King Fahd University of Petroleum and Minerals Department of Mathematics Math 101 Major Exam II 213 July 25, 2022 Net Time Allowed: 120 Minutes

## MASTER VERSION

(correct)

## 1. Consider the function

$$f(x) = \begin{cases} x^2 + b & \text{if } x \le 2\\ ax + 3 & \text{if } x \ge 2 \end{cases},$$

If f differentiable everywhere, then a + b =

- (a) 11
- (b) 8
- (c) 15
- (d) 10
- (e) 13

2. If 
$$g(x) = \frac{x}{e^x}$$
, then  $g^{(101)}(0) =$ 

- (a) 101
- (b) 100
- (c) -101
- (d) -100
- (e) 0

3. If 
$$f(x) = \frac{xe^x}{x^2 + e^x}$$
, then  $f'(0) =$ 

- (a) 1(b) e(c) 0
- (d)  $e^{-1}$
- (e)  $e^2$

- 4. The equations of the tangent line(s) to the graph of  $f(x) = x^2 6x + 9$ that pass through the origin (0,0) are:
  - (a) y = 0 and y = -12x
  - (b) y = x and y = -12x
  - (c) y = 0 and y = 12x
  - (d) y = x and y = 12
  - (e) y = 0 and y = x

5. 
$$\lim_{x \to \frac{\pi}{6}} \frac{2\sin x - 1}{x - \frac{\pi}{6}} =$$
(a)  $\sqrt{3}$ 
(b)  $\frac{1}{2}$ 
(c) 0

(e) DNE

6. If 
$$f(x) = \frac{(\tan x) - 1}{\sec x}$$
, then  $f'\left(\frac{\pi}{4}\right) =$ 

(a) 
$$\sqrt{2}$$
  
(b) 1  
(c) 0  
(d)  $-\frac{2}{\sqrt{2}}$   
(e)  $\frac{1}{\sqrt{2}}$ 

7. If F(x) = f(3f(4f(x))), where f(0) = 0 and f'(0) = 2, then F'(0) = 0

- (a) 96
- (b) 48
- (c) 32
- (d) 24
- (e) 192

8. The slope of the line tangent to the curve  $\tan xy = xy^3 + 2y^2 - 8$  at the point (0, 2) is

(a)  $-\frac{3}{4}$  (correct) (b) 2 (c) 0 (d)  $\frac{3}{4}$ (e)  $\frac{4}{3}$ 

MASTER

(correct)

9. If  $5x^2 + 2xy + 2y^2 = 9$ , then y'' at the point (1, 1) is

(a) -3 (correct)

- (b) 16
- (c) 8
- (d) -4
- (e) 4

10. For any 
$$x > 0$$
,  $\lim_{n \to \infty} \left(1 + \frac{x}{n}\right)^n$ 

- (a)  $e^x$
- (b) *e*
- (c)  $e^n$
- (d) 1
- (e) DNE

11. If 
$$y = (\sqrt{x})^x$$
, then  $y'(2) =$ 

(a) 
$$1 + \ln 2$$
 (correct)

(b) 
$$2\sqrt{2}$$

(c) 
$$1$$

(d) 
$$1 + \ln(\sqrt{2})$$
  
(e)  $\frac{1}{2} + \ln(\sqrt{2})$ 

- 12. A particle is moving according to a law of motion  $s(t) = \sin\left(\frac{\pi t}{2}\right)$  where t is measured in seconds and s in meters. Then, the total distance, in meters, traveled by the particle during the time interval  $0 \le t \le 2$  is equal to
  - (a) 2
  - (b) 1
  - (c) = 0
  - (d) 3
  - (e) 4

MASTER

(correct)

13. A spotlight on the ground shines on a wall 10 meters away. A woman 2m tall walks from the spotlight towards the wall at a speed of 3 m/s. When the woman is 4m from the building, the length of her shadow on the wall is decreasing at a rate of

(a) 
$$\frac{5}{3}m/s$$
 (correct)  
(b)  $\frac{30}{8}m/s$   
(c)  $\frac{5}{9}m/s$   
(d)  $\frac{5}{4}m/s$ 

(e) 
$$\frac{10}{9}m/s$$

- 14. If we use linear approximation (or differentials) to estimate  $(1.009)^9$ , then we get  $(1.009)^9 \approx$ 
  - (a) 1.081
  - (b) 18.1
  - (c) 1.81
  - (d) 1.0081
  - (e) 1.00081

15. The curve  $y = x^2 - 2x + \cos(\ln x)$  has a horizontal tangent line at x =

$$(a) \quad 1$$

- (b) 0
- (c) *e* (d) 2
- (u) 2 1
- (e)  $\frac{1}{e}$

16. If 
$$y = x \sin^{-1} x + \sqrt{1 - x^2}$$
, then  $\frac{dy}{dx} =$ 

(a) 
$$\sin^{-1} x$$
 (correct)  
(b)  $x \sin^{-1} x$   
(c)  $\sin^{-1} x + \frac{2x}{\sqrt{1-x^2}}$   
(d)  $\sin^{-1} x - \frac{2x}{\sqrt{1-x^2}}$   
(c)  $2x$ 

(e) 
$$\frac{2x}{\sqrt{1-x^2}}$$

MASTER

MASTER

17. Consider the function  $y = f(x) = 2^{x^2+1} + \log_2 x$ . The rate of change of y with respect to x when x = 1 is

(a) 
$$8 \ln 2 + \frac{1}{\ln 2}$$
 (correct)  
(b)  $64 \ln 2 + \frac{1}{\ln 2}$   
(c)  $1 + 128 \ln 2$   
(d)  $16 + \frac{1}{\ln 2}$   
(e)  $128 + \frac{1}{\ln 2}$ 

18. A linearization L(x) of the function  $f(x) = \sqrt{x} + \sin(x-1)$  at a = 1 is

(a) 
$$L(x) = \frac{3}{2}x - \frac{1}{2}$$
 (correct)  
(b)  $L(x) = \frac{3}{2}x + \frac{1}{2}$ 

(c) 
$$L(x) = \frac{3}{2}x - \frac{3}{2}$$
  
(d)  $L(x) = \frac{3}{2}x + \frac{3}{2}$   
(e)  $L(x) = \frac{1}{2}x - \frac{1}{2}$