1. Let

$$f(x) = \begin{cases} x + b & \text{if } 0 < x < 2 \\ ax^2 + 1 & \text{if } 2 \leq x \leq 4 \end{cases}$$

If f is continuous at $x = 2$, then $4a - b =$

(a) 1 (correct)
(b) 3
(c) -1
(d) 4
(e) 2

2. The sum of all values of x at which the tangent lines to the graph of $y = \dfrac{2x-1}{3x+1}$ are parallel to the line $4x - 5y + 1 = 0$ is

(a) $-\dfrac{2}{3}$ (correct)
(b) $-\dfrac{4}{5}$
(c) -1
(d) $-\dfrac{1}{4}$
(e) $-\dfrac{3}{5}$

3. If $h(x) = xf(x)g(x)$, with $f(1) = 2$, $g(1) = 2$, $f'(1) = 1$ and $g'(1) = 1$, then $h'(1) =$

(a) 8 (correct)

(b) 4

(c) 1

(d) 6

(e) 5

4. If $\lim_{x \to 0} \dfrac{\sqrt{b - ax} - 2}{x} = 3$, then $a + b =$

(a) -8 (correct)

(b) 8

(c) -6

(d) 6

(e) 0

5. If $f(x) = \frac{1}{2}x + 2$, then the largest number $\delta > 0$ such that $|f(x) - 3| < 0.45$ whenever $0 < |x - 2| < \delta$ is equal to

(a) $\frac{9}{10}$ (correct)

(b) 1

(c) $\frac{11}{10}$

(d) $\frac{12}{10}$

(e) $\frac{8}{10}$

6. If M and N are the numbers of relative maximum points and relative minimum points of the function $f(x) = 5x^{4/5}(x-1)^2$ respectively, then $2M + N =$

(a) 4 (correct)

(b) 3

(c) 6

(d) 5

(e) 2

7. The absolute maximum of the function $f(x) = \dfrac{x^2}{x-2}$ on $[-2, 1]$ is

(a) 0 (correct)
(b) -1
(c) 8
(d) 1
(e) 2

8. If $x^2y - 4x = 5$, then $\dfrac{d^2y}{dx^2}$ at the point $(-1, 1)$ is equal to

(a) 22 (correct)
(b) 38
(c) 14
(d) 6
(e) 24

9. The sum of all values of x at which the function $f(x) = \dfrac{-4x}{\sqrt{2x-1}}$ has a horizontal tangent line is

(a) 1 (correct)
(b) $\frac{3}{2}$
(c) -1
(d) $-\frac{3}{2}$
(e) 0

10. The sum of all $x-$coordinates of the points at which the curve

$$x^2 - 5x + 3y^3 + 6 = 0$$

has vertical tangent line is

(a) 5 (correct)
(b) 6
(c) 4
(d) 3
(e) 2

11. The sum of all values of x at which the graph of $f(x) = \dfrac{x}{e \ln x}$ has a point of inflection is

(a) e^2 (correct)
(b) e^{-2}
(c) $\ln 2$
(d) e
(e) e^{-1}

12. If $(-12, 0)$ is the largest open interval over which the graph of

$$f(x) = x^4 + ax^3$$

is concave downward, then $a =$

(a) 24 (correct)
(b) 12
(c) 15
(d) 27
(e) 19

13. $\lim_{x\to-\infty}\left(2x+\sqrt{4x^2-x}\right)=$

(a) $\frac{1}{4}$ (correct)

(b) 0

(c) $\frac{1}{2}$

(d) 2

(e) does not exist

14. If $\lim_{x\to-\infty}\left(\frac{3x-2}{\sqrt{4x^2+5}}\right)=L_1$ and $\lim_{x\to\infty}\left(\frac{3x-2}{\sqrt{4x^2+5}}\right)=L_2$, then $L_1-L_2=$

(a) -3 (correct)

(b) $-\infty$

(c) ∞

(d) $-\frac{3}{2}$

(e) 0

15. If $f(x) = \dfrac{2x^3 - 3x + 5}{x^2 + x}$, then an equation of the slant asymptote for the graph of f is

(a) $y = 2x - 2$ (correct)
(b) $y = 2x - 3$
(c) $y = x + 3$
(d) $y = 3x + 1$
(e) $y = 3x - 2$

16. The function

$$f(x) = \frac{2}{x \ln x}$$

has

(a) two vertical and one horizontal asymptotes (correct)
(b) one vertical and one horizontal asymptotes
(c) no vertical and one horizontal asymptote
(d) one vertical and no horizontal asymptote
(e) two vertical and no horizontal asymptotes

17. If $y = 0$ and $y = 5$ are two horizontal asymptotes of the function

$$f(x) = \frac{ae^x + be^{-x}}{2e^x + 3e^{-x}}, \quad b > 0,$$

then $5a + b =$

(a) 15 (correct)
(b) 5
(c) 6
(d) 10
(e) 30

18. If an open box of maximum volume is to be made from a square piece of material, 30 cm on a side, by cutting equal squares from the corners and turning up the sides, then the sum of the dimensions of the box is

(a) 45 cm (correct)
(b) 42 cm
(c) 48 cm
(d) 40 cm
(e) 30 cm

19. The area of the largest rectangle that can be inscribed in the region bounded by the x-axis, the y-axis and the line $y = 3 - \frac{1}{2}x$ is equal to

(a) $\frac{9}{2}$ (correct)

(b) 3

(c) $\frac{7}{2}$

(d) 6

(e) 9

20. Using the tangent line approximation of $f(x) = \sqrt{x+9}$ at the point $(0, 3)$, $\sqrt{9.03}$ can be approximated as

(a) 3.005 (correct)

(b) 3.004

(c) 3.003

(d) 3.002

(e) 3.001

21. The radius of a spherical balloon is measured as 8 cm, with a possible error of 0.02 cm. Using differentials, the percent error in computing the volume of the sphere is

(Hint: $V = \frac{4}{3}\pi r^3$)

(a) 0.75 % (correct)

(b) 0.25 %

(c) 0.50 %

(d) 0.05 %

(e) 0.45 %

22. If $f(x) + f''(x) = 1$ for all x, and $F(x)$ is an anti-derivative of $f(x)$ such that $F(0) = -1$, $F(1) = 2$ and $f'(0) = 2$, then $f'(1) =$

(a) 0 (correct)

(b) -1

(c) 1

(d) 2

(e) -2

23. If $f''(x) = \dfrac{2}{x^2}$, $f'(1) = 4$ and $f(1) = 3$, then $2f(2) - f(4) =$

(a) -3 (correct)
(b) $8\ln 2 + 3$
(c) $-4\ln 2 + 21$
(d) $4\ln 2 + 18$
(e) 15

24. $\lim\limits_{x \to 3^+} [4(x-3)]^{(x-3)} =$

(a) 1 (correct)
(b) 2
(c) 3
(d) 4
(e) 5

25. If $f''(x)$ is a continuous function, then

$$\lim_{h\to 0} \frac{3f(x+4h) - 7f(x) + 4f(x-3h)}{6h^2} =$$

(a) $7f''(x)$ (correct)
(b) $3f'(x)$
(c) $4f''(x)$
(d) $3f''(x) + 4f'(x)$
(e) $f''(x)$

26. If $f(x) = \cosh(\ln x)$, then $f'(1/5) =$

(a) -12 (correct)
(b) -10
(c) 24
(d) -26
(e) -1

27. The function $f(x) = x\sinh(x-1) - \cosh(x-1)$ has

(a) one relative minimum (correct)
(b) one relative maximum
(c) two relative extrema
(d) no relative minimum
(e) two critical points

28. $\lim_{x\to 0} \dfrac{1-\cos x}{x^2} =$

(a) $\dfrac{1}{2}$ (correct)
(b) 0
(c) 1
(d) ∞
(e) $-\infty$

Math 101 , 221, Final Exam

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	A	C	D	E
2	A	A	B	D	E
3	A	D	B	E	A
4	A	E	D	E	C
5	A	B	B	A	A
6	A	C	A	E	A
7	A	E	D	A	A
8	A	C	E	A	E
9	A	E	A	C	C
10	A	B	A	D	B
11	A	E	E	A	C
12	A	E	A	A	E
13	A	D	A	D	B
14	A	D	B	C	D
15	A	B	D	B	A
16	A	C	B	D	B
17	A	E	E	C	D
18	A	C	C	E	E
19	A	B	E	C	B
20	A	D	A	C	A
21	A	A	B	B	E
22	A	A	D	A	C
23	A	B	E	D	C
24	A	B	C	D	C
25	A	E	C	D	A
26	A	A	A	B	C
27	A	B	D	A	B
28	A	E	C	C	C