King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 101

Final Exam

212

May 24, 2022

Net Time Allowed: 170 Minutes

MASTER VERSION

#33 §2.6 1. $\lim_{x \to -\infty} (x^2 + 2x^7) =$

$$\lim_{x \to -\infty} x^7 \left(\frac{1}{x^5} + 2\right)$$
$$= -\infty \, (0 + 2)$$
$$= -\infty$$

(a) $-\infty$ (correct)

(b) ∞

(c) 0

(d) 2

(e) -2

#33 §3.1 2. An equation of the **tangent line** to the curve

$$y = 2x^3 - x^2 + 2$$

at the point $(1, 3)$ is

$y' = 6x^2 - 2x$

slope $= y'|_{x=1} = 6(1)^2 - 2(1)$
$= 6 - 2 = 4$

Eq :
$y - 3 = 4(x - 1)$
$y - 3 = 4x - 4$
$y = 4x - 1$

(a) $y = 4x - 1$ (correct)

(b) $y = -4x + 7$

(c) $y = 2x + 1$

(d) $y = 3x$

(e) $y = -3x + 6$

Product rule & the Chain rule

3. If $y = \tan x \cdot \sec^2 x$, then $y' =$ $\tan x \cdot 2\sec x \cdot \sec x \tan x + \sec^2 x \cdot \sec^2 x$

$= 2\tan^2 x \cdot \sec^2 x + \sec^4 x$

$= (2\tan^2 x + \sec^2 x)\sec^2 x$

(a) $(2\tan^2 x + \sec^2 x)\sec^2 x$ (correct)

(b) $(\tan^2 x + 2\sec^2 x)\sec^2 x$

(c) $(\tan^2 x + \sec^2 x)\sec^2 x$

(d) $(2 + \tan^2 x)\sec^2 x$

(e) $\sec^4 x$

The chain Rule

4. If $y = u + \dfrac{1}{u}$ and $u = x - \dfrac{1}{x}$, then the value of $\dfrac{dy}{dx}$ at $x = 2$ is

by the Chain Rule

(a) $\dfrac{25}{36}$ (correct)

(b) $\dfrac{5}{9}$

(c) $\dfrac{25}{4}$

(d) $\dfrac{1}{36}$

(e) $\dfrac{5}{18}$

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$= \left(1 - \frac{1}{u^2}\right) \cdot \left(1 + \frac{1}{x^2}\right)$$

$$x = 2 \Rightarrow u = 2 - \frac{1}{2} = \frac{3}{2}$$

$$\left.\frac{dy}{dx}\right|_{x=2} = \left(1 - \frac{4}{9}\right) \cdot \left(1 + \frac{1}{4}\right)$$

$$= \frac{5}{9} \cdot \frac{5}{4} = \frac{25}{36}$$

~ #25 Review ch 3

5. If y is defined **implicitly** as differentiable function of x by the equation

$$xy = \sin(xy),$$

then $\frac{dy}{dx} =$ (Assume $\cos(xy) \neq 1$)

(a) $-\frac{y}{x}$ (correct)

(b) $\frac{x}{1-\cos(xy)}$

(c) $-\frac{1}{x}$

(d) $\frac{1}{y}$

(e) $\frac{y}{x}$

$$xy' + y \cdot 1 = \cos(xy) \cdot [xy' + y \cdot 1]$$

$$xy' + y = xy' \cos(xy) + y\cos(xy)$$

$$xy' - xy'\cos(xy) = y\cos(xy) - y$$

$$xy'(1-\cos(xy)) = y(\cos(xy)-1)$$

$$y' = \frac{y(\cos(xy)-1)}{x(1-\cos(xy))}$$

$$y' = -\frac{y}{x}$$

Note:
$\cos(xy) \neq 1$
$\Rightarrow xy \neq 0$
$\Rightarrow x \neq 0$ & $y \neq 0$

~ #38 Review Ch 3

6. If $f(x) = \tan^{-1}(\tanh(\tan x))$, then $f'(0) =$

(a) 1 (correct)

(b) $\frac{1}{2}$

(c) 0

(d) -2

(e) 2

$$f'(x) = \frac{1}{1+[\tanh(\tan x)]^2} \cdot [\operatorname{sech}^2(\tan x) \cdot \sec^2 x]$$

$$f'(0) = \frac{1}{1+[\tanh(0)]^2} \cdot [\operatorname{sech}^2(0) \cdot 1]$$

$$= \frac{1}{1+0} \cdot 1$$

$$= 1$$

#7 §4.8 7. If **Newton's Method** is used to approximate the root of the equation

$$\frac{2}{x} - x^2 + 1 = 0$$

with first approximation $x_1 = 2$, then the second approximation $x_2 =$

$f(x) = \frac{2}{x} - x^2 + 1$; $f'(x) = -\frac{2}{x^2} - 2x$

(a) $\frac{14}{9}$ (correct)

(b) $\frac{8}{3}$

(c) $\frac{20}{9}$

(d) $\frac{17}{3}$

(e) $\frac{22}{3}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$; $f(2) = \frac{2}{2} - 4 + 1 = -2$

$= 2 - \frac{f(2)}{f'(2)}$; $f'(2) = -\frac{2}{4} - 4 = -\frac{1}{2} - 4 = -\frac{9}{2}$

$= 2 - \frac{-2}{-\frac{9}{2}}$

$= 2 - \frac{4}{9} = \frac{18-4}{9} = \frac{14}{9}$

#41 §4.1 8. The set of all **critical numbers** of

$$f(\theta) = 2\cos\theta + \sin^2\theta$$

is

$f'(\theta) = -2\sin\theta + 2\sin\theta \cdot \cos\theta$

$= 2\sin\theta\,(-1 + \cos\theta)$

. f' exists at all θ

. $f'(\theta) = 0 \Rightarrow \sin\theta = 0$ or $\cos\theta = 1$

(a) $\{n\pi : n \text{ is integer}\}$ (correct)

(b) $\{2n\pi : n \text{ is integer}\}$

(c) $\{\frac{(2n+1)\pi}{2} : n \text{ is integer}\}$

(d) $\{(2n+1)\pi : n \text{ is integer}\}$

(e) $\{\frac{n\pi}{2} : n \text{ is integer}\}$

$\Rightarrow \theta = n\pi$ or $\theta = 2n\pi$

$\Rightarrow \theta = n\pi$, n is an integer

~ #34
§ 3.10

9. The radius of a circular disk is given as $12\,cm$ with a maximum error in measurement of $0.1\,cm$. Using **differentials**, the maximum error in the calculated **area** of the disk is approximately equal to

$r = 12$, $\Delta r = 0.1$

(a) $2.4\,\pi\,cm^2$ (correct)

(b) $1.2\,\pi\,cm^2$

(c) $144\,\pi\,cm^2$

(d) $0.1\,\pi\,cm^2$

(e) $2.1\,\pi\,cm^2$

$A = \pi r^2$

$dA = 2\pi r\,dr$

$\Delta A \approx dA$

$\Delta A \approx 2\pi r\,dr = 2\pi(12)(0.1)$

$\Delta A \approx 2.4\pi$

~ Example 9 (b) & #31
§ 2.5

10. Where is the function $f(x) = \sqrt{2 - \ln(x-1)}$ **continuous** ?

It is continuous in its domain:

(a) $(1, e^2 + 1]$ (correct)

(b) $(1, \infty)$

(c) $[e^2, \infty)$

(d) $(2, \infty)$

(e) $(1, (e+1)^2]$

$2 - \ln(x-1) \geq 0$ & $x - 1 > 0$

$\Rightarrow \ln(x-1) \leq 2$ & $x > 1$

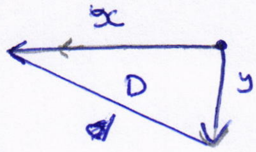
$\Rightarrow x - 1 \leq e^2$ & $x > 1$

$\Rightarrow x \leq e^2 + 1$ & $x > 1$

Domain : $(1, e^2+1]$.

~ #17
§3.9

11. Two cars start moving from the same point. One travels south at $30\,km/h$ and the other travels west at $60\,km/h$. At what rate is the **distance** between the cars increasing two hours later?



(a) $30\sqrt{5}\,km/h$

(b) $45\sqrt{5}\,km/h$

(c) $50\sqrt{5}\,km/h$

(d) $35\sqrt{5}\,km/h$

(e) $40\sqrt{5}\,km/h$

$D^2 = x^2 + y^2$

$\cancel{2}D\frac{dD}{dt} = \cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}$

$\frac{dD}{dt} = \frac{120 \cdot 60 + 60 \cdot 30}{60\sqrt{5}}$

$= \frac{150}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$= 30\sqrt{5}$

$\frac{dy}{dt} = 30$
(correct)

$\frac{dx}{dt} = 60$

$\left.\frac{dD}{dt}\right|_{t=2} = ?$

two hours later,

$x = 60 \cdot 2 = 120$

$y = 30 \cdot 2 = 60$

$D = \sqrt{x^2+y^2}$

$= \sqrt{(120)^2+(60)^2}$

$= 60\sqrt{5}$

~ #6, #12
§4.9

12. The **most general antiderivative** of $f(x) = (x - \sqrt{x})^2$ is

$= x^2 - 2x\sqrt{x} + x$

$= x^2 - 2x^{3/2} + x$

(a) $F(x) = \frac{1}{3}x^3 - \frac{4}{5}x^{5/2} + \frac{1}{2}x^2 + C$

(b) $F(x) = \frac{1}{3}x^3 - \frac{2}{5}x^{5/2} + \frac{1}{2}x^2 + C$

(c) $F(x) = x^3 + 2x^{3/2} + x^2 + C$

(d) $F(x) = 3x^3 + \frac{4}{5}x^{5/2} + 2x^2 + C$

(e) $F(x) = \frac{1}{3}(x - \sqrt{x})^3 + C$

(correct)

$\Rightarrow F(x) = \frac{x^{2+1}}{2+1} - 2\frac{x^{3/2+1}}{\frac{3}{2}+1} + \frac{x^{1+1}}{1+1} + C$

$= \frac{1}{3}x^3 - 2 \cdot \frac{2}{5}x^{5/2} + \frac{1}{2}x^2 + C$

$= \frac{1}{3}x^3 - \frac{4}{5}x^{5/2} + \frac{1}{2}x^2 + C$

~ Example 6
§ 4.5

13. The function $f(x) = \dfrac{x^2}{x+1}$ is **increasing** on the interval(s)

(a) $(-\infty, -2)$ and $(0, \infty)$

(b) $(-\infty, -1)$ and $(-1, \infty)$

(c) $(-\infty, 0)$

(d) $(-1, \infty)$

(e) $(-2, -1)$ and $(-1, 0)$

$$f'(x) = \frac{(x+1)(2x) - x^2(1)}{(x+1)^2} = \frac{x^2+2x}{(x+1)^2} = \frac{x(x+2)}{(x+1)^2}$$

(correct)

$f'(x) = 0 \Rightarrow x = 0, -2$

$f'(x)$ DNE $\Rightarrow x = -1$

$f'(x)$: $+$ | -2 | $-$ | -1 | $-$ | 0 | $+$

f: $\nearrow$ | -2 | $\searrow$ | -1 | $\searrow$ | 0 | $\nearrow$

increasing on $(-\infty, -2)$ & $(0, \infty)$.

~ #3 & #10
§ 3.11

14. $[\sinh(\ln 3) - \cosh(\ln 3)]^3 =$

(a) $-\dfrac{1}{27}$

(b) $\dfrac{1}{27}$

(c) -27

(d) $-(\ln 3)^3$

(e) $(\ln 3)^3$

- $\sinh(\ln 3) = \dfrac{e^{\ln 3} - e^{-\ln 3}}{2} = \dfrac{1}{2}\left(3 - \dfrac{1}{3}\right) = \dfrac{1}{2} \cdot \dfrac{8}{3} = \dfrac{4}{3}$
- $\cosh(\ln 3) = \dfrac{e^{\ln 3} + e^{-\ln 3}}{2} = \dfrac{1}{2}\left(3 + \dfrac{1}{3}\right) = \dfrac{1}{2} \cdot \dfrac{10}{3} = \dfrac{5}{3}$ (correct)
- $[\sinh(\ln 3) - \cosh(\ln 3)]^3 = \left(\dfrac{4}{3} - \dfrac{5}{3}\right)^3 = \left(-\dfrac{1}{3}\right)^3 = -\dfrac{1}{27}$

<u>OR</u>: $\cosh x - \sinh x = e^{-x}$

$\Rightarrow \sinh x - \cosh x = -e^{-x}$

$\Rightarrow \sinh(\ln 3) - \cosh(\ln 3) = -e^{-\ln 3} = -\dfrac{1}{3}$

$\Rightarrow [\sinh(\ln 3) - \cosh(\ln 3)]^3 = \left(-\dfrac{1}{3}\right)^3 = -\dfrac{1}{27}$

~ #47 §3.6

15. If $y = (\cos x)^{x^2}$, then $\dfrac{dy}{dx} =$

$\ln y = x^2 \cdot \ln(\cos x)$

$\frac{1}{y} \cdot y' = x^2 \cdot \frac{-\sin x}{\cos x} + \ln(\cos x) \cdot 2x$

(a) $-x \cdot (\cos x)^{x^2}(x \tan x - 2 \ln \cos x)$ (correct)

(b) $-x \cdot (\cos x)^{x^2}(x \tan x + 2 \ln \cos x)$

(c) $(\cos x)^{x^2}(-2x \tan x + x^2 \ln \cos x)$

(d) $-x^2 \cdot (\cos x)^{x^2}(\tan x - \ln \cos x)$

(e) $x^2 \cdot (\cos x)^{x^2-1} \cdot (-\sin x) \cdot (2x)$

$y' = y(-x^2 \tan x + 2x \ln(\cos x))$

$= -x \cdot (\cos x)^{x^2}(x \tan x - 2 \ln(\cos x))$

16. Which one of the following statements is **TRUE**? Assume f and g are differentiable functions.

Rolle's Theorem : $[-1, 1]$

T (a) If $f(-1) = f(1)$, then there is a number c such that $|c| < 1$ and $f'(c) = 0$ (correct)

$f(x) = x^3$; F (b) If $f'(c) = 0$, then f has a local maximum or minimum at c

$f(x) = x$, $g(x) = x + 1$ F (c) If $f'(x) = g'(x)$ for all x, then $f(x) = g(x)$

F (d) If $f''(2) = 0$, then $(2, f(2))$ is an inflection point of the curve $y = f(x)$

$f(x) = (x-2)^4$

(e) f has at least one critical number

F : $f(x) = x$ has no critical numbers

$\sim \# 20$
§3.6

17. If $y = \ln \sqrt{\dfrac{1-\cos x}{1+\cos x}}$, then $\dfrac{dy}{dx} =$

$$y = \frac{1}{2}\left[\ln(1-\cos x) - \ln(1+\cos x)\right]$$

(a) $\csc x$ (correct)

(b) $\dfrac{\sin x}{1+\cos x}$

(c) $\sec x$

(d) 1

(e) $\tan x \sec x$

$$\frac{dy}{dx} = \frac{1}{2}\left[\frac{\sin x}{1-\cos x} - \frac{-\sin x}{1+\cos x}\right]$$

$$= \frac{1}{2}\,\frac{\sin x + \cancel{\sin x \cos x} + \sin x - \cancel{\sin x \cos x}}{1-\cos^2 x}$$

$$= \frac{1}{2}\,\frac{2\sin x}{\sin^2 x} = \frac{1}{\sin x} = \csc x$$

$\sim \# 49$
§4.1

18. If M and m are respectively the **maximum** and **minimum** values of

$$f(x) = 2x^3 - 3x^2 - 12x$$

on $[-2, 3]$, then $M + m =$

$$f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2)$$
$$= 6(x-2)(x+1)$$
$$f'(x) = 0 \Rightarrow x = 2, -1$$

(a) -13 (correct)

(b) 27

(c) 1

(d) -20

(e) 3

- $f(-1) = -2 - 3 + 12 = 7 \longrightarrow M$
- $f(2) = 16 - 12 - 24 = -20 \longrightarrow m$
- $f(-2) = -16 - 12 + 24 = -4$
- $f(3) = 54 - 27 - 36 = -9$

$$M + m = 7 - 20 = -13$$

~ #41
§4.9

19. If $f''(x) = 3x^2 + 2\cos x$, $f(0) = 2$, $f'(0) = 1$, then $f(2) =$

(a) $10 - 2\cos(2)$ (correct)

(b) $8 - 2\cos(2)$

(c) $5 - \cos(2)$

(d) $6 + 2\cos(2)$

(e) $8 + 2\cos(2)$

$\cdot\ f'(x) = x^3 + 2\sin x + C$

$f'(0) = 0 + 2(0) + C$

$\boxed{1 = C}$

$\cdot\ f'(x) = x^3 + 2\sin x + 1$

$f(x) = \frac{x^4}{4} - 2\cos x + x + D$

$f(0) = 0 - 2(1) + 0 + D$

$2 = -2 + D \Rightarrow \boxed{D = 4}$

$f(x) = \frac{x^4}{4} - 2\cos x + x + 4$

$f(2) = 4 - 2\cos 2 + 2 + 4 = 10 - 2\cos 2$

#20 §4.4
~ #18 §2.5
#71

20. If

$$f(x) = \begin{cases} \dfrac{\ln x}{\sin(\pi x)}, & x \neq 1 \\ -2k, & x = 1 \end{cases}$$

is **continuous** at $x = 1$, then $k =$

(a) $\frac{1}{2\pi}$ (correct)

(b) $\frac{2}{\pi}$

(c) π

(d) $-\frac{2}{\pi}$

(e) $-\frac{1}{\pi}$

for continuity at $x=1$, we must have

$\lim_{x\to 1} f(x) = f(1)$

$\cdot\ \lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{\ln x}{\sin(\pi x)}$, $\frac{0}{0}$

$\underline{\underline{L'R}}\ \lim_{x\to 1} \frac{\frac{1}{x}}{\cos(\pi x)\cdot\pi}$

$= \frac{1}{-\pi}$

$-\frac{1}{\pi} = -2K$

$\Rightarrow K = \frac{1}{2\pi}$

$\sim \#24$
§3.10

21. Using a suitable **linear approximation**, we get $\frac{1}{5.01} \approx$

Take $f(x) = \frac{1}{x}$, $a = 5$ $f'(x) = -\frac{1}{x^2}$

(a) 0.1996 (correct)

(b) 0.1997

(c) 0.1999

(d) 0.1936

(e) 0.1937

$L(x) = f(a) + f'(a)(x-a)$
$= f(5) + f'(5)(x-5)$
$= \frac{1}{5} - \frac{1}{25}(x-5)$

$f(x) \approx L(x)$, when x is near 5

$f(x) \approx \frac{1}{5} - \frac{1}{25}(x-5)$

$f(5.01) \approx \frac{1}{5} - \frac{1}{25}(5.01-5) = \frac{1}{5} - \frac{1}{2500} = \frac{1}{5} - \frac{4}{10000}$

$\frac{1}{5.01} \approx \frac{2000}{10000} - \frac{4}{10000} = \frac{1996}{10000} = 0.1996$

$\sim \#66$
§4.5

22. If $y = Ax + B$ is a **slant asymptote** and $x = C$ is a **vertical asymptote** for the curve

$$y = \frac{3x^2 + x + 1}{2x - 1}$$

then $ABC =$

. $2x - 1 = 0 \Rightarrow x = \frac{1}{2} \Rightarrow C = \frac{1}{2}$

$$\begin{array}{r|l} & \frac{3}{2}x + \frac{5}{4} \\ 2x-1 & 3x^2 + x + 1 \\ & 3x^2 - \frac{3}{2}x \\ \hline & \frac{5}{2}x + 1 \\ & \frac{5}{2}x - \frac{5}{4} \\ \hline & \frac{9}{4} \end{array}$$

(a) $\frac{15}{16}$ (correct)

(b) $\frac{15}{8}$

(c) $\frac{3}{4}$

(d) $\frac{5}{8}$

(e) $\frac{5}{2}$

$y = \frac{3}{2}x + \frac{5}{4}$, is slant asymptote

$A = \frac{3}{2}$, $B = \frac{5}{4}$

$ABC = \frac{3}{2} \cdot \frac{5}{4} \cdot \frac{1}{2} = \frac{15}{16}$

21
§ 4.3

23. The **local minimum value** of $f(x) = \sqrt{x} - \sqrt[4]{x}$ is

Domain : $[0, \infty)$

$f'(x) = \frac{1}{2} x^{-1/2} - \frac{1}{4} x^{-3/4}$

$= \frac{1}{4} x^{-3/4} (2 x^{1/4} - 1) = \frac{2\sqrt[4]{x} - 1}{4\sqrt[4]{x^3}}$

(a) $-\frac{1}{4}$ (correct)

(b) $-\frac{1}{2}$

(c) $\frac{3}{2}$

(d) $\frac{1}{2}$

(e) $-\frac{1}{3}$

$f'(x) = 0 \Rightarrow 2\sqrt[4]{x} - 1 \Rightarrow \sqrt[4]{x} = \frac{1}{2} \Rightarrow x = \frac{1}{16}$

$f'(x)$ DNE $\Rightarrow \sqrt[4]{x^3} = 0 \Rightarrow x = 0$

f': 0 [$\quad$ $1/81$: $-$ $\quad$ $\frac{1}{16}$ $\quad$ 1: $+$

f: $\searrow$ $\frac{1}{16}$ $\nearrow$

Local min

$f(\frac{1}{16}) = \frac{1}{4} - \frac{1}{2} = -\frac{1}{4}$

~ Example 5
§ 4.2

24. If $f(0) = 2$ and $f'(x) \leq 4$ for all values of x, then the **largest** possible value for $f(5)$ is **(Hint**: Use the Mean Value Theorem)

Apply MVT to f on $[0, 5]$: Conditions are satisfied.

(a) 22 (correct)

(b) 4

(c) 10

(d) 20

(e) 16

$\frac{f(5) - f(0)}{5 - 0} = f'(c)$, $c \in (0, 5)$

$\Rightarrow \frac{f(5) - 2}{5} = f'(c) \leq 4$

$\Rightarrow f(5) - 2 \leq 20$

$\Rightarrow f(5) \leq 22$

So the largest possible value of $f(5)$ is 22.

Example 8
§ 4.3

25. Which one of the following statements is **TRUE** about the graph of $f(x) = e^{1/x}$.

$f'(x) = -\frac{1}{x^2} e^{1/x}$

$f''(x) = -\frac{1}{x^2} \cdot e^{1/x} \cdot -\frac{1}{x^2} + e^{1/x} \cdot \frac{2}{x^3}$

$= \left(\frac{1}{x^4} + \frac{2}{x^3}\right) e^{1/x}$

$= \frac{(1+2x)\, e^{1/x}}{x^4}$

$f''(x) = 0 \Rightarrow 1+2x = 0 \Rightarrow x = -\frac{1}{2}$

$f''(x)$ DNE $\Rightarrow x = 0 \notin$ domain

f'' − + +

∩ $-1/2$ ∪ 0 ∪

(a) It is concave downward on $\left(-\infty, -\frac{1}{2}\right)$ (correct)

(b) It is concave downward on $(0, \infty)$

(c) It is concave upward on $(-\infty, 0)$

(d) It is concave upward on $(-\infty, \infty)$

(e) It has two inflection points

#62
§ 4.4

26. $\lim_{x\to\infty} x^{(\ln 2)/(1+\ln x)} =$

∞^0, indeterminate

$y = x^{(\ln 2)/(1+\ln x)}$

$\ln y = \frac{\ln 2}{1+\ln x} \cdot \ln x = \ln 2 \frac{\ln x}{1+\ln x}$

$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \ln 2 \cdot \frac{\ln x}{1+\ln x}$; $\frac{\infty}{\infty}$

$\overset{L'H}{=} \lim_{x\to\infty} \ln 2 \cdot \frac{\frac{1}{x}}{0+\frac{1}{x}}$

$= \lim_{x\to\infty} \ln 2 \cdot 1 = \ln 2$

$\Rightarrow \lim_{x\to\infty} y = e^{\ln 2} = 2$

(a) 2 (correct)

(b) 1

(c) ∞

(d) $\ln 2$

(e) e^2

~ #78
§3.4

27. If $f(x) = xe^{-x}$, then $f^{(100)}(x) =$

Differentiate repeatedly & Discover a pattern:

(a) $(x - 100)\, e^{-x}$ (correct)

(b) $(-x + 100)\, e^{-x}$

(c) $(x + 100)\, e^{-x}$

(d) $(-x - 100)\, e^{-x}$

(e) $x^{100}\, e^{-x}$

$f'(x) = -xe^{-x} + e^{-x} = (-x+1)e^{-x}$

$f''(x) = -(-x+1)e^{-x} - e^{-x} = (x-2)e^{-x}$

$f'''(x) = -(x-2)e^{-x} + e^{-x} = (-x+3)e^{-x}$ even

$f^{(4)}(x) = -(-x+3)e^{-x} - e^{-x} = (x-4)e^{-x}$

$\vdots$

$f^{(100)}(x) = (x-100)e^{-x}$

~ #54, 58
§ 4.7

28. What is the **smallest possible area** of the triangle that is cut off from the first quadrant by a line through the point $(2, 4)$?

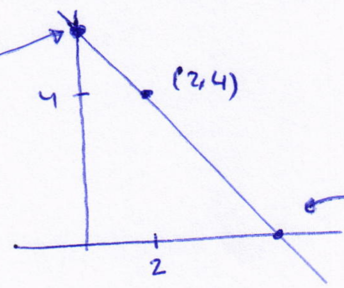
(a) 16 (correct)

(b) 8

(c) 32

(d) 12

(e) 20



• Slope $= m < 0$, Eq: $y - 4 = m(x-2)$

• x-intercept: $y = 0 \Rightarrow x = 2 - \frac{4}{m}$

• y-intercept: $x = 0 \Rightarrow y = 4 - 2m$

• Area of the triangle

$A = \frac{1}{2}\left(2 - \frac{4}{m}\right)(4 - 2m)$

$A(m) = 2\left(1 - \frac{2}{m}\right)(2 - m) = 2\left(4 - m - \frac{4}{m}\right)$

$A'(m) = 2\left(0 - 1 + \frac{4}{m^2}\right) = 0 \Rightarrow m^2 = 4 \Rightarrow m = -2$ (as $m<0$)

$A''(m) = 2(-8m^{-3}) \Rightarrow A''(-2) > 0 \Rightarrow$ L. min at $m = -2$

($\Rightarrow$ abs min as it is the only critical point)

So the smallest possible area is

$A(-2) = 2(4 + 2 + 2) = 16$