King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 101
Final Exam
223
August 15, 2023
Net Time Allowed: 180 Minutes

MASTER VERSION

1. If $L(x)$ is the tangent line approximation to the graph of $f(x) = 3^x$ at the point $(2, 9)$, then $L(1) =$

    (a) $9 - 9 \ln 3$ ————————————————————————————(correct)

    (b) $8 - 8 \ln 3$

    (c) $9 + 27 \ln 3$

    (d) 9

    (e) $9 + 9 \ln 3$

2. $\displaystyle \int (4t^2 + 3)^2 \, dt =$

    (a) $\dfrac{16t^5}{5} + 8t^3 + 9t + c$ ————————————————(correct)

    (b) $\dfrac{16t^5}{5} + 8t^3 + 9t$

    (c) $3t^5 + 8t^3 + 9t$

    (d) $\dfrac{16t^5}{5} + 6t^3 + 9t + c$

    (e) $\dfrac{16t^5}{5} + 8t^3 + 6t + c$

3. The value of c which satisfies the conclusion of the Mean Value Theorem for the function $f(x) = \sin x$ on $[0, \pi]$ is

(a) $\dfrac{\pi}{2}$ ————————————————————————————(correct)

(b) $\dfrac{\pi}{4}$

(c) $\dfrac{\pi}{3}$

(d) $\dfrac{\pi}{6}$

(e) $\dfrac{\pi}{5}$

4. $\displaystyle\lim_{x\to 0} \dfrac{\cos x - \sin x - 1}{2x} =$

(a) $-\dfrac{1}{2}$ ————————————————————————(correct)

(b) $\dfrac{1}{2}$

(c) $-\dfrac{1}{4}$

(d) $\dfrac{1}{4}$

(e) -1

5. The function $f(x) = 12x - x^3$ is increasing on the interval

(a) $(-2, 2)$ ————————————————————————————(correct)

(b) $(-\infty, -2)$

(c) $(2, \infty)$

(d) $(-\infty, \infty)$

(e) $(-\infty, 2)$

6. The graph of the function $f(x) = \dfrac{24}{x^2 + 12}$ is concave downward on the interval:

(a) $(-2, 2)$ ————————————————————————————(correct)

(b) $(-\infty, -2)$

(c) $(2, \infty)$

(d) $(-12, -2)$

(e) $(-12, 12)$

7. The number of inflection points for the graph of the function $f(x) = \dfrac{6 - x}{\sqrt{x}}$ is:

    (a) 0 ——————————————————————————————(correct)

    (b) 1

    (c) 2

    (d) 3

    (e) 4

8. If $y = Ax + B$ is the slant asymptote for the graph of the function
$f(x) = \dfrac{x^3 - 3x^2 + 2}{x(x - 3)}$, then $A + B =$

    (a) 1 ——————————————————————————————(correct)

    (b) 0

    (c) −1

    (d) 2

    (e) −2

Question 37/ Section 4.8

9. The measurement of the side of a square floor tile is 10 inches, with a possible error of $\frac{1}{32}$ inch. The percent error in computing the area of the square is:

(a) $\frac{5}{8}\%$ ──────────────(correct)

(b) $\frac{8}{5}\%$

(c) $\frac{3}{8}\%$

(d) $\frac{8}{3}\%$

(e) 1%

Example 9/ Section 5.1

10. A ball is thrown upward with an initial velocity of 64 feet per second from an initial height of 80 feet. Assume the acceleration is $a(t) = -32$ feet per second per second. The ball will hit the ground after

(a) 5 seconds ──────────────(correct)

(b) 4 seconds

(c) 6 seconds

(d) 3 seconds

(e) 7 seconds

11. $\lim\limits_{x\to\infty} \dfrac{e^{x^2}}{1-x^3} =$

    (a) $-\infty$ ——————————————————————————(correct)

    (b) ∞

    (c) 0

    (d) 1

    (e) e

12. $\lim\limits_{x\to 0^+} 3x^{x/2} =$

    (a) 3 ——————————————————————————(correct)

    (b) 2

    (c) 1

    (d) 0

    (e) $\ln 2$

13. $\displaystyle\lim_{x\to 2^+}\left(\frac{1}{x^2-4}-\frac{\sqrt{x-1}}{x^2-4}\right)=$

   (a) $-\dfrac{1}{8}$ ————————————————————————————(correct)

   (b) $\dfrac{1}{8}$

   (c) $-\dfrac{1}{4}$

   (d) $\dfrac{1}{4}$

   (e) 1

14. If $y=\ln\left(\tanh\dfrac{x}{2}\right)$, then $y'(\ln 2)=$

   (a) $\dfrac{4}{3}$ ————————————————————————————(correct)

   (b) $\dfrac{3}{4}$

   (c) $\dfrac{2}{5}$

   (d) $\dfrac{3}{5}$

   (e) $\dfrac{4}{5}$

Question 37/ Section 5.9

15. The equation of the tangent line to the graph of the function $f(x) = (\cosh x - \sinh x)^2$ at the point $(0, 1)$ is $y =$

   (a) $-2x + 1$ ——————————————————————————(correct)

   (b) $2x + 1$

   (c) $-2x - 1$

   (d) $2x - 1$

   (e) $-x + 1$

Question 41/ Section 4.1

16. The absolute maximum of the function $f(x) = \arctan(x^2)$ on the interval $[-2, 1]$ is:

   (a) $\arctan 4$ ——————————————————————————(correct)

   (b) $\dfrac{\pi}{4}$

   (c) 0

   (d) $\arctan 3$

   (e) $\arctan 2$

17. The equation of the tangent line to the graph of the equation
$\arctan(xy) = \arcsin(x + y)$ at the point $(0, 0)$ is

   (a) $y = -x$ ————————————————————————————(correct)

   (b) $y = -2x$

   (c) $y = x$

   (d) $y = 2x$

   (e) $y = \dfrac{1}{2}x$

18. If $4xy + \ln(x^2 y) = 7$, then $\dfrac{dy}{dx} =$

   (a) $\dfrac{-4xy^2 - 2y}{4x^2 y + x}$ ————————————————————(correct)

   (b) $\dfrac{4xy^2 - 2y}{4x^2 y - x}$

   (c) $\dfrac{2y - 4xy^2}{4x^2 y + x}$

   (d) $\dfrac{-4xy^2 + 2y}{4x^2 y + x}$

   (e) $\dfrac{-2xy^2 - y}{4x^2 y + x}$

Question 94/ Section 3.4

19. If $y = \ln\left(\dfrac{1 + e^x}{1 - e^x}\right)$, then $y' =$

    (a) $\dfrac{2e^x}{1 - e^{2x}}$ ————————————————————————————(correct)

    (b) $\dfrac{2e^x}{1 + e^{2x}}$

    (c) $\dfrac{e^{2x}}{1 + e^x}$

    (d) $\dfrac{e^{2x}}{1 - e^x}$

    (e) $\dfrac{2e^{2x}}{1 - e^{2x}}$

Question 83/ Section 3.2

20. The graph of the function $f(x) = 3x + \sin x + 2$

    (a) does not have a horizontal tangent line ————————————————————(correct)

    (b) has one horizontal tangent line

    (c) has two horizontal tangent lines

    (d) has three horizontal tangent lines

    (e) has infinitely many horizontal tangent lines

Question 65/ section 3.3

21. The slope of the graph of the function $f(t) = \dfrac{\sec t}{t}$ at the point $\left(\pi, \dfrac{-1}{\pi}\right)$ is:

(a) $\dfrac{1}{\pi^2}$ ————————————————————————————————(correct)

(b) $\dfrac{1}{\pi}$

(c) $\dfrac{-1}{\pi^2}$

(d) $\dfrac{-1}{\pi}$

(e) 1

Question 89/ Section 3.1

22. If the function $f(x) = \begin{cases} x^2 + 1, & x \le A \\ 4x - 3, & x > A \end{cases}$ is differentiable everywhere, then $A =$

(a) 2 ————————————————————————————————(correct)

(b) 1

(c) -1

(d) -2

(e) 0

23. The number of vertical asymptotes of the graph of the function $f(x) = \dfrac{x^2 - 9}{x^3 + 3x^2 - x - 3}$ is

    (a) 2 ————————————————————————————————(correct)

    (b) 3

    (c) 4

    (d) 1

    (e) 0

24. The function $f(x) = \sec \dfrac{\pi x}{4}$ is continuous when

    (a) $x \neq 2 + 4n$ where n is an integer ————————————————(correct)

    (b) $x \neq 1 + 4n$ where n is an integer

    (c) $x \neq 4 + 2n$ where n is an integer

    (d) $x \neq 4 + 4n$ where n is an integer

    (e) $x \neq 1 + n$ where n is an integer

Question 62/ Section 2.3

25. $\displaystyle\lim_{\triangle x \to 0} \frac{(x + \triangle x)^3 - x^3}{\triangle x} =$

    (a) $3x^2$ ————————————————————————————(correct)

    (b) x^2

    (c) $2x^2$

    (d) $2x^3$

    (e) $-3x^2$

Question 10/ Section 4.7

26. Let x and y be two positive numbers such that the sum of the first number cubed and the second number is 500 and the product is a maximum. Then $x + y =$

    (a) 380 ————————————————————————————(correct)

    (b) 360

    (c) 340

    (d) 400

    (e) 420

Example 1/ Section 4.7

27. A manufacturer wants to design an open box having a square base and a surface area of 108 square inches. If we want to produce a box with maximum volume, then the volume in cubic inches is

  (a) 108 ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯(correct)

  (b) 72

  (c) 144

  (d) 36

  (e) 180

Question 18/ Section 3.7

28. The volume of oil in a cylindrical container is increasing at a rate of 150 cubic inches per second. The height of the cylinder is ten times the radius.
    At what rate (in inches per second) is the height of the oil changing when the oil is 35 inches high?

    (Hint: The volume of the cylinder is $V = \pi r^2 h$)

  (a) $\dfrac{200}{49\pi}$ ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯(correct)

  (b) $\dfrac{200}{9\pi}$

  (c) $\dfrac{100}{49\pi}$

  (d) $\dfrac{200}{7\pi}$

  (e) $\dfrac{100}{7\pi}$