqrt{1+t^2}} = \dfrac{-2}{1+1} = -1$

~ #49
§2.3

5. $\lim_{x\to -3^-} \dfrac{x^2 - x - 12}{|x+3|} =$ $\lim_{x\to -3^-} \dfrac{(x-4)(x+3)}{-(x+3)}$

$= \lim_{x\to -3^-} -(x-4) = -(-3-4) = 7$

(a) 7 (correct)

(b) -7

(c) 4

(d) -3

(e) 3

~ Example 9
§2.2

6. $\lim_{x\to -2^-} \dfrac{x-4}{x(x+2)} =$ $\longrightarrow \dfrac{-6}{0}$, check the sign of $\dfrac{x-4}{x(x+2)}$ to the left of -2:

(a) $-\infty$ (correct)

$\dfrac{-}{-(-)} = -$

(b) ∞

(c) 0

(d) -2

(e) 2

$\lim_{x\to -2^-} \dfrac{x-4}{x(x+2)} = -\infty$

OR $\lim_{x\to -2^-} \dfrac{x-4}{x} \cdot \dfrac{1}{x+2} = \dfrac{-6}{-2} \cdot -\infty = -\infty$

~ #20
§2.5

7. Which one of the following statements is **TRUE** about

$$f(x) = \begin{cases} \dfrac{4-x^2}{2-x} & \text{if} \quad x \neq 2 \\ 1 & \text{if} \quad x = 2 \end{cases}$$

$\lim_{x \to 2} f(x) = \lim_{x\to 2} \frac{4-x^2}{2-x}$
$= \lim_{x\to 2}(2+x) = 4$ exists
$\neq f(2) = 1$
So f has a removable discontinuity at $x=2$ (correct)

(a) f has a removable discontinuity at $x = 2$
(b) f has a jump discontinuity at $x = 2$
(c) f has an infinite discontinuity at $x = 2$
(d) f is continuous at $x = 2$
(e) f is not defined at $x = 2$

~ #53, 54, 55
§2.3

8. $\lim_{x\to \frac{1}{3}^+} [\![-6x]\!] =$ ($[\![\cdot]\!]$ greatest integer function)

Let $z = -6x$
Since $x \to \frac{1}{3} \Rightarrow z \to -2$
& $x > \frac{1}{3}$ & $-6x < -2$ (correct)
$z < -2$

-2

then
$\lim_{x\to\frac{1}{3}^+} [\![-6x]\!] = \lim_{z\to -2^+} [\![z]\!]$
$= \underline{\underline{-3}}$

(a) -1
$\boxed{\text{(b)}}$ -2
(c) does not exist
(d) 2
(e) 3

~ Example 6, #27 §2.5

9. Where is the function $f(x) = \dfrac{\sqrt{x-1}}{x^3 - 3x^2}$ **continuous**?

$\sqrt{x-1} \Rightarrow x-1 \geq 0 \Rightarrow x \geq 1$

$x^3 - 3x^2 \neq 0 \Rightarrow x^2(x-3) \neq 0$

$\Rightarrow x \neq 0,\ x \neq 3$

(a) $[1, 3) \cup (3, \infty)$ (correct)

(b) $[1, \infty)$

(c) $(0, 1) \cup (1, \infty)$

(d) $(-\infty, 0) \cup (0, 1]$

(e) $(-\infty, 0) \cup (0, 1] \cup [1, 3) \cup (3, \infty)$.

0 1 3

$[1, 3) \cup (3, \infty)$

~ #45 §2.5

10. If

$$f(x) = \begin{cases} ax^2 + b & \text{if} \quad x < 2 \\ 2bx + a - 3 & \text{if} \quad x \geq 2 \end{cases}$$

is **continuous** at $x = 2$, then $b - a =$

f is continuous at $x = 2 \Rightarrow \lim_{x \to 2} f(x)$ exists

$\Rightarrow \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$

$\Rightarrow \lim_{x \to 2^-} (ax^2 + b) = \lim_{x \to 2^+} (2bx + a - 3)$

$\Rightarrow 4a + b = 4b + a - 3$

$\Rightarrow 3a - 3b = -3$

$\Rightarrow a - b = -1$

$\Rightarrow b - a = 1$

(a) 1 (correct)

(b) -1

(c) 3

(d) 2

(e) -5

~ #19
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Ch 2 Review

11. $\lim_{x\to 0^+} \tan^{-1}\left(\frac{-1}{x}\right)$

Let $z = -\frac{1}{x}$

If $x \to 0^+$, then $z \to -\infty$

$= \lim_{z\to -\infty} \tan^{-1}(z) = -\frac{\pi}{2}$

(a) $-\frac{\pi}{2}$ (correct)

(b) $\frac{\pi}{2}$

(c) $-\infty$

(d) ∞

(e) 0

~ #20
§ 2.4

12. Let $f(x) = 10 - 7x$ and $\lim_{x\to 2} f(x) = L$. Then the **largest** number δ such that "If $0 < |x-2| < \delta$, then $|f(x) - L| < 0.1$" is

$L = -4$

$|f(x) - L| < 0.1 \iff |10 - 7x - (-4)| < 0.1$

$\iff |14 - 7x| < 0.1$

$\iff 7|2 - x| < 0.1$

$\iff |2 - x| < \frac{0.1}{7}$

$\iff |x - 2| < \frac{1}{70}$

we may take $\delta = \frac{1}{70}$ (or any smaller value)

$\Rightarrow$ the largest value of δ is $\frac{1}{70}$

(a) $\delta = \frac{1}{70}$ (correct)

(b) $\delta = \frac{1}{60}$

(c) $\delta = \frac{1}{80}$

(d) $\delta = \frac{1}{40}$

(e) $\delta = \frac{1}{10}$

$\boxed{\sim \#22 \ \S 2.6}$

13. $\displaystyle \lim_{x\to-\infty} \frac{\sqrt{4x^6 - x}}{x^3+2}$ $\displaystyle = \lim_{x\to-\infty} \frac{\sqrt{x^6\left(4-\frac{1}{x^5}\right)}}{x^3\left(1+\frac{2}{x^3}\right)}$

$\displaystyle = \lim_{x\to-\infty} \frac{|x^3|\sqrt{4-\frac{1}{x^5}}}{x^3\left(1+\frac{2}{x^3}\right)}$

$\displaystyle = \lim_{x\to-\infty} \frac{-x^3\sqrt{4-\frac{1}{x^5}}}{x^3\left(1+\frac{2}{x^3}\right)}$

$\displaystyle = \lim_{x\to-\infty} -\frac{\sqrt{4-\frac{1}{x^5}}}{1+\frac{2}{x^3}} = -\frac{\sqrt{4-0}}{1+0} = -2$

(a) -2 (correct)

(b) 2

(c) 1

(d) $-\infty$

(e) $\frac{1}{2}$

14. Which one of the following statements is **TRUE**?

T (a) If $\lim_{x\to 0} f(x) = 0$, then there is a number δ such that if $0 < |x| < \delta$ then $|f(x)| < 0.25$ (correct)

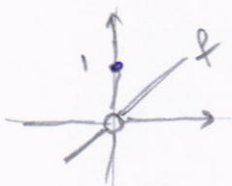
the precise definition of limit with $\varepsilon = 0.25$

F (b) If $\lim_{x\to 0} f(x) = 0$, then $f(0) = 0$

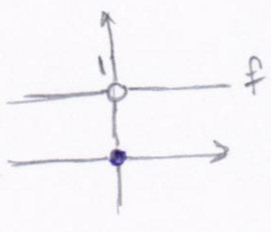
F (c) If $f(0) = 0$, then $\lim_{x\to 0} f(x) = 0$

F (d) If $f(0)$ is undefined, then $\lim_{x\to 0} f(x)$ does not exist

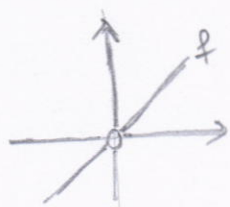
F (e) If $\lim_{x\to 0} f(x)$ exists, then $f(0)$ is defined.



(b)



(c)



(d) & (e)

~ #49
§ 2.6

15. Which one of the following statements is **TRUE** about the curve $y = \dfrac{4x^2+5x}{2x^2+x}$ $= \dfrac{x(4x+5)}{x(2x+1)} = \dfrac{4x+5}{2x+1}$, $x \neq 0$

(**V.A.** : Vertical asymptote, **H.A.**: Horizontal asymptote)

$\lim_{x \to \infty} \dfrac{4x+5}{2x+1} = \dfrac{4}{2} = 2$

$\lim_{x \to -\infty} \dfrac{4x+5}{2x+1} = \dfrac{4}{2} = 2$

H.A.: $y = 2$

(a) It has $y = 2$ as a **H.A.** and $x = -\frac{1}{2}$ as a **V.A.** (correct)

(b) It has $x = 0$ and $x = -\dfrac{1}{2}$ as **V.A.**

(c) It has $y = 4$ as a **H.A.**

(d) It has no **V.A.**

(e) It has no **H.A.**

, $2x+1 = 0 \Rightarrow x = -\frac{1}{2}$

$\lim_{x \to -\frac{1}{2}^+} \dfrac{4x+5}{2x+1} = \infty$

V.A.: $x = -\frac{1}{2}$

~ # 8
§ 2.7

16. The **slope** of the tangent line to the curve $y = \dfrac{3x+1}{x+3}$ at the point $(1, 1)$ is equal to

(a) $\dfrac{1}{2}$ (correct)

(b) 2

(c) 3

(d) $\dfrac{1}{3}$

(e) 1

$$m = \lim_{x \to 1} \frac{f(x) - f(1)}{x-1}$$

$$= \lim_{x \to 1} \frac{\frac{3x+1}{x+3} - 1}{x-1}$$

$$= \lim_{x \to 1} \frac{\frac{3x+1-(x+3)}{x+3}}{x-1}$$

$$= \lim_{x \to 1} \frac{2x-2}{(x+3)(x-1)}$$

$$= \lim_{x \to 1} \frac{2(x-1)}{(x+3)(x-1)}$$

$$= \lim_{x \to 1} \frac{2}{x+3} = \frac{2}{1+3} = \frac{2}{4} = \frac{1}{2}$$

~ #53 §2.5

17. Which of the following conditions will **guarantee** that there is a root of the equation

$$4x^3 - 6x^2 + ax + b = 0$$

between -1 and 1?

(a) $a = -6$, $b = 6$
(b) $a = -1$, $b = 1$
(c) $a = 0$, $b = 1$
(d) $a = -5$, $b = 8$
(e) $a = 0$, $b = -1$

$f(x) = 4x^3 - 6x^2 - 6x + 6$ diff. sign
$f(-1) = -4 - 6 + 6 + 6 = 2 > 0$ ✓
$f(1) = 4 - 6 - 6 + 6 = -2 < 0$

(correct)

$f(x) = 4x^3 - 6x^2 - x + 1$
$f(-1) = -4 - 6 + 1 + 1 = -8 < 0$ same sign X
$f(1) = 4 - 6 - 1 + 1 = -2 < 0$

$f(x) = 4x^3 - 6x^2 + 1$
$f(-1) = -4 - 6 + 1 = -9 < 0$ same sign X
$f(1) = 4 - 6 + 1 = -1 < 0$

$f(x) = 4x^3 - 6x^2 - 5x + 8$ same sign X
$f(-1) = -4 - 6 + 5 + 8 = 3 > 0$
$f(1) = 4 - 6 - 5 + 8 = 1 > 0$

$f(x) = 4x^3 - 6x^2 - 1$
$f(-1) = -4 - 6 - 1 = -11 < 0$
$f(1) = 4 - 6 - 1 = -3 < 0$ X
same sign

~ #65 §2.3

18. If a and L are real numbers and

$$\lim_{x \to -3} \frac{x^2 - ax + a - 13}{x^2 + x - 6} = L,$$

then $L =$

(a) $\frac{7}{5}$
(b) $\frac{13}{6}$
(c) $-\frac{13}{6}$
(d) 1
(e) 0

$$\frac{9 + 3a + a - 13}{9 - 3 - 9} = \frac{4a - 4}{0}$$

For the limit to exist, we must have $4a - 4 = 0 \Rightarrow a = 1$

So $\lim_{x \to -3} \frac{x^2 - x - 12}{x^2 + x - 6}$

$= \lim_{x \to -3} \frac{(x+3)(x-4)}{(x+3)(x-2)}$

(correct)

$= \lim_{x \to -3} \frac{x - 4}{x - 2} = \frac{-3-4}{-3-2} = \frac{-7}{-5}$

$= \frac{7}{5}$