King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 101
Major Exam I
213
June 21, 2022

EXAM COVER

Number of versions: 4
Number of questions: 18
Number of Answers: 5

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Net Time Allowed: 120 Minutes

MASTER VERSION

1. Using the definition of the limit for $\lim_{x\to 3}\sqrt{x+1}=2$, the largest number δ, which corresponds to $\epsilon = 0.1$ is

   (a) 0.39 (correct)
   (b) 0.4
   (c) 0.41
   (d) 0.2
   (e) 0.1

2. Let

$$f(x)=\begin{cases} \dfrac{6}{(5-x)(4+2x)} & \text{if} \quad x\le 1 \\ \dfrac{1}{2\sqrt{x}} & \text{if} \quad x>1. \end{cases}$$

   Then the number of points of discontinuity of f is

   (a) 2 (correct)
   (b) 0
   (c) 1
   (d) 3
   (e) 4

3. Let

$$f(x) = \begin{cases} \dfrac{x^2-4}{x+2} & \text{if} \quad x < -2 \\ ax+b & \text{if} \quad -2 \le x \le 2 \\ \dfrac{x^2-4}{x-2} & \text{if} \quad x > 2. \end{cases}$$

If f is continuous everywhere, then $a + b =$

(a) 2 (correct)

(b) -2

(c) 0

(d) 4

(e) -4

4. The height, in feet, of a ball thrown into the air after t seconds is given by $y = 40t - 16t^2$. The average velocity of the ball on the interval $[1, 2]$ is

(a) $-8\, ft/s$ (correct)

(b) $8\, ft/s$

(c) $4\, ft/s$

(d) $-4\, ft/s$

(e) $-10\, ft/s$

5. The function $f(x) = \dfrac{\sqrt{4x^2+3}-x}{x-1}$ has

(a) Two horizontal asymptotes $y = 1$ and $y = -3$ and one vertical asymptote. (correct)

(b) Only one horizontal asymptote $y = 1$ and one vertical asymptote.

(c) Only one horizontal asymptote $y = -3$ and one vertical asymptote.

(d) One vertical asymptote and no horizontal asymptotes.

(e) No horizontal asymptotes and no vertical asymptote.

6. $\lim_{x\to-\infty}(\sqrt{4x^2+4x}+2x) =$

(a) -1 (correct)

(b) 4

(c) -2

(d) 0

(e) DNE

7. $\lim_{x\to 2}([\![2x]\!] + [\![-x]\!]) =$

(a) 1 (correct)
(b) 6
(c) 5
(d) 4
(e) DNE

8. $\lim_{x\to 1}(x^2 - 2x + 1)\cos\dfrac{1}{x-1} =$

(a) 0 (correct)
(b) 1
(c) 1/2
(d) $-\infty$
(e) ∞

9. If $\lim_{x\to 2} \dfrac{10+x-g(x)}{x-2} = 3$, then $\lim_{x\to 2} g(x) =$

(a) 12 (correct)

(b) 10

(c) 3

(d) 0

(e) DNE

10. Using the Intermediate Value Theorem, we conclude that the two curves $y = x^3 - x^2 - 1$ and $y = x^2 - 3$ intersect in the interval

(a) $(-1, 0)$ (correct)

(b) $(-2, -1)$

(c) $(0, 1)$

(d) $(1, 2)$

(e) $(2, 3)$

11. $\lim_{h\to 0} \dfrac{(2+h)^6 - 64}{h} =$

(a) $f'(2)$ where $f(x) = x^6$ (correct)

(b) $f'(2)$ where $f(x) = (x+h)^6$

(c) 64

(d) 0

(e) 32

12. Suppose that f satisifes the equation $f(x+y) = f(x)+f(y)+x^2y+xy^2$ for all real numbers x and y. Suppose also that $\lim_{x\to 0} \dfrac{f(x)}{x} = 1$. Then $f'(x) =$

(a) $1 + x^2$ (correct)

(b) $x + x^2$

(c) x^2

(d) $1 + x + x^2$

(e) $1 + x$

13. If $f(x) = \dfrac{1}{\sqrt{x-1}}$, then $f'(2) =$

(a) $-1/2$ (correct)
(b) $1/2$
(c) 1
(d) 2
(e) -2

14. Consider the function

$$f(x) = \begin{cases} x^2 & \text{if} \quad x \geq 0 \\ x^3 & \text{if} \quad x < 0 \end{cases}$$

Which of the following is true

(a) The domain of $f'(x)$ is $(-\infty, \infty)$ (correct)
(b) The domain of $f'(x)$ is $(-\infty, 0) \cup (0, \infty)$
(c) The domain of $f''(x)$ is $(-\infty, \infty)$
(d) The domain of $f'''(x)$ is $(-\infty, \infty)$
(e) $f(x)$ is discontinuous at 0

15. $\lim_{x\to 1} arc\sin\left(\frac{1-\sqrt{x}}{1-x}\right) =$

(a) $\pi/6$ (correct)
(b) $\pi/3$
(c) $-\pi/6$
(d) $-\pi/3$
(e) $\pi/4$

16. The function $y = \sqrt[3]{x}$ has

(a) a vertical tangent at $x = 0$ (correct)
(b) a horizontal tangent at $x = 0$
(c) a vertical asymptote at $x = 0$
(d) a horizontal asymptote
(e) a removable discontinuity at $x = 0$

17. Consider the function $f(x) = \dfrac{x^2 - 2x + 1}{x^3 - x}$.
Which of the following statements is **FALSE**:

(a) f has a removable discontinuity at $x = -1$ (correct)
(b) f has infinite discontinuity at $x = -1$
(c) f has infinite discontinuity at $x = 0$
(d) f has a removable discontinuity at $x = 1$
(e) f has two vertical asymptotes

18. If the equation of the tangent line to the curve $y = f(x)$ at the point where $x = 2$ is $y = 4x - 5$, then $f(2) + f'(2) =$

(a) 7 (correct)
(b) 6
(c) 5
(d) 8
(e) 3