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44
p.166
Review/ch2

1. If $g(x) = \begin{cases} \sqrt{1-x}, & x \le 1 \\ x+1, & x > 1 \end{cases}$ then $\lim_{x \to 1} g(x)$

(a) does not exist ________________ (correct)

(b) is equal to 0

(c) is equal to 2

(d) is equal to 1

(e) is equal to $\frac{1}{2}$

$\lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} \sqrt{1-x} = \sqrt{1-1} = 0$

$\lim_{x \to 1^+} g(x) = \lim_{x \to 1^+} (x+1) = 1+1 = 2$

Since $\lim_{x \to 1^-} g(x) \neq \lim_{x \to 1^+} g(x)$, then

$\lim_{x \to 1} g(x)$ dos not exist.

~ #37/2.5

2. $\lim_{x \to 5^-} \frac{x}{x-5} =$

$\longrightarrow \frac{5}{0}$; the sign of $\frac{x}{x-5}$ on the left of $x=5$ is negative

(a) $-\infty$ ________________ (correct)

(b) ∞

(c) 5

(d) $\frac{1}{5}$

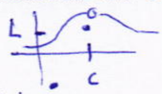
(e) 1

So $\lim_{x \to 5^-} \frac{x}{x-5} = -\infty$

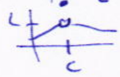
3. Which one of the following statements is **TRUE**? (c and L are real numbers)

(a) If $\lim_{x\to c} f(x) = f(c)$, then f is continuous at c. Definition ______ (correct)

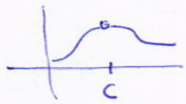
(b) If $f(c) = L$, then $\lim_{x\to c} f(x) = L$



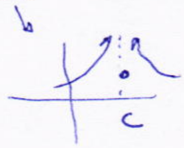
(c) If $\lim_{x\to c} f(x) = L$, then $f(c) = L$



(d) If $f(c)$ is undefined, then $\lim_{x\to c} f(x)$ does not exist.



(e) If $\lim_{x\to c} f(x) = \infty$, then $f(c)$ is undefined.



~#37 & Example 7

§4.5 4. $\lim_{x\to -\infty} (5e^x - 3x^3) =$ $5(0) - 3(-\infty) = 0 + \infty = \infty$

(a) ∞ ______ (correct)

(b) $-\infty$

(c) 2

(d) 0

(e) -2

~ #46
§2.3

5. $\lim_{x\to 0} \dfrac{e^{3x}-1}{e^x-1} =$ $\lim_{x\to 0} \dfrac{(e^x-1)(e^{2x}+e^x+1)}{e^x-1} = \lim_{x\to 0} (e^{2x}+e^x+1) = 1+1+1 = 3$

(a) 3 ——— (correct)

(b) 2

(c) 0

(d) 1

(e) ∞

~ #53 & #78
§2.3

6. $\lim_{x\to 4} \dfrac{x-2\sqrt{x}}{x^2-3x-4} =$ $\lim_{x\to 4} \dfrac{x-2\sqrt{x}}{x^2-3x-4} \cdot \dfrac{x+2\sqrt{x}}{x+2\sqrt{x}}$

$= \lim_{x\to 4} \dfrac{x^2-4x}{x^2-3x-4} \cdot \dfrac{1}{x+2\sqrt{x}}$

$= \lim_{x\to 4} \dfrac{x(x-4)}{(x+1)(x-4)} \cdot \dfrac{1}{x+2\sqrt{x}}$

$= \lim_{x\to 4} \dfrac{x}{x+1} \cdot \dfrac{1}{x+2\sqrt{x}}$

$= \dfrac{4}{5} \cdot \dfrac{1}{4+4} = \dfrac{1}{10}$

(a) $\dfrac{1}{10}$ ——— (correct)

(b) $\dfrac{1}{20}$

(c) $\dfrac{1}{5}$

(d) $\dfrac{4}{5}$

(e) $\dfrac{5}{8}$

~#75,78
§2.4

7. Find the interval(s) on which $f(x) = \dfrac{\sqrt{x+3}}{x^2+4x}$ is continuous?

(a) $[-3,0) \cup (0,\infty)$ ______(correct)

(b) $(-4,-3) \cup (-3,0) \cup (0,\infty)$

(c) $(-\infty,-4) \cup (0,\infty)$

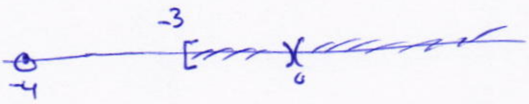
(d) $(-\infty,-4) \cup [-3,\infty)$

(e) $(-4,-3]$

• $\sqrt{x+3}$, $x+3 \geq 0 \Rightarrow x \geq -3$

• x^2+4x : $x^2+4x \neq 0 \Rightarrow x(x+4) \neq 0$

$\Rightarrow x \neq 0, x = -4$



$[-3,0) \cup (0,\infty)$

~#64
§2.4

8. If the function $f(x) = \begin{cases} \dfrac{\tan(3x)}{x}, & x < 0 \\ 6a - x^2, & x \geq 0 \end{cases}$ is continuous at $x = 0$, then $a =$

(a) $\dfrac{1}{2}$ ______(correct)

(b) 0

(c) 2

(d) -1

(e) $\dfrac{1}{3}$

$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

$\lim_{x\to 0^-} \dfrac{\tan(3x)}{x} = \lim_{x\to 0^+} 6a - x^2$

$3 = 6a - 0$

$\Rightarrow a = \dfrac{1}{2}$

$\lim_{x\to 0^-} \dfrac{\sin(3x)}{x \cdot \cos(3x)} = \lim_{x\to 0^-} 3 \cdot \dfrac{\sin(3x)}{3x} \cdot \dfrac{1}{\cos(3x)} = 3 \cdot 1 \cdot \dfrac{1}{1} = 3$

~#47

§2.4

9. Which one of the following statements is **TRUE** about $f(x) = \dfrac{x^3 + x}{x^2 + x}$?

(a) f has a nonremovable discontinuity at $x = -1$. ____________ (correct)

(b) f has a nonremovable discontinuity at $x = 0$.

(c) f has a nonremovable discontinuity at $x = 1$.

(d) f has a removable discontinuity at $x = -1$.

(e) f has a removable discontinuity at $x = 1$.

$$f(x) = \frac{x(x^2+1)}{x(x+1)} = \frac{x^2+1}{x+1}, \quad x \neq 0 \rightarrow x = 0 \text{ is a removable discontinuity}$$

$\rightarrow x+1 = 0 \Rightarrow x = -1$ a nonremovable discontinuity

~#30

§4.5

10. $\displaystyle\lim_{x\to\infty} \frac{\sqrt{x^4 - 1}}{x^3 - 1} = \lim_{x\to\infty} \frac{\sqrt{x^4(1-\frac{1}{x^4})}}{x^3(1-\frac{1}{x^3})} = \lim_{x\to\infty} \frac{x^2\sqrt{1-\frac{1}{x^4}}}{x^3(1-\frac{1}{x^3})}$

(a) 0 ____________ (correct)

(b) 1

(c) ∞

(d) $-\infty$

(e) -1

$$= \lim_{x\to\infty} \frac{1}{x} \cdot \frac{\sqrt{1-\frac{1}{x^4}}}{1-\frac{1}{x^3}}$$

$$= 0 \cdot \frac{\sqrt{1-0}}{1-0}$$

$$= 0 \cdot 1$$

$$= 0$$

11. $\lim_{x\to\frac{1}{2}}([\![2x]\!]+[\![-2x]\!])$

~ #27, #28

§2.4

(a) is equal to -1 ______________________________ (correct)

(b) is equal to 1

(c) is equal to 0

(d) is equal to 2

(e) does not exist

$\cdot \lim_{x\to\frac{1}{2}^-}([\![2x]\!]+[\![-2x]\!]) = 0+(-1) = -1$

$\cdot \lim_{x\to\frac{1}{2}^+}([\![2x]\!]+[\![-2x]\!]) = 1+(-2) = -1$ $] =$

$\Rightarrow \lim_{x\to\frac{1}{2}}([\![2x]\!]+[\![-2x]\!]) = -1$

~ #77, p.116

Review/Ch2

12. The graph of $f(x)=\dfrac{x^4}{x^3-9x}$ has (V.A: vertical asymptote(s); H.A: horizontal asymptote(s))

(a) 2 V. A. and no H.A. ______________________________ (correct)

(b) 2 V. A. and 1 H. A.

(c) 3 V. A. and no H. A.

(d) 3 V. A. and 1 H. A.

(e) no V. A. and 1 H. A.

$f(x)=\dfrac{x^4}{x(x^2-9)}$

$=\dfrac{x^3}{(x-3)(x+3)}, \ x\neq 0$

$\cdot$ Deno $=0 \Rightarrow x=\pm 3 \Rightarrow 2$ V.A.

$\cdot \lim_{x\to+\infty} f(x)=\infty \ ; \ \lim_{x\to-\infty} f(x)=-\infty \Rightarrow$ no H.A.

~#44
#51
§3.2

13. If $y = Ax + B$ is an equation of the **tangent line** to the graph of $f(x) = 8\sqrt{x} + \frac{3}{x^2}$ at the point $(1, 11)$, then $A - B =$

$f(x) = 8\sqrt{x} + 3x^{-2}$

(a) -15 (correct)

$f'(x) = 8 \cdot \frac{1}{2\sqrt{x}} + 3(-2)x^{-3}$

(b) -11

(c) 0

slope $= f'(1) = 4 - 6 = -2$

(d) -9

Eq: $y - 11 = -2(x-1)$

(e) 4

$\Rightarrow y = -2x + 2 + 11$

$\Rightarrow y = -2x + 13$

$A = -2, \ B = 13$

$A - B = -2 - 13 = -15$

~#72
§3.2

14. If the line $y = 3x + 1$ is **tangent** to the graph of $f(x) = kx^2$, then

point of tangency is (x_0, y_0)

(a) $k = -\frac{9}{4}$ (correct)

- (x_0, y_0) is on both graphs:

$3x_0 + 1 = kx_0^2$ — (1)

(b) $k = -\frac{3}{2}$

(c) $k = \frac{5}{4}$

- slope is the same at (x_0, y_0):

$\underbrace{3 = 2kx_0}_{(2)} \quad (\Rightarrow k \neq 0 \ \& \ x_0 \neq 0)$

(d) $k = \frac{9}{2}$

(e) $k = \frac{3}{2}$

$(2) \Rightarrow k = \frac{3}{2x_0}$

$\overset{(1)}{\Longrightarrow} 3x_0 + 1 = \frac{3}{2x_0} x_0^2$

$\Rightarrow 3x_0 + 1 = \frac{3}{2} x_0$

$\Rightarrow \frac{3}{2} x_0 = -1$

$\Rightarrow x_0 = -\frac{2}{3}$

$\Rightarrow k = \frac{3}{2x_0} = \frac{3}{2\left(-\frac{2}{3}\right)} = -\frac{9}{4}$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	A $_{2}$	E $_{1}$	B $_{3}$	B $_{4}$
2	A	C $_{3}$	B $_{4}$	B $_{1}$	D $_{1}$
3	A	C $_{4}$	B $_{2}$	E $_{2}$	B $_{2}$
4	A	A $_{1}$	D $_{3}$	D $_{4}$	A $_{3}$
5	A	A $_{7}$	E $_{5}$	A $_{7}$	A $_{6}$
6	A	D $_{9}$	B $_{6}$	D $_{5}$	C $_{7}$
7	A	C $_{8}$	A $_{8}$	C $_{8}$	A $_{9}$
8	A	B $_{5}$	C $_{9}$	A $_{6}$	D $_{8}$
9	A	B $_{6}$	C $_{7}$	C $_{9}$	C $_{5}$
10	A	D $_{13}$	E $_{14}$	E $_{10}$	E $_{12}$
11	A	B $_{11}$	A $_{12}$	C $_{14}$	E $_{11}$
12	A	A $_{12}$	E $_{10}$	A $_{11}$	A $_{10}$
13	A	C $_{10}$	B $_{11}$	E $_{13}$	E $_{14}$
14	A	D $_{14}$	A $_{13}$	A $_{12}$	D $_{13}$