#62
§3.3

1. If $f(x) = \dfrac{\cos x}{1 - \sin x}$, then $f'(x) =$

(a) $\dfrac{1}{1-\sin x}$ ______ (correct)

(b) $\dfrac{-\cos x}{(1-\sin x)^2}$

(c) $\dfrac{1+\cos x}{(1-\sin x)^2}$

(d) $\dfrac{\sin x}{1-\sin x}$

(e) $\dfrac{1-\cos x}{1-\sin x}$

$$f'(x) = \frac{(1-\sin x)(-\sin x) - \cos x\,(0-\cos x)}{(1-\sin x)^2} = \frac{-\sin x + \underbrace{\sin^2 x + \cos^2 x}_{=1}}{(1-\sin x)^2} = \frac{1-\sin x}{(1-\sin x)^2} = \frac{1}{1-\sin x}$$

#73
§3.3

2. If $y = Ax + B$ is an equation of the tangent line to the graph of $f(x) = (x-1)e^x$ at the point $(1, 0)$, then $A - B =$

(a) $2e$ ______ (correct)

(b) $0$

(c) $e + 1$

(d) $-2e$

(e) $e$

$$f'(x) = (x-1)e^x + e^x \cdot 1 = xe^x - e^x + e^x = xe^x$$

slope: $f'(1) = 1e^1 = e$

Eq: $y - 0 = e(x-1)$

$\Rightarrow y = ex - e$

$A = e,\ B = -e$

$\Rightarrow A - B = e - (-e) = e + e = 2e$

~#112
§3.3

3. If $f(t) = t\sin t$, then $f^{(3)}(t) =$

(a) $-t\cos t - 3\sin t$ ____________________(correct)

(b) $t\cos t - \sin t$

(c) $-t\sin t - 3\cos t$

(d) $2t\cos t - \sin t$

(e) $t^3 \sin^3 t$

$f'(t) = t\cos t + \sin t$

$f''(t) = -t\sin t + \cos t + \cos t$

$= -t\sin t + 2\cos t$

$f'''(t) = -t\cos t - \sin t - 2\sin t$

$= -t\cos t - 3\sin t$

Example 4
§3.4

4. The **sum** of the $x$-coordinates of the points at which the graph of $f(x) = (3x - 2x^2)^3$ has a horizontal tangent line is equal to

(a) $\frac{9}{4}$ ____________________(correct)

(b) $\frac{9}{2}$

(c) 3

(d) $\frac{3}{4}$

(e) 2

Horizontal tangent $\Rightarrow f'(x) = 0$

$\Rightarrow 3(3x-2x^2)^2 \cdot (3-4x) = 0$

$\Rightarrow 3x - 2x^2 = 0$ or $3-4x = 0$

$\Rightarrow x(3-2x) = 0$ or $3-4x=0$

$\Rightarrow x = 0, \ x = \frac{3}{2}, \ x = \frac{3}{4}$

Sum $= 0 + \frac{3}{2} + \frac{3}{4}$

$= \frac{6}{4} + \frac{3}{4}$

$= \frac{9}{4}$

#146
§ 3.4

5. If $h(x) = \log_3\left(\frac{x\sqrt{x-1}}{2}\right)$, then $h'(2) =$

(a) $\frac{1}{\ln 3}$ ______________________________ (correct)

(b) $\frac{3}{2\ln 3}$

(c) 0

(d) $-\frac{2}{\ln 3}$

(e) $\frac{2}{\ln 3}$

$$h(x) = \log_3 x + \frac{1}{2}\log_3(x-1) - \log_3 2$$

$$h'(x) = \frac{1}{x \cdot \ln 3} + \frac{1}{2}\cdot\frac{1}{(x-1)\ln 3} - 0$$

$$h'(2) = \frac{1}{2\ln 3} + \frac{1}{2\ln 3}$$

$$= \frac{1}{\ln 3}$$

~ #26, 27
§ 3.4

6. If $g(x) = \frac{x^2}{\sqrt{1-x^2}}$, then $g'(x) =$ $\frac{\sqrt{1-x^2}\cdot 2x - x^2 \cdot \frac{1}{2\sqrt{1-x^2}}\cdot(-2x)}{\left(\sqrt{1-x^2}\right)^2}$

(a) $\frac{2x - x^3}{(1-x^2)^{3/2}}$ ______________________________ (correct)

(b) $\frac{2x - 3x^3}{(1-x^2)^{3/2}}$

(c) $\frac{x}{\sqrt{1-x^2}}$

(d) $\frac{-2x}{\sqrt{1-x^2}}$

(e) $\frac{x}{(1-x^2)^{3/2}}$

$$= \frac{2x\sqrt{1-x^2} + \frac{x^3}{\sqrt{1-x^2}}}{1-x^2}$$

$$= \frac{\frac{2x(1-x^2) + x^3}{\sqrt{1-x^2}}}{1-x^2}$$

$$= \frac{2x - 2x^3 + x^3}{(1-x^2)^{3/2}}$$

$$= \frac{2x - x^3}{(1-x^2)^{3/2}}$$

~ Examples

§3.5

7. If $y$ is defined implicitly as a differentiable function of $x$ by the equation $(x^2+y^2)^3 = 8xy$, then $\dfrac{dy}{dx} =$

$\Rightarrow 3(x^2+y^2)^2 \cdot (2x + 2y \cdot y') = 8xy' + 8y$

(a) $\dfrac{4y - 3x(x^2+y^2)^2}{-4x + 3y(x^2+y^2)^2}$ ______ (correct)

$\Rightarrow 6x(x^2+y^2)^2 + 6y(x^2+y^2)^2 \cdot y' = 8xy' + 8y$

(b) $\dfrac{8y - 3x(x^2+y^2)^2}{-8x + 3y(x^2+y^2)^2}$

(c) $\dfrac{4x - 3y(x^2+y^2)^2}{-4y + 3x(x^2+y^2)^2}$

$\Rightarrow [-8x + 6y(x^2+y^2)^2]\, y' = 8y - 6x(x^2+y^2)^2$

(d) $\dfrac{4y + 3x(x^2+y^2)^2}{-4x + 3y(x^2+y^2)^2}$

$\Rightarrow y' = \dfrac{8y - 6x(x^2+y^2)^2}{-8x + 6y(x^2+y^2)^2}$

(e) $\dfrac{4y - 3x(x^2+y^2)}{-4x - 3y(x^2+y^2)}$

$= \dfrac{4y - 3x(x^2+y^2)^2}{-4x + 3y(x^2+y^2)^2}$

~ #80

§3.5

8. If $y = (\sin x)^{\ln(\cos x)}$, $0 < x < \dfrac{\pi}{2}$, then $y' =$

(a) $(\sin x)^{\ln(\cos x)}\,[\cot x \cdot \ln(\cos x) - \tan x \cdot \ln(\sin x)]$ ______ (correct)

(b) $(\sin x)^{\ln(\cos x)}\left[\dfrac{\ln(\cos x)}{\sin x} + \dfrac{\ln(\sin x)}{\cos x}\right]$

(c) $(\sin x)^{\ln(\cos x)}\,[\tan x \cdot \ln(\cos x) + \cot x \cdot \ln(\sin x)]$

(d) $(\sin x)^{\ln(\cos x)}[\ln(\cos x) - \ln(\sin x)]$

(e) $\ln(\cos x) \cdot (\sin x)^{\ln(\cos x) - 1} \cdot \cos x$

$\ln y = \ln(\cos x) \cdot \ln(\sin x)$

$\dfrac{1}{y} \cdot y' = \ln(\cos x) \cdot \dfrac{\cos x}{\sin x} + \ln(\sin x) \cdot \dfrac{-\sin x}{\cos x}$

$\Rightarrow y' = y\,[\cot x \cdot \ln(\cos x) - \tan x \cdot \ln(\sin x)]$

#38
§3.6

9. The **slope** of the tangent line to the graph of

$$f(x) = x\arctan(2x) - \frac{1}{4}\ln(1+4x^2)$$

at the point with $x$-coordinate $\frac{\sqrt{3}}{2}$ is equal to

(a) $\frac{\pi}{3}$ ______________________________ (correct)

(b) $\frac{\pi}{6}$

(c) $\frac{\pi}{4}$

(d) $\frac{2\pi}{3}$

(e) $\frac{\pi}{2}$

$$f'(x) = x \cdot \frac{1}{1+(2x)^2} \cdot 2 + \arctan(2x) \cdot 1 - \frac{1}{4} \cdot \frac{1}{1+4x^2} \cdot 8x$$

$$= \frac{2x}{1+4x^2} + \arctan(2x) - \frac{2x}{1+4x^2}$$

$$= \arctan(2x)$$

$$\text{Slope} = f'\left(\tfrac{\sqrt{3}}{2}\right) = \arctan\left(2 \cdot \tfrac{\sqrt{3}}{2}\right)$$

$$= \arctan(\sqrt{3}) = \frac{\pi}{3}$$

~ #153, 154
§ 3.4

10. Let $h(4) = 3$, $h'(4) = -\frac{1}{4}$, $g(3) = 3$, $g'(3) = -4$.
If $f(x) = [g(h(x^2))]^2$, then $f'(2) =$

(a) 24 ______________________________ (correct)

(b) $-24$

(c) $-6$

(d) 12

(e) $-8$

$$f'(x) = 2[g(h(x^2))] \cdot g'(h(x^2)) \cdot h'(x^2) \cdot 2x$$

$$f'(2) = 2[g(h(4))] \cdot g'(h(4)) \cdot h'(4) \cdot 4$$

$$= 2 \cdot g(3) \cdot g'(3) \cdot h'(4) \cdot 4$$

$$= 2 \cdot 3 \cdot -4 \cdot -\frac{1}{4} \cdot 4$$

$$= 24$$

#105, p. 202, Rev. Ex. for Chapter 3

11. If $y = Ax + B$ is an equation of the **normal line** to the graph of the equation $y \ln x + y^2 = 0$ at the point $(e, -1)$, then $A^2 + B =$

$\Rightarrow y \cdot \frac{1}{x} + \ln x \cdot y' + 2y \cdot y' = 0$

(a) $-1$ ________________________________(correct)

(b) 1

(c) 0

(d) $-e$

(e) $e$

$\xrightarrow[y=-1]{x=e} -1 \cdot \frac{1}{e} + 1 \cdot y' - 2y' = 0$

$\Rightarrow -\frac{1}{e} - y' = 0 \Rightarrow y' = -\frac{1}{e}$

$\Rightarrow$ Slope of the normal line is $e$.

Eq. of the normal line is

$y + 1 = e(x - e)$

$\Rightarrow y = ex - e^2 - 1$

$A = e, \quad B = -e^2 - 1$

$\Rightarrow A^2 + B = e^2 - e^2 - 1 = -1.$

~ #12
§3.7

12. If the length $s$ of each side of an equilateral triangle is increasing at a rate of $5\,m/min$, then the rate at which the area $A$ of the triangle changes when $s = 4m$ is equal to $\left(A = \frac{\sqrt{3}}{4}s^2\right)$

$\frac{ds}{dt} = 5$ m/min

$\frac{dA}{dt} = ?$ when $s = 4$ m.

(a) $10\sqrt{3}\ m^2/\min$ ________________________________(correct)

(b) $5\sqrt{3}\ m^2/\min$

(c) $\frac{\sqrt{3}}{5}\ m^2/\min$

(d) $\frac{5\sqrt{3}}{2}\ m^2/\min$

(e) $4\sqrt{3}\ m^2/\min$

$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \cdot 2s \frac{ds}{dt}$

$= \frac{\sqrt{3}}{2} \cdot 4 \cdot 5$

$= 10\sqrt{3}$ m²/min

~ #29

§ 3.7

13. A man 2 meters tall walks at a rate of 2.5 meters per second away from a light that is 7 meters above the ground. At what rate is the **tip of his shadow** moving when he is 5 meters from the base of the light?

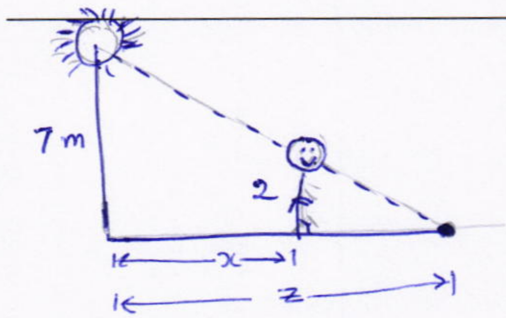
(a) 3.5 meter per second ____(correct)

(b) 0.5 meter per second

(c) 0.7 meter per second

(d) 2.7 meter per second

(e) 2.5 meter per second




$\frac{dx}{dt} = 2.5$ m/s

$\frac{dz}{dt} = ?$ when $x = 5$

$$\frac{z}{7} = \frac{z-x}{2} \Rightarrow 2z = 7z - 7x \Rightarrow 7x = 5z \Rightarrow z = \frac{7}{5}x$$

$$\Rightarrow \frac{dz}{dt} = \frac{7}{5}\frac{dx}{dt}$$

$$= \frac{7}{5}\cdot(2.5) = \frac{7}{5}\cdot\frac{25}{10} = \frac{7}{1}\cdot\frac{5}{10} = \frac{35}{10} = 3.5 \text{ m/s}$$

~ # 9

§ 3.8

14. **Newton's Method** is used to approximate a zero of the function $f(x) = x^3 + x - 3$. If we choose $x_1 = 1$, then $x_2 =$

(a) $\frac{5}{4}$ ____(correct)

(b) $\frac{3}{4}$

(c) $\frac{1}{2}$

(d) $\frac{1}{4}$

(e) $\frac{3}{2}$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1 - \frac{f(1)}{f'(1)}$$

$$= 1 - \frac{-1}{4}$$

$$= 1 + \frac{1}{4}$$

$$= \frac{5}{4}$$

. $f(1) = 1 + 1 - 3 = -1$

. $f'(x) = 3x^2 + 1$

$f'(1) = 3 + 1 = 4$

~ #17

§4.1 15. The **sum** of the critical numbers of

Domain of f is $(-\infty, \infty)$

$$f(x) = x\sqrt[3]{4-x}$$

is equal to

$f'(x) = x \cdot \frac{1}{3}(4-x)^{-2/3} \cdot (-1) + \sqrt[3]{4-x} \cdot 1$

$= (4-x)^{-2/3}\left[-\frac{x}{3} + (4-x)\right]$

(a) 7 ______________________________ (correct)

(b) 3

(c) $\frac{1}{3}$

(d) $\frac{13}{3}$

(e) 0

$= (4-x)^{-2/3}\left[-\frac{4}{3}x + 4\right]$

$= \dfrac{-\frac{4}{3}x+4}{(4-x)^{2/3}}$

C.# : $-\frac{4}{3}x + 4 = 0$ , $4-x=0$

$\Rightarrow$ $x = 3$ , $x = 4$ both in domain of f

Sum $= 3+4 = 7$

#33

§4.1 16. On the closed interval $[-2, 1]$, if

$$f(x) = \frac{6x^2}{x-2}$$

has an **absolute maximum** at the point $(a, b)$, then $b =$

(a) 0 ______________________________ (correct)

(b) −6

(c) 48

(d) −1

(e) 10

• $f'(x) = 6 \cdot \dfrac{(x-2)\cdot 2x - x^2 \cdot 1}{(x-2)^2}$

$= 6 \cdot \dfrac{2x^2 - 4x - x^2}{(x-2)^2} = 6 \cdot \dfrac{x^2-4x}{(x-2)^2} = 6 \cdot \dfrac{x(x-4)}{(x-2)^2}$

Cr.# : $x = 0$, $\underbrace{x = 2, x = 4}_{\text{rejected as they are not in } [-2,1)}$

$f(0) = 0$ ⟵ max at $(0, 0)$

$\Rightarrow b = 0$

$f(-2) = \frac{24}{-4} = -6$

$f(1) = \frac{6}{-1} = -6$

~ #6, 16, 27

§4.2 17. Which one of the following functions can **Rolle's Theorem** be applied to on the interval $[-1, 1]$.

(a) $f(x) = 2 + \sin(\pi x)$ ______________________________ (correct)

(b) $f(x) = |x| - 1 \to$ not diff. at $0 \in (-1, 1)$; a corner at $x=0$

(c) $f(x) = |x - 2| \to f(-1) \neq f(1)$

(d) $f(x) = \dfrac{1}{x^4 + x^2} \to$ not conts at $0 \in [-1, 1]$

(e) $f(x) = 2 - x^{2/3} \to f'(x) = -\frac{2}{3}\frac{1}{\sqrt[3]{x}}$, not diff at $0 \in (-1, 1)$

#48

§4.2 18. The value of $c$ that satisfies the **Mean Value Theorem** when applied to $f(x) = \dfrac{x}{x - 5}$ on $[1, 4]$ is equal to

(a) 3 ______________________________ (correct)

(b) 2

(c) 1

(d) $\frac{1}{2}$

(e) $\frac{1}{3}$

$$f'(x) = \frac{(x-5)\cdot 1 - x \cdot 1}{(x-5)^2} = \frac{-5}{(x-5)^2}$$

$$f'(c) = \frac{f(4) - f(1)}{4 - 1} \quad ; \quad f(4) = \frac{4}{4-5} = -4$$

$$f(1) = \frac{1}{1-5} = -\frac{1}{4}$$

$$\frac{-5}{(c-5)^2} = \frac{-4 + \frac{1}{4}}{3}$$

$$\frac{-5}{(c-5)^2} = \frac{\frac{-15}{4}}{3} = -\frac{15}{12} = -\frac{5}{4}$$

$$\Rightarrow (c-5)^2 = 4 \Rightarrow c - 5 = \pm 2$$

$$\Rightarrow c = 5 \pm 2$$

$$\Rightarrow c = 3 \text{ or } c = 7 \notin (1, 4)$$

$$\Rightarrow c = 3 \in (1, 4)$$

**Answer KEY**

| Q | MASTER | CODE01 | CODE02 | CODE03 | CODE04 |
|---|---|---|---|---|---|
| 1 | A | A $_{2}$ | A $_{4}$ | C $_{1}$ | B $_{1}$ |
| 2 | A | E $_{5}$ | D $_{1}$ | E $_{6}$ | A $_{2}$ |
| 3 | A | D $_{4}$ | D $_{5}$ | E $_{3}$ | C $_{5}$ |
| 4 | A | A $_{6}$ | B $_{3}$ | E $_{4}$ | C $_{6}$ |
| 5 | A | D $_{1}$ | C $_{6}$ | A $_{2}$ | B $_{3}$ |
| 6 | A | A $_{3}$ | C $_{2}$ | D $_{5}$ | B $_{4}$ |
| 7 | A | B $_{12}$ | D $_{8}$ | A $_{8}$ | C $_{12}$ |
| 8 | A | C $_{7}$ | B $_{9}$ | D $_{11}$ | C $_{7}$ |
| 9 | A | A $_{10}$ | C $_{12}$ | C $_{12}$ | E $_{10}$ |
| 10 | A | B $_{8}$ | D $_{7}$ | E $_{10}$ | A $_{8}$ |
| 11 | A | D $_{11}$ | E $_{10}$ | B $_{7}$ | D $_{9}$ |
| 12 | A | B $_{9}$ | D $_{11}$ | D $_{9}$ | B $_{11}$ |
| 13 | A | A $_{15}$ | A $_{16}$ | D $_{18}$ | A $_{14}$ |
| 14 | A | C $_{17}$ | B $_{15}$ | A $_{15}$ | A $_{16}$ |
| 15 | A | B $_{16}$ | B $_{17}$ | E $_{17}$ | E $_{18}$ |
| 16 | A | A $_{18}$ | B $_{18}$ | E $_{13}$ | A $_{13}$ |
| 17 | A | B $_{14}$ | B $_{14}$ | A $_{16}$ | C $_{15}$ |
| 18 | A | C $_{13}$ | C $_{13}$ | E $_{14}$ | B $_{17}$ |