#52
§2.3

1. $\lim_{x\to 2} \dfrac{x^2+2x-8}{x^2-x-2} =$ $\lim_{x\to 2} \dfrac{(x+4)(x-2)}{(x+1)(x-2)} = \lim_{x\to 2} \dfrac{x+4}{x+1} = \dfrac{6}{3} = 2$

(a) 2 ______ (correct)
(b) 1
(c) 0
(d) 4
(e) -3

#33
§2.2

2. If $f(x) = \begin{cases} x^2, & x \le 2 \\ 8-2x, & 2 < x < 4 \\ 4, & x \ge 4 \end{cases}$

then $\lim_{x\to 2} f(x)$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} x^2 = 2^2 = 4$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} 8-2x = 8-4 = 4$

Since $\lim_{x\to 2^-} f(x) = \lim_{x\to 2^+} f(x) = 4$,

then $\lim_{x\to 2} f(x) = 4$

(a) is equal to 4 ______ (correct)
(b) is equal to 2
(c) is equal to 6
(d) is equal to 1
(e) does not exist

#54
§2.3

3. $\lim_{x\to 3} \dfrac{\sqrt{x+1}-2}{x-3}$ $= \lim_{x\to 3} \dfrac{\sqrt{x+1}-2}{x-3} \cdot \dfrac{\sqrt{x+1}+2}{\sqrt{x+1}+2}$

$= \lim_{x\to 3} \dfrac{(x+1)-4}{(x-3)[\sqrt{x+1}+2]}$

(a) $\dfrac{1}{4}$ ——— (correct)

(b) 0

$= \lim_{x\to 3} \dfrac{x-3}{(x-3)[\sqrt{x+1}+2]}$

(c) $\dfrac{2}{3}$

(d) $\dfrac{-1}{3}$

$= \lim_{x\to 3} \dfrac{1}{\sqrt{x+1}+2} = \dfrac{1}{2+2} = \dfrac{1}{4}$

(e) $\dfrac{1}{2}$

#22
§4.5

4. $\lim_{x\to -\infty} \dfrac{5x^3+1}{10x^3-3x^2+7}$ $= \lim_{x\to -\infty} \dfrac{x^3[5+\frac{1}{x^3}]}{x^3[10-\frac{3}{x}+\frac{7}{x^3}]}$

(a) $\dfrac{1}{2}$ ——— (correct)

(b) 0

$= \lim_{x\to -\infty} \dfrac{5+\frac{1}{x^3}}{10-\frac{3}{x}+\frac{7}{x^3}}$

(c) $-\infty$

(d) 5

(e) $-\dfrac{1}{2}$

$= \dfrac{5+0}{10-0+0} = \dfrac{5}{10} = \dfrac{1}{2}$

~#75
§2.3

5. $\lim_{t \to 0} \frac{\sin(6t)}{2t} =$ $\lim_{t \to 0} \frac{6}{2} \cdot \frac{\sin(6t)}{6t} = \frac{6}{2} \cdot 1 = 3$

(a) 3 ______________________________ (correct)

(b) 6

(c) $\frac{1}{2}$

(d) 1

(e) ∞

~Example 8
§2.4

6. The **Intermediate Value Theorem** guarantees that the function $f(x) = x^3 - x^2 + 2x - 3$ has a zero in the interval

(a) $[0, 2]$ ______ $f(0) = -3 < 0$, $f(2) = 8 - 4 + 4 - 3 = 5 > 0$ ✓ opposite sign ______ (correct)

(b) $[0, 1]$ $f(0) = -3 < 0$, $f(1) = 1 - 1 + 2 - 3 = -1 < 0$ ✗ same sign

(c) $[-1, 1]$ $f(-1) = -1 - 1 - 2 - 3 = -7 < 0$, $f(1) = -1 < 0$ ✗ "

(d) $[2, 3]$ $f(2) = 5 > 0$, $f(3) = 27 - 9 + 6 - 3 = 21 > 0$ ✗ "

(e) $[-1, 0]$ $f(-1) = -7 < 0$, $f(0) = -3 < 0$ ✗ "

~ Example 3
§ 2.5

7. $\lim\limits_{x\to -5^-} \dfrac{x+3}{x^2+8x+15} = \lim\limits_{x\to -5^-} \dfrac{x+3}{(x+3)(x+5)} = \lim\limits_{x\to -5^-} \dfrac{1}{x+5}$

$= -\infty$

$- \quad +$ -5

(a) $-\infty$ ______ (correct)

(b) ∞

(c) 0

(d) 1

(e) 3

~ #72
§ 2.3

8. $\lim\limits_{x\to 0} \dfrac{2-3\tan(3x)-2\cos x}{2x} = \lim\limits_{x\to 0} \dfrac{2-2\cos x - 3\tan(3x)}{2x}$

$= \lim\limits_{x\to 0} \dfrac{2-2\cos x}{2x} - \dfrac{3\tan(3x)}{2x}$

(a) $-\dfrac{9}{2}$ ______ (correct)

(b) -3

(c) $\dfrac{3}{2}$

(d) -1

(e) 3

$= \lim\limits_{x\to 0} \dfrac{1-\cos x}{x} - \dfrac{3}{2} \cdot \dfrac{\tan(3x)}{3x} \cdot 3$

$= 0 - \dfrac{3}{2} \cdot 1 \cdot 3$

$= -\dfrac{9}{2}$

#26, 27
§2.4

9. $\lim_{x\to -1^-}\left(\llbracket \frac{x}{3}\rrbracket - \llbracket x\rrbracket\right) =$

($\llbracket\cdot\rrbracket$ is the greatest integer function).

$\cdot\ x \to -1^-$
$\Rightarrow x < -1$
$\Rightarrow \frac{x}{3} < -\frac{1}{3} \Rightarrow [\frac{x}{3}] \to -1$
$\cdot\ x \to -1^- \Rightarrow x < -1 \Rightarrow [x] \to -2$

(a) 1 ______________________________ (correct)

(b) 0

(c) 2

(d) -1

(e) $\frac{2}{3}$

So $\lim_{x\to -1^-}\left(\llbracket \frac{x}{3}\rrbracket - \llbracket x\rrbracket\right) = -1-(-2) = -1+2$
$= 1$

#50 §4.5

10. $\lim_{x\to\infty}(x-\sqrt{x^2+x}) = \lim_{x\to\infty} x-\sqrt{x^2+x} \cdot \frac{x+\sqrt{x^2+x}}{x+\sqrt{x^2+x}}$

(a) $-\frac{1}{2}$ ______________________________ (correct)

(b) 0

(c) -1

(d) $-\infty$

(e) $\frac{1}{2}$

$= \lim_{x\to\infty} \frac{x^2-(x^2+x)}{x+\sqrt{x^2+x}}$

$\cdot\ \sqrt{x^2+x} = \sqrt{x^2(1+\frac{1}{x})}$
$= |x|\sqrt{1+\frac{1}{x}}$

$= \lim_{x\to\infty} \frac{-x}{x+|x|\sqrt{1+\frac{1}{x}}}$

$= \lim_{x\to\infty} \frac{-x}{x+x\sqrt{1+\frac{1}{x}}}$, $x\to\infty \to x>0$. So $|x|=x$

$= \lim_{x\to\infty} \frac{-1}{1+\sqrt{1+\frac{1}{x}}}$

$= \frac{-1}{1+1} = -\frac{1}{2}$

~ Example 3, § 2.5
~ #7, § 4.5

11. The graph of $f(x) = \dfrac{x^2+2x-8}{x^3-4x}$ has

(V. A.: vertical asymptote; H. A.: horizontal asymptote)

(a) 2 V.A and 1 H. A. ______________________________ (correct)

(b) 3 V.A and 1 H. A.

(c) 1 V.A and 2 H. A.

(d) no V.A and 1 H. A.

(e) no V.A and 2 H. A.

$$f(x) = \frac{(x-2)(x+4)}{x(x-2)(x+2)}$$

$$= \frac{x+4}{x(x+2)}, \quad x \neq 2$$

V.A.: $x(x+2) = 0 \Rightarrow x = 0, \; x = -2$

H.A.: $\lim_{x\to\infty} f(x) = 0$; $\lim_{x\to-\infty} f(x) = 0 \Rightarrow y = 0$

#127, § 2.4

12. If $f(x) = \begin{cases} 1-x^2, & x \le c \\ x, & x > c \end{cases}$ is continuous at $x = c$, then $(2c+1)^2 =$

(a) 5 ______________________________ (correct)

(b) 9

(c) 7

(d) 3

(e) 1

f is Cont at $c \Rightarrow \lim_{x\to c} f(x) = f(c)$

$\Rightarrow \lim_{x\to c^-} f(x) = \lim_{x\to c^+} f(x) = f(c)$

$\Rightarrow \lim_{x\to c^-} (1-x^2) = \lim_{x\to c^+} x$

$\Rightarrow 1-c^2 = c$

$\Rightarrow c^2+c-1 = 0$

$\Rightarrow c = \dfrac{-1 \pm \sqrt{1-4(1)(-1)}}{2(1)}$

$\Rightarrow \boxed{c = \dfrac{-1 \pm \sqrt{5}}{2}}$

$\Rightarrow 2c = -1 \pm \sqrt{5}$

$\Rightarrow 2c+1 = \pm\sqrt{5}$

$\Rightarrow (2c+1)^2 = 5$

~ #78

§2.4 13. The function $f(x) = \dfrac{\sqrt{x+3}}{x}$ is continuous on

$x+3 \geq 0$ & $x \neq 0$

$\Rightarrow$ $x \geq -3$ & $x \neq 0$

(a) $[-3,0) \cup (0,\infty)$ (correct)

(b) $[-3,\infty)$

(c) $(-\infty,0) \cup (0,\infty)$

(d) $(-\infty,-3]$

(e) $(-\infty,\infty)$

$\Rightarrow$ $[-3,0) \cup (0,\infty)$

#45 §2.4

14. Which one of the following statements is **TRUE** about $f(x) = \dfrac{x}{x^2 - x}$?

(a) f has a removable discontinuity at $x = 0$. (correct)

(b) f has a nonremovable discontinuity at $x = 0$.

(c) f has a removable discontinuity at $x = 1$.

(d) f has a removable discontinuity at $x = -1$.

(e) f has a nonremovable discontinuity at $x = 2$.

$f(x) = \dfrac{x}{x(x-1)}$

- $\boxed{f \text{ is continuous everywhere except at } x=0 \text{ \& } x=1}$

$= \dfrac{1}{x-1}$, $x \neq 0$

- $\lim_{x\to 0} f(x) = \lim_{x \to 0} \dfrac{1}{x-1} = -1$ $\Rightarrow$ f has a remov. disc. at $x=0$
- $\lim_{x\to 1^+} f(x) = \infty$; $\lim_{x\to 1^-} f(x) = -\infty$ $\Rightarrow$ f has a nonremovable disc. at $x=1$

#53
§3.2

15. If $f(x) = 6\sqrt{x} + 5\cos x + 11$, then

(a) $f'(x) = \dfrac{3}{\sqrt{x}} - 5\sin x$ ______ (correct)

(b) $f'(x) = \dfrac{3}{\sqrt{x}} + 5\sin x$

(c) $f'(x) = \dfrac{6}{\sqrt{x}} - 5\cos x$

(d) $f'(x) = \dfrac{-3}{\sqrt{x}} - 5\sin x$

(e) $f'(x) = 2\sqrt{x} + 5\sin x$

$f'(x) = 6 \cdot \dfrac{1}{2\sqrt{x}} + 5(-\sin x) + 0$

$= \dfrac{3}{\sqrt{x}} - 5\sin x$

85, §3.1

16. Find the **derivative from the left** of $f(x) = |x-1|$ at $x = 1$.

$= \begin{cases} x-1 & \text{if } x \geq 1 \\ -(x-1) & \text{if } x < 1 \end{cases}$

(a) -1 ______ (correct)

(b) 1

(c) 0

(d) $-\infty$

(e) ∞

$= \begin{cases} x-1 & \text{if } x \geq 1 \\ -x+1 & \text{if } x < 1 \end{cases}$

The derivative from the left of f at $x=1$ is

$\lim\limits_{x \to 1^-} \dfrac{f(x) - f(1)}{x-1}$, $f(1) = 0$

$= \lim\limits_{x \to 1^-} \dfrac{-x+1-0}{x-1}$

$= \lim\limits_{x \to 1^-} \dfrac{-(x-1)}{x-1} = \lim\limits_{x \to 1^-} -1 = -1$

#60
§3.2

17. If $y = Ax + B$ is an equation of the tangent line to the graph of $f(x) = \dfrac{2}{\sqrt[4]{x^3}}$ at the point $(1, 2)$, then $B - A =$

(a) 5 ______ (correct)

(b) 4

(c) -3

(d) $-\dfrac{5}{2}$

(e) -2

$f(x) = 2\,x^{-3/4}$

$f'(x) = 2 \cdot \frac{-3}{4}\, x^{-7/4}$

. slope $= f'(1) = 2 \cdot \frac{-3}{4} \cdot 1 = -\frac{3}{2}$

. Eq: $y - 2 = -\frac{3}{2}(x-1)$

$\Rightarrow y = -\frac{3}{2}x + \frac{3}{2} + 2$

$\Rightarrow y = -\frac{3}{2}x + \frac{7}{2}$

$A = -\frac{3}{2}$, $B = \frac{7}{2}$

$\Rightarrow B - A = \frac{7}{2} - (-\frac{3}{2}) = \frac{7}{2} + \frac{3}{2} = \frac{10}{2} = 5$

#73
§3.2

18. If the line $y = -\dfrac{3}{4}x + 3$ is tangent to the graph of $f(x) = \dfrac{k}{x}$, then

Let (x_0, y_0) be the point of tangency. Then

(a) $k = 3$ ______ (correct)

(b) $k = -\dfrac{1}{3}$

(c) $k = -\dfrac{3}{4}$

(d) $k = 1$

(e) $k = -\dfrac{2}{3}$

(i) (x_0, y_0) lies on the line & the graph, so

$-\frac{3}{4}x_0 + 3 = \frac{k}{x_0}$ $(= y_0)$

$\Rightarrow k = -\frac{3}{4}x_0^2 + 3x_0$ — (1)

(ii) At (x_0, y_0), the slope of the graph equals the slope of the line.

So $f'(x_0) = -\frac{3}{4}$

$\Rightarrow -\frac{k}{x_0^2} = -\frac{3}{4}$

$\Rightarrow k = \frac{3}{4}x_0^2$ — (2)

From (1) & (2): $\frac{3}{4}x_0^2 = -\frac{3}{4}x_0^2 + 3x_0 \Rightarrow \frac{3}{2}x_0^2 - 3x_0 = 0 \Rightarrow 3x_0\left(\frac{x_0}{2} - 1\right) = 0$

$\Rightarrow x_0 = 0$ (rejected: it is not in the domain of f)

or $\boxed{x_0 = 2}$

$\overset{(2)}{\Longrightarrow} k = \frac{3}{4}x_0^2 = \frac{3}{4}(2)^2$

$\Rightarrow \boxed{k = 3}$

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	A $_{2}$	A $_{1}$	D $_{3}$	A $_{3}$
2	A	A $_{1}$	D $_{3}$	D $_{4}$	A $_{2}$
3	A	B $_{4}$	B $_{2}$	D $_{2}$	E $_{1}$
4	A	B $_{3}$	B $_{4}$	D $_{1}$	B $_{4}$
5	A	C $_{5}$	B $_{8}$	C $_{9}$	B $_{9}$
6	A	C $_{9}$	E $_{9}$	B $_{6}$	E $_{6}$
7	A	E $_{7}$	C $_{7}$	B $_{5}$	C $_{7}$
8	A	C $_{6}$	A $_{6}$	D $_{8}$	C $_{8}$
9	A	C $_{8}$	A $_{5}$	E $_{7}$	C $_{5}$
10	A	E $_{11}$	A $_{13}$	D $_{12}$	D $_{11}$
11	A	D $_{12}$	B $_{10}$	C $_{11}$	B $_{13}$
12	A	E $_{13}$	C $_{11}$	A $_{13}$	C $_{12}$
13	A	C $_{10}$	E $_{12}$	E $_{10}$	C $_{10}$
14	A	D $_{14}$	D $_{16}$	E $_{17}$	A $_{15}$
15	A	B $_{18}$	E $_{18}$	C $_{16}$	D $_{14}$
16	A	C $_{16}$	B $_{14}$	E $_{15}$	D $_{17}$
17	A	B $_{15}$	A $_{15}$	E $_{14}$	A $_{16}$
18	A	E $_{17}$	B $_{17}$	B $_{18}$	C $_{18}$