King Fahd University of Petroleum and Minerals
Department of Mathematics
Math 101
Final Exam
233
August 15, 2024
Net Time Allowed: 120 Minutes

MASTER VERSION

1. The function $f(x) = x + \dfrac{9}{x}$ is decreasing on the interval

    (a) $(-3, 0)$ ————————————————————————(correct)

    (b) $(-9, 9)$

    (c) $(3, \infty)$

    (d) $(-\infty, -3)$

    (e) $(-9, 0)$

2. If the function $f(x) = (x-1)^2(x+3)$ has a relative maximum at the point (a, b), then $a =$

    (a) $\dfrac{-5}{3}$ ————————————————————————(correct)

    (b) $\dfrac{5}{3}$

    (c) $\dfrac{-3}{5}$

    (d) $\dfrac{3}{5}$

    (e) -1

3. The graph of the function $f(x) = \dfrac{2x^2}{3x^2 + 1}$ is concave upward on the interval

(a) $\left(-\dfrac{1}{3}, \dfrac{1}{3}\right)$ ——————————————————————(correct)

(b) $\left(-\infty, -\dfrac{1}{3}\right)$

(c) $\left(\dfrac{1}{3}, \infty\right)$

(d) $(-1, 1)$

(e) $\left(-\dfrac{1}{3}, \infty\right)$

4. The number of inflection points for the graph of the function $f(x) = x + 2\cos x$ over $[0, 2\pi]$ is

(a) 2 ——————————————————————(correct)

(b) 3

(c) 1

(d) 0

(e) 4

5. If $y = Ax + B$ is the slant asymptote for the graph of the function $f(x) = \dfrac{-x^2 - 4x - 7}{x + 3}$, then $A + B =$

(a) -2 ————————————————————————————(correct)

(b) 2

(c) -1

(d) 1

(e) 0

6. Let x and y be two positive numbers such that their product is 147 and the sum of the first number plus three times the second number is a minimum. Then $x + y =$

(a) 28 ————————————————————————————(correct)

(b) 26

(c) 30

(d) 32

(e) 34

7. If $L(x)$ is the tangent line approximation to the graph of $f(x) = \log_2 x$ at the point $(2, 1)$, then $L(1) =$

(a) $1 - \dfrac{1}{2 \ln 2}$ ————————————————————————————————(correct)

(b) $2 - \dfrac{1}{\ln 2}$

(c) 0

(d) $1 + \dfrac{1}{\ln 2}$

(e) $2 + \dfrac{1}{2 \ln 2}$

8. The radius of a spherical ballon is measured as 8 inches, with a possible error of 0.02 inch. The percent error, approximately, in computing the volume of the sphere is, (Note, the volume of the sphere is $V = \dfrac{4}{3}\pi r^3$).

(a) $\dfrac{3}{4}\%$ ————————————————————————————————(correct)

(b) $\dfrac{1}{2}\%$

(c) $\dfrac{1}{4}\%$

(d) 1%

(e) 2%

9. If $f''(x) = \dfrac{2}{x^2}$, $f'(1) = 4$, and $f(1) = 3$, then $f(-1) =$

    (a) -9 ————————————————————————————————(correct)

    (b) 9

    (c) -6

    (d) 6

    (e) -11

10. If $\lim\limits_{x \to 0} \dfrac{a - \cos bx}{x^2} = 2$, where $b > 0$, then $a + b =$

    (a) 3 ————————————————————————————————(correct)

    (b) 2

    (c) 1

    (d) 0

    (e) 4

11. $\lim\limits_{x \to 0^+} (e^x + x)^{2/x} =$

    (a) e^4 ———————————————————————————(correct)

    (b) e^3

    (c) e^2

    (d) e

    (e) 1

12. If $f(x) = \arctan(\sinh x)$, then $f'(\ln 2) =$

    (a) $\dfrac{4}{5}$ ———————————————————————————(correct)

    (b) $\dfrac{5}{4}$

    (c) $\dfrac{3}{4}$

    (d) $\dfrac{4}{3}$

    (e) 2

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13. Which one of the following functions is continuous on the entire real number line?

(a) $f(x) = \begin{cases} x \sin \dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ ————————————————(correct)

(b) $f(x) = \begin{cases} \sin \dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

(c) $f(x) = \begin{cases} x^2 \sin \dfrac{1}{x}, & x \neq 0 \\ 1, & x = 0 \end{cases}$

(d) $f(x) = \tan x$

(e) $f(x) = [\![x]\!]$

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14. If $x \sin y = y \cos x$, defines y as an implicit function of x, then $\dfrac{dy}{dx} =$

(a) $\dfrac{y \sin x + \sin y}{\cos x - x \cos y}$ ————————————————(correct)

(b) $\dfrac{y \sin x - \sin y}{\cos x + x \cos y}$

(c) $\dfrac{y \cos x + \cos y}{\sin x - x \sin y}$

(d) $\dfrac{y \cos x - \cos y}{\sin x + x \sin y}$

(e) $\dfrac{y \sin x}{x \cos y}$

15. If $y = \dfrac{(2x+1)^3(x^2-1)^2}{x+3}$, then $\dfrac{dy}{dx}$ at $x=0$ equals

(a) $\dfrac{17}{9}$ ————————————————————————————————(correct)

(b) $\dfrac{-17}{9}$

(c) $\dfrac{9}{17}$

(d) $\dfrac{-9}{17}$

(e) 1

16. The slope of the graph of the function $f(x) = \dfrac{3x+1}{(4x-3)^3}$ at the point $(1,4)$ is

(a) -45 ————————————————————————————————(correct)

(b) 45

(c) -54

(d) 54

(e) 55

17. $\lim\limits_{x \to \infty} \left(\dfrac{-4}{3 + 3e^{2x}} + \dfrac{6}{5 + 2e^{-4x}} \right)$

    (a) $\dfrac{6}{5}$ ──────────────────────────────(correct)

    (b) $\dfrac{-4}{3}$

    (c) $\dfrac{-12}{5}$

    (d) $\dfrac{2}{15}$

    (e) 0

18. $\lim\limits_{x \to 0^+} \left(\ln(\sin x) - \dfrac{\cos^2 x}{x} \right) =$

    (a) $-\infty$ ──────────────────────────(correct)

    (b) ∞

    (c) 0

    (d) 1

    (e) -1

19. If $f(x) = x \sin^2 x$, then $f''\left(\dfrac{\pi}{4}\right) =$

    (a) 2 ————————————————————————————————(correct)

    (b) 2π

    (c) $2 + \pi$

    (d) $2 - \pi$

    (e) π

20. The equation of the normal line to the graph of the equation $\ln(x + y) = x$ at the point $(0, 1)$ is

    (a) $x = 0$ ————————————————————————————(correct)

    (b) $y = 1$

    (c) $x = 1$

    (d) $y = 0$

    (e) $y = x + 1$