~ #13
§ 3.2

1. If $f(t) = \dfrac{t^2+2}{3t^2+4}$, then $f'(t) =$

$$= \frac{(3t^2+4)\cdot(2t) - (t^2+2)\cdot(6t)}{(3t^2+4)^2}$$

$$= \frac{6t^3+8t-6t^3-12t}{(3t^2+4)^2}$$

$$= \frac{-4t}{(3t^2+4)^2}$$

(a) $\dfrac{-4t}{(3t^2+4)^2}$ (correct)

(b) $\dfrac{4t^3-4t}{(3t^2+4)^2}$

(c) $\dfrac{t^3+4t}{(3t^2+4)^2}$

(d) $\dfrac{3t}{(3t^2+4)^2}$

(e) $\dfrac{2t}{(3t^2+4)^2}$

~ #26
§ 3.1

2. If $y = x^2 + \dfrac{1}{x^2}$, then $x^3\dfrac{dy}{dx} - 2x^2y + 6 =$

$$= x^3\left(2x - \frac{2}{x^3}\right) - 2x^2\left(x^2+\frac{1}{x^2}\right) + 6$$

$$= 2x^4 - 2 - 2x^4 - 2 + 6$$

$$= -4 + 6$$

$$= 2$$

(a) 2 (correct)

(b) $2x^4$

(c) $x^3 + \dfrac{1}{x}$

(d) 0

(e) $x - \dfrac{1}{x^3}$

~ #49
§ 3.4

3. If $y = \sqrt{1 + \cot^2 x}$, then $y'\left(\frac{\pi}{4}\right) =$

$$y'(x) = \frac{1}{2\sqrt{1+\cot^2 x}}\left[0 + 2\cot x \cdot (-\csc^2 x)\right]$$

$$= \frac{-\cot x \cdot \csc^2 x}{\sqrt{1+\cot^2 x}}$$

$$y'\left(\frac{\pi}{4}\right) = \frac{-(1)\cdot(\sqrt{2})^2}{\sqrt{1+(1)^2}} = \frac{-2}{\sqrt{2}} = -\sqrt{2}$$

(a) $-\sqrt{2}$ (correct)

(b) $\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$

(d) $\frac{1}{2}$

(e) $-\frac{1}{2}$

#4
§ 3.9

4. The length of a rectangle is increasing at a rate of $8\,cm/min$ and its width is increasing at a rate of $3\,cm/min$. How fast is **the area** of the rectangle increasing when its length is $20\,cm$ and its width is $10\,cm$.

• $\frac{dL}{dt} = 8$, $\frac{dW}{dt} = 3$; $\frac{dA}{dt} = ?$ when $L = 20$, $W = 10$

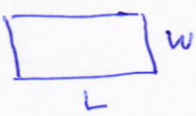
(a) $140\,cm^2/min$ (correct)

(b) $120\,cm^2/min$

(c) $60\,cm^2/min$

(d) $80\,cm^2/min$

(e) $100\,cm^2/min$



• $A = LW$

$$\frac{dA}{dt} = L\frac{dW}{dt} + W\frac{dL}{dt}$$

$$= (20)(3) + (10)(8)$$

$$= 60 + 80$$

$$= 140$$

~ # 37
§ 3.5

5. If y is defined implicity as a function of x by the equation $x + y = \sin y$, then the value of y'' at $(-\pi, \pi)$ is equal to

(a) 0 (correct)

(b) 1

(c) -1

(d) $\frac{1}{2}$

(e) $-\frac{1}{2}$

$x + y = \sin y$

$\Rightarrow 1 + y' = \cos y \cdot y'$

$\Rightarrow y'' = \cos y \cdot y'' + y' \cdot (-\sin y)$

$\Rightarrow y''\Big|_{(-\pi,\pi)} = \cos\pi \cdot y''\Big|_{(-\pi,\pi)} + \underbrace{y'\Big|_{(-\pi,\pi)} \cdot (-\sin\pi)}_{=0}$

$\Rightarrow y''\Big|_{(-\pi,\pi)} = -y''\Big|_{(-\pi,\pi)}$

$\Rightarrow 2y''\Big|_{(-\pi,\pi)} = 0 \Rightarrow y''\Big|_{(-\pi,\pi)} = 0.$

~ #50
§ 3.3

6. $\lim\limits_{x \to -3} \dfrac{\sin(x+3)}{x^2 + 8x + 15} = \lim\limits_{x \to -3} \dfrac{\sin(x+3)}{(x+3)\cdot(x+5)}$

(a) $\frac{1}{2}$ (correct)

(b) 0

(c) 1

(d) 2

(e) ∞

$= \lim\limits_{x \to -3} \dfrac{\sin(x+3)}{x+3} \cdot \dfrac{1}{x+5}$

$= 1 \cdot \dfrac{1}{-3+5}$

$= \dfrac{1}{2}$

Let $w = x+3$

$\lim\limits_{w \to 0} \dfrac{\sin w}{w} = 1$

~#133
§3.3

7. The function $f(x) = x - 2\cos x$, on $[0, 2\pi]$, has a **horizontal tangent** line at $x =$

$f'(x) = 1 + 2\sin x = 0 \Rightarrow \sin x = -\frac{1}{2}$

$\Rightarrow x = \frac{7\pi}{6}, \frac{11\pi}{6} \in [0, 2\pi]$.

(a) $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$ (correct)

(b) $\frac{\pi}{2}$

(c) π

(d) $\frac{\pi}{2}$ and $\frac{3\pi}{2}$

(e) $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

Definition (§2.8)
+ ~#13 §3.4

8. If $f(x) = xe^{x^2-x^3}$, then $\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} =$ $f'(x)$

$= x \cdot e^{x^2-x^3}(2x - 3x^2) + e^{x^2-x^3} \cdot 1$

$= e^{x^2-x^3}(2x^2 - 3x^3 + 1)$

$= (1 + 2x^2 - 3x^3)\, e^{x^2-x^3}$.

(a) $(1 + 2x^2 - 3x^3)\, e^{x^2-x^3}$ (correct)

(b) $(2x^2 - 3x^3)\, e^{x^2-x^3}$

(c) $(2x - 3x^2)\, e^{x^2-x^3}$

(d) $(1 + 2x - 3x^2)\, e^{x^2-x^3}$

(e) $(1 + x - x^2)\, e^{x^2-x^3}$

~ #1(i)
§3.7

9. The position function of a particle moving in a straight line is given by

$$s(t) = t^3 - 12t^2 + 10,\ t \geq 0$$

$v(t) = 3t^2 - 24t = 3t(t-8)$
$a(t) = 6t - 24 = 6(t-4)$

The particle is **slowing down** when

	0 to 4	4 to 8	8 and up
$v(t)$	$-$	$-$	$+$
$a(t)$	$-$	$+$	$+$

(a) $4 < t < 8$ (correct)

(b) $0 < t < 4$ and $8 < t < \infty$

(c) $t > 4$

(d) $t > 0$

(e) $0 < t < 8$

$v(t)$ & $a(t)$ have different signs $\Rightarrow$ slowing down

~ #47
§3.3

10. $\displaystyle\lim_{x \to 0} \frac{2\sin x - \sin(2x)}{3x^3} = \lim_{x\to 0} \frac{2\sin x}{3x} \cdot \frac{1-\cos x}{x^2} \cdot \frac{1+\cos x}{1+\cos x}$

$\displaystyle = \lim_{x\to 0} \frac{2}{3} \cdot \frac{\sin x}{x} \cdot \frac{\sin^2 x}{x^2} \cdot \frac{1}{1+\cos x}$

$\displaystyle = \frac{2}{3} \cdot 1 \cdot (1)^2 \cdot \frac{1}{2}$

$\displaystyle = \frac{1}{3}$

(a) $\frac{1}{3}$ (correct)

(b) 0

(c) does not exist

(d) $\frac{2}{3}$

(e) $-\frac{4}{3}$

~#71
§3.1

11. If $f(x) = x|x-4|$, then $f'_{-}(4) =$ **[the left-hand derivative]**

$$f(x) = \begin{cases} x(x-4) & x \geq 4 \\ -x(x-4) & x < 4 \end{cases}$$

$$= \begin{cases} x^2-4x & x \geq 4 \\ -x^2+4x & x < 4 \end{cases}$$

(a) -4 (correct)

(b) 4

(c) Does not exist

(d) 0

(e) -8

$$f'_{-}(4) = \frac{d}{dx}(-x^2+4x)\Big|_{x=4}$$

$$= -2x+4\Big|_{x=4} = -8+4 = -4$$

~#74
§3.4

12. If $f(1) = 2$, $f'(1) = 4$; $f(2) = 3$, $f'(2) = 5$; and

$$F(x) = f(x\,f(x^2)),$$

then $F'(1) =$

By the chain rule,

$$F'(x) = f'(xf(x^2)) \cdot [x \cdot f'(x^2) \cdot 2x + f(x^2) \cdot 1]$$

$$F'(1) = f'(1f(1)) \cdot [1 \cdot f'(1) \cdot 2 + f(1)]$$

$$= f'(2)\ [2f'(1) + f(1)]$$

$$= 5\ [2(4) + 2]$$

$$= 5\ (10)$$

$$= 50$$

(a) 50 (correct)

(b) 30

(c) 40

(d) 20

(e) 10

~ #45
§ 3.6

13. Which of the following is an equation of a **horizontal tangent line** to the curve $y = x^{\ln x}$.

$\ln y = \ln x \cdot \ln x = (\ln x)^2$

(a) $y = 1$ (correct)

$\frac{1}{y} \cdot y' = 2(\ln x) \cdot \frac{1}{x}$

(b) $y = 2$

$y' = 2y \cdot \frac{\ln x}{x} = 0 \Rightarrow \ln x = 0 \quad , \quad y \neq 0$

(c) $y = 0$

(d) $x = 2$

$\Rightarrow x = 1$

$\Rightarrow$ point $(1,1)$

(e) $y = \frac{1}{2}$

Equation : $y = 1$

Example 4 §3.3
+
chain rule

14. If $f(t) = \dfrac{\cos(2022t)}{2022}$, then $f^{(2022)}(t) =$

$2022 = 4(505) + 2 = 2020 + 2$

(a) $-2022^{2021}\cos(2022t)$ (correct)

(b) $2022^{2021}\cos(2022t)$

(c) $2022^{2021}\sin(2022t)$

(d) $-2022^{2021}\sin(2022t)$

(e) $2022^{2022}\cos(2022t)$

$f^{(2020)}(t) = \frac{1}{2022} \cdot 2022^{2020} \cos(2022t)$

$f^{(2021)}(t) = \frac{2022^{2020}}{\cancel{2022}} \cdot -\sin(2022t) \cdot \cancel{2022}$

$f^{(2022)}(t) = 2022^{2020} \cdot -\cos(2022t) \cdot 2022$

$= -2022^{2021} \cdot \cos(2022t)$

~ #67
p. 268

15. The **slope** of the tangent line to the curve

$$y = (1+\sqrt{x})(2+\sqrt[3]{x})(3-\sqrt[4]{x})$$

at $x = 1$ is equal to

$\ln y = \ln(1+\sqrt{x}) + \ln(2+\sqrt[3]{x}) + \ln(3-\sqrt[4]{x})$

$\frac{1}{y}\cdot y' = \frac{1}{1+\sqrt{x}}\left(\frac{1}{2\sqrt{x}}\right) + \frac{1}{2+\sqrt[3]{x}}\left(\frac{1}{3}x^{-2/3}\right) + \frac{1}{3-\sqrt[4]{x}}\left(-\frac{1}{4}x^{-3/4}\right)$

(a) $\frac{17}{6}$ (correct)

(b) $\frac{4}{3}$

(c) $\frac{5}{9}$

(d) $\frac{7}{8}$

(e) $\frac{12}{5}$

at $x=1$

$\frac{y'(1)}{2\cdot 3\cdot 2} = \frac{1}{2}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{3} + \frac{1}{2}\left(-\frac{1}{4}\right)$

$\frac{y'(1)}{12} = \frac{1}{4} + \frac{1}{9} - \frac{1}{8}$

$y'(1) = 3 + \frac{4}{3} - \frac{3}{2} = \frac{18+8-9}{6} = \frac{17}{6}$

~ #60
§3.1

16. If (a, b) and (c, d) are the points on the curve

$$y = 2x^3 + 3x^2 - 12x + 1$$

$\Rightarrow m = 60$

at which the tangent line is **parallel** to the line $y = 60x + 7$, then $a^2 + c^2 =$

$y' = 60$

$6x^2 + 6x - 12 = 60$

$x^2 + x - 2 = 10$

$x^2 + x - 12 = 0$

$(x+4)(x-3) = 0$

$\Rightarrow x = -4, \; x = 3$

$a \qquad c$

(a) 25 (correct)

(b) 16

(c) 36

(d) 30

(e) 49

$a^2 + c^2 = 16 + 9 = 25$

~ #12 §3.3 + #60 §3.5

17. If $f(x) = \tan^{-1}\left(\dfrac{\cos x}{1-\sin x}\right)$, then $f'(x) =$

$$= \frac{1}{1+\left[\frac{\cos x}{1-\sin x}\right]^2} \cdot \frac{(1-\sin x)(-\sin x) - \cos x(-\cos x)}{(1-\sin x)^2}$$

(a) $\dfrac{1}{2}$ (correct)

(b) 1

(c) $\dfrac{1}{1-\sin x}$

(d) $\dfrac{2}{1+\cos^2 x}$

(e) 2

$$= \frac{1}{(1-\sin x)^2+\cos^2 x} \cdot (-\sin x + \underbrace{\sin^2 x + \cos^2 x}_{1})$$

$$= \frac{1-\sin x}{1-2\sin x+\underbrace{\sin^2 x+\cos^2 x}_{=1}}$$

$$= \frac{1-\sin x}{2-2\sin x} = \frac{1-\sin x}{2(1-\sin x)} = \frac{1}{2}$$

~ #78 §3.5

18. If $f(x) = x + 2e^x$, then $(f^{-1})'(2) = \dfrac{1}{f'(f^{-1}(2))}$

(a) $\dfrac{1}{3}$ (correct)

(b) $-\dfrac{1}{2}$

(c) $\dfrac{1}{2+2e^2}$

(d) 1

(e) $\dfrac{1}{4}$

• $f^{-1}(2) = a$

$\Rightarrow f(a) = 2 \Rightarrow a + 2e^a = 2$

$\Rightarrow a = 0$, by inspection

$$= \frac{1}{f'(0)}$$

, $f'(x) = 1 + 2e^x$

$f'(0) = 1 + 2e^0 = 1+2$

$= 3$

$$= \frac{1}{3}$$