1. If the slope of the tangent line to the curve $y = \dfrac{e^x - a}{e^x + a}$ at $x = 0$ is $\dfrac{1}{2}$, then the value of a is equal to

Similar to Q # 67 / page 164

(a) 1 ______________________________ (correct)

(b) -1

(c) $\dfrac{1}{2}$

(d) $\dfrac{-1}{2}$

(e) 0

$$y' = \frac{e^x(e^x+a) - e^x(e^x-a)}{(e^x+a)^2} = \frac{2ae^x}{(e^x+a)^2}$$

$$\therefore \; y'\big|_{x=0} = \frac{1}{2} \Rightarrow \frac{2a}{(1+a)^2} = \frac{1}{2}$$

$$\Rightarrow a^2 + 2a + 1 - 4a = 0$$

$$\Rightarrow a^2 - 2a + 1 = 0$$

$$\Rightarrow (a-1)^2 = 0 \Rightarrow a - 1 = 0 \Rightarrow \boxed{a = 1}$$

2. If $y = \ln\left|\sec \frac{x}{2} + \tan \frac{x}{2}\right|$, then $y'(0) =$

Similar to Q # 92 / page 165

(a) $\dfrac{1}{2}$ ______________________________ (correct)

(b) $\dfrac{1}{4}$

(c) 0

(d) $\dfrac{-1}{4}$

(e) $\dfrac{-1}{2}$

$$y' = \frac{\left(\sec \frac{x}{2} + \tan \frac{x}{2}\right)'}{\sec \frac{x}{2} + \tan \frac{x}{2}} = \frac{\frac{1}{2}\sec \frac{x}{2} \tan \frac{x}{2} + \frac{1}{2}\sec^2 \frac{x}{2}}{\sec \frac{x}{2} + \tan \frac{x}{2}}$$

$$y'(0) = \frac{\frac{1}{2}(1)(0) + \frac{1}{2}(1)^2}{(1) + (0)} = \frac{1}{2}.$$

3. The x-intercept of the tangent line to the curve of $x^{2/3} + y^{2/3} = 5$ at the point $(1, 8)$ is

Similar to Q # 46 / page 175

(a) $(5, 0)$ (correct)

(b) $(4, 0)$

(c) $(6, 0)$

(d) $(7, 0)$

(e) $(9, 0)$

$$\frac{d}{dx}\left(x^{2/3} + y^{2/3}\right) = \frac{d}{dx}(5)$$

$$\Rightarrow \frac{2}{3}x^{-1/3} + \frac{2}{3}y^{-1/3}y' = 0 \quad \Rightarrow y' = -\frac{x^{-1/3}}{y^{-1/3}} = -\sqrt[3]{\frac{y}{x}}$$

at $(1,8)$, we have $y'\big|_{(1,8)} = -\sqrt[3]{\frac{8}{1}} = -2.$

$\therefore$ The eq. of the tangent line is

$$y - 8 = -2(x-1) \quad \Rightarrow y = -2x + 10$$

$\therefore$ The x-intercept $(y=0)$ is $x = 5 \quad \Rightarrow (5, 0).$

4. The slope of the tangent line to the curve of $x^4 + 2x^2y^2 + y^4 = 4x^2y$ at $(-1, 1)$ is

Similar to Q # 41 / page 175

(a) 0 (correct)

(b) 1

(c) -1

(d) 2

(e) -2

Using implicit differentiation we get

$$4x^3 + 4xy^2 + 4x^2yy' + 4y^3y' = 8xy + 4x^2y'$$

at $(-1,1)$ we have $(x=-1, y=1)$

$$\therefore \quad 4(-1)^3 + 4(-1)(1)^2 + 4(-1)^2(1)y' + 4(1)^3y' = 8(-1)(1) + 4(-1)^2y'$$

$$\Rightarrow -4 - 4 + 4y' + 4y' = -8 + 4y'$$

$$\Rightarrow \quad 4y' = 0 \quad \Rightarrow y' = 0$$

5. If $y(x) = \ln\sqrt{\dfrac{1-x}{1+x}} + \arctan x$, then $y'(0) =$

Similar to Q#33 / page 182

(a) 0 ________________ (correct)

(b) 2

(c) -2

(d) -1

(e) 1

$y = \frac{1}{2}\left(\ln 1-x - \ln 1+x\right) + \arctan x$

$\therefore\ y' = \frac{1}{2}\left(\frac{-1}{1-x} - \frac{1}{1+x}\right) + \frac{1}{1+x^2}$

$y'(0) = \frac{1}{2}(-1-1) + 1 = -1+1 = 0$

6. If $f(x) = x^3 + 3x - 1$, then $(f^{-1})'(-5) =$

Q#4 / page 182

(a) $\dfrac{1}{6}$ ________________ (correct)

(b) $\dfrac{2}{3}$

(c) $\dfrac{1}{3}$

(d) $\dfrac{1}{2}$

(e) $\dfrac{2}{5}$

Let $f^{-1}(-5) = x \Rightarrow f(x) = -5$

$\Rightarrow x^3 + 3x - 1 = -5$

$\Rightarrow x^3 + 3x + 4 = 0$

Rational zeros theorem $\Rightarrow$ possible zeros $-1, -2, -4$

$\because\ (-1)^3 + 3(-1) + 4 = 0 \Rightarrow x = -1$

$\therefore\ (f^{-1})'(-5) = \dfrac{1}{f'(f^{-1}(-5))} = \dfrac{1}{f'(-1)}$

$\because\ f'(x) = 3x^2 + 3 \quad \therefore (f^{-1})'(-5) = \frac{1}{6}$

7. If a point moves along the curve $y = \sqrt[3]{x^2}$ such that the y-component of the point is increasing at a rate of 2 units per second, then the rate at which the x-component is changing when $x = 1$ is equal to

Similar to Q 7-10 | Page 190 & Q#119 | page 202.

(a) 3 ______ (correct)

(b) $\frac{2}{3}$

(c) 1

(d) $\frac{3}{2}$

(e) 6

$\because \; y = \sqrt[3]{x^2} = x^{2/3}$

$\frac{dy}{dt} = \frac{2}{3} x^{-1/3} \frac{dx}{dt}$

when $x=1$, we have $\frac{dy}{dt} = 2$

$\therefore \; 2 = \frac{2}{3}(1)\frac{dx}{dt} \Rightarrow \frac{dx}{dt} = 3$

8. Using Newton's Method with initial guess $x_1 = 1$, the approximated zero of $f(x) = x^3 - 7$ after two iterations is equal to

Similar to Q # 4 | page 198

(a) $\frac{61}{27}$ ______ (correct)

(b) $\frac{101}{27}$

(c) $\frac{55}{27}$

(d) $\frac{52}{27}$

(e) $\frac{88}{27}$

$\because \; f'(x) = 3x^2$

$\therefore \; x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$= x_n - \frac{x_n^3 - 7}{3x_n^2}$

$\because \; x_1 = 1$

$x_2 = 1 - \frac{-6}{3} = 3$

$x_3 = 3 - \frac{27-7}{27} = \frac{81-20}{27} = \frac{61}{27}$

9. If M and m are the absolute maximum and absolute minimum values of the function $f(x) = 3x^4 - 4x^3$ where $-1 \le x \le 2$ respectively, then $M + m =$

(a) 15 ______ Example #2 / page 209 ______(correct)

(b) 25

(c) 10

(d) 12

(e) 23

$f'(x) = 12x^3 - 12x^2$

$= 12x^2(x-1) = 0$

$x = 0$ or $x = 1$

x	-1	0	1	2
$f(x)$	7	0	-1	16

$\therefore M = 16$ & $m = -1$ $\therefore M + m = 16 - 1 = 15$

10. The function $f(x) = x^3 + 2x^2 + x + 1$ has

Similar to Q#27-28 / page 227

(a) a local maximum at $x = -1$ and a local minimum at $x = -\dfrac{1}{3}$ ______(correct)

(b) a local minimum at $x = -1$ and a local maximum at $x = -\dfrac{1}{3}$

(c) a local minimum at $x = -\frac{2}{3}$ and a local maximum at $x = -\dfrac{1}{3}$

(d) a local maximum at $x = -\frac{2}{3}$ and a local minimum at $x = -\dfrac{1}{3}$

(e) a local maximum at $x = -\frac{4}{3}$ and a local minimum at $x = -\dfrac{2}{3}$

$f'(x) = 3x^2 + 4x + 1 = 0 \Rightarrow (3x+1)(x+1) = 0$

$\Rightarrow x = -\frac{1}{3}, x = -1$

f': $+$ -1 $-$ $-1/3$ $+$

f: $\nearrow$ $\searrow$ $\nearrow$

f is a poly. $\Rightarrow$ cont. in $\mathbb{R}$. $\therefore f$ has a local max. at $x = -1$ and a local min. at $x = -1/3$.

11. The slope of the tangent line to the curve $4e^{xy} = x^2 - y^2$ at the point $(\mathbf{2}, 0)$ is

Similar to Q#16 / page 175

(a) $\frac{1}{2}$ ________________ (correct)

(b) 1

(c) $\frac{1}{3}$

(d) $\frac{1}{4}$

(e) $\frac{1}{5}$

Differentiate implicitly the eq. of the curve

$4e^{xy}(xy)' = 2x - 2yy'$

$\Rightarrow 4e^{xy}(y + xy') = 2x - 2yy'$

at $(2,0)$, $4e^{0}(0 + 2y') = 4 - 0$

$\Rightarrow 8y' = 4 \Rightarrow y' = \frac{4}{8} = \frac{1}{2}$

12. If $f(x) = 2\sin x - \cos 2x$ is defined on $[0, 2\pi]$, then the sum of all its critical numbers is

Example # 4 / page 210

(a) 5π ________________ (correct)

(b) $\frac{9\pi}{2}$

(c) 3π

(d) $\frac{5\pi}{3}$

(e) 2π

$f'(x) = 2\cos x + 2\sin 2x$

$= 2(\cos x + 2\sin x\cos x)$

$= 2\cos x(1 + 2\sin x)$

$\therefore f$ is differentiable for all $x \in \mathbb{R}$.

The critical # are all x at which $f'(x) = 0$.

$\Rightarrow$ on $[0, 2\pi]$, $2\cos x(1 + 2\sin x) = 0$

$\Rightarrow \cos x = 0 \Rightarrow x = \frac{\pi}{2}, x = \frac{3\pi}{2}$

or $1 + 2\sin x = 0 \Rightarrow \sin x = -\frac{1}{2}$

$\Rightarrow x = \frac{7\pi}{6}, x = \frac{11\pi}{6}$

$\therefore$ The sum $= \frac{\pi}{2} + \frac{3\pi}{2} + \frac{7\pi}{6} + \frac{11\pi}{6} = 2\pi + 3\pi = 5\pi$.

13. If $f(x) = \dfrac{(2x+1)^3(x^2-1)^2}{x+3}$, then $f'(0) =$

(a) $\dfrac{17}{9}$ ________________ (correct)

Q # 108 / page 202

(b) $\dfrac{13}{3}$

let $y = \dfrac{(2x+1)^3(x^2-1)^2}{x+3}$

(c) $\dfrac{19}{7}$

$\ln y = 3\ln(2x+1) + 2\ln(x^2-1) - \ln(x+3)$

(d) $\dfrac{15}{7}$

(e) $\dfrac{21}{5}$

$\therefore \dfrac{y'}{y} = \dfrac{6}{2x+1} + \dfrac{4x}{x^2-1} - \dfrac{1}{x+3}$

$y'\big|_{x=0} = \left(\frac{1}{3}\right)\left(6+0-\frac{1}{3}\right) = \left(\frac{1}{3}\right)\left(\frac{17}{3}\right) = \frac{17}{9}.$

14. Which one of the following statements is always true

(a) Newton's Method fails when the initial guess x_1 corresponds to a horizontal tangent line for the graph of f at x_1 ________________ (correct)

(b) If the graph of a function has three x-intercepts, then it must have at least two points at which its tangent line is horizontal

(c) If f and g are increasing functions then fg is an increasing function

(d) If f is continuous on $[a, b]$, differentiable on (a, b) and there exists $c \in (a, b)$ such that $f'(c) = 0$, then $f(a) = f(b)$

(e) The maximum value of $f(x) = x^2$ on the interval $(-3, 3)$ is 9

15. If f is a function such that $f'(x) \geq 4$ for all $x \in (-2, 2)$ and $f(1) = 3$, then the largest value that $f(0)$ can take is

(a) -1 ______ Application of the Mean Value Theorem ______(correct)

(b) 0

(c) 2

(d) 3

(e) 4

$f'(x) \geq 4 \ \forall x \in (-2,2) \Rightarrow f$ is contin on $[0,1]$ and differentiable on $(0,1)$.

By M.V.T, $\exists\, c \in (0,1)$ s.t. $f'(c) = \dfrac{f(1)-f(0)}{1-0}$

$\therefore f'(c) = 3 - f(0) \geq 4$

$\Rightarrow 3 - 4 \geq f(0)$

$\Rightarrow f(0) \leq -1$

$\therefore$ The Largest value of $f(0)$ is -1.

16. If $f(x) = x \log_2 x$ is defined on the closed interval $[1, 2]$, then a number c in the open interval $(1, 2)$ such that $f'(c) = f(2) - f(1)$ is equal to

(a) $\frac{4}{e}$ ______ Q # 55 / Page 219 ______(correct)

(b) $\frac{5}{e}$

(c) $\frac{e}{2}$

(d) $\frac{3}{e}$

(e) $\frac{e}{3}$

By M.V.T, $\exists\, c \in (1,2)$ s.t. $f'(c) = \dfrac{f(2)-f(1)}{2-1}$

$\therefore f'(x) = \log_2 x + x \cdot \dfrac{1}{x \ln 2}$

$= \log_2 x + \dfrac{1}{\ln 2}$

$\therefore f'(c) = \log_2 c + \dfrac{1}{\ln 2} = \dfrac{2-0}{1} = 2$

$\Rightarrow \log_2 c = 2 - \dfrac{1}{\ln 2}$

$\Rightarrow \dfrac{\ln c}{\ln 2} = 2 - \dfrac{1}{\ln 2}$

$\Rightarrow \ln c = 2 \ln 2 - 1 = \ln 4 - \ln e = \ln \dfrac{4}{e}$

$\Rightarrow c = \dfrac{4}{e}$

17. If a man 1.5 meters tall walks at a rate of 3 meters per second away from a light this is 6 meters above the ground, then the rate at which the length of his shadow is changing when he is 6 meters from the base of the light is equal to

Similar to Q#29 (b) / page 192

(a) 1 m/sec ________ (correct)

(b) 1.5 m/sec

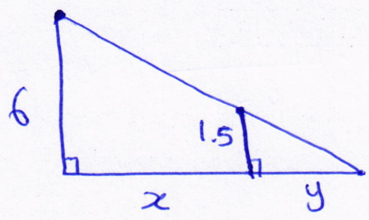
(c) 2 m/sec

(d) 2.5 m/sec

(e) 3 m/sec

$\because \frac{dx}{dt} = 3$ m/s

$\frac{dy}{dt} = ?$



From the similarity of triangles, we have

$$\frac{y}{x+y} = \frac{1.5}{6} = \frac{15}{60} = \frac{1}{4}$$

$$\Rightarrow 4y = x + y \Rightarrow 3y = x$$

$$\Rightarrow 3\frac{dy}{dt} = \frac{dx}{dt} = 3 \Rightarrow \frac{dy}{dt} = 1 \text{ m/s}$$

18. If the normal line to the parabola $y = \frac{1}{2}(x-2)^2 + 1$ at the point $\left(3, \frac{3}{2}\right)$ intersects the parabola at another point (a, b) then $4a + 2b =$

Similar to Q # 95 / page 177

(a) 7 ________ (correct)

(b) 13

(c) 0

(d) 3

(e) 1

The slope of the tangent line at $x=3$ is $y'\big|_{x=3} = x-2 = 1$

$\therefore$ The slope of the normal line is -1.

$\therefore$ the eq. of the normal line is $y - \frac{3}{2} = (-1)(x-3)$

$$\Rightarrow y = -x + \frac{9}{2}$$

$\therefore$ The points of intersections of the normal line & the parabola are the solutions of the eq. $-x + \frac{9}{2} = \frac{1}{2}(x-2)^2 + 1$

$$\Rightarrow -2x + 9 = x^2 - 4x + 6$$

$$\Rightarrow x^2 - 2x - 3 = (x-3)(x+1) = 0 \Rightarrow x = 3 \text{ or } x = -1$$

$x=3, y = \frac{3}{2} \Rightarrow (3, \frac{3}{2})$, $x = -1, y = \frac{11}{2} \Rightarrow$ the other point $(-1, \frac{11}{2})$

$\therefore a = -1, b = \frac{11}{2} \Rightarrow 4a + 2b = -4 + 11 = 7.$

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	D $_{12}$	E $_{9}$	A $_{11}$	A $_{10}$
2	A	C $_{6}$	E $_{1}$	A $_{9}$	A $_{5}$
3	A	B $_{11}$	A $_{7}$	E $_{10}$	D $_{2}$
4	A	A $_{7}$	D $_{8}$	C $_{2}$	E $_{9}$
5	A	A $_{9}$	C $_{10}$	E $_{5}$	D $_{4}$
6	A	D $_{1}$	E $_{3}$	A $_{8}$	C $_{6}$
7	A	E $_{5}$	C $_{5}$	B $_{6}$	E $_{3}$
8	A	A $_{2}$	E $_{12}$	A $_{3}$	C $_{12}$
9	A	A $_{3}$	B $_{6}$	D $_{1}$	B $_{7}$
10	A	C $_{8}$	A $_{4}$	B $_{4}$	C $_{8}$
11	A	D $_{10}$	D $_{2}$	B $_{7}$	C $_{1}$
12	A	A $_{4}$	D $_{11}$	B $_{12}$	B $_{11}$
13	A	E $_{15}$	A $_{14}$	C $_{18}$	C $_{18}$
14	A	A $_{18}$	C $_{13}$	D $_{17}$	C $_{13}$
15	A	A $_{17}$	D $_{15}$	B $_{13}$	C $_{15}$
16	A	E $_{14}$	D $_{18}$	C $_{14}$	D $_{16}$
17	A	C $_{16}$	B $_{16}$	D $_{15}$	C $_{14}$
18	A	A $_{13}$	E $_{17}$	E $_{16}$	D $_{17}$