Code-wise key in last page

1. If $y = Ax + B$ is an equation of the **tangent line** to the graph of $f(x) = 1 + x\cos x$ at the point $(0, 1)$, then $AB =$

~#21
§3.3

(a) 1 (correct)
(b) -1
(c) 0
(d) 2
(e) $-\frac{1}{2}$

$f'(x) = 0 + x(-\sin x) + \cos x \cdot 1$

slope $= f'(0) = 0 + 1 = 1$

Eq: $y - 1 = 1(x - 0)$

$\Rightarrow y = x + 1$

$A = 1,\ B = 1 \Rightarrow AB = 1$

2. The **slope** of the tangent line to the graph of $f(x) = \dfrac{4x}{x^2+6}$ at the point $\left(2, \dfrac{4}{5}\right)$ is equal to

#78
§3.3

(a) $\frac{2}{25}$ (correct)
(b) $\frac{4}{25}$
(c) $\frac{1}{25}$
(d) 0
(e) $\frac{3}{25}$

$f'(x) = \dfrac{(x^2+6)\cdot 4 - 4x(2x)}{(x^2+6)^2}$

$= \dfrac{24 - 4x^2}{(x^2+6)^2}$

slope $= f'(2) = \dfrac{24-16}{(10)^2} = \dfrac{8}{100} = \dfrac{2}{25}$

~#26
§3.4

3. If $y = x^2\sqrt{4-x^2}$, then $\dfrac{dy}{dx} =$

(a) $\dfrac{8x-3x^3}{\sqrt{4-x^2}}$ ________ (correct)

(b) $\dfrac{8x+x^3}{\sqrt{4-x^2}}$

(c) $\dfrac{2x-x^3}{\sqrt{4-x^2}}$

(d) $\dfrac{4x+3x^3}{\sqrt{4-x^2}}$

(e) $\dfrac{-x^3}{\sqrt{4-x^2}}$

$$\frac{dy}{dx} = x^2 \cdot \frac{1}{2}(4-x^2)^{-1/2} \cdot (-2x) + \sqrt{4-x^2} \cdot 2x$$

$$= (4-x^2)^{-1/2}\left[-x^3 + (4-x^2)\cdot 2x\right]$$

$$= \frac{-x^3+8x-2x^3}{(4-x^2)^{1/2}}$$

$$= \frac{8x-3x^3}{\sqrt{4-x^2}}$$

#~~2~~3
§3.5

4. If $x^2 + y^2 - 3\ln y = 10$, then, by implicit differentiation, $\dfrac{dy}{dx} =$

(a) $\dfrac{2xy}{3-2y^2}$ ________ (correct)

(b) $\dfrac{2xy}{3-2y}$

(c) $\dfrac{2x}{3-y^2}$

(d) $\dfrac{xy}{1-2y^2}$

(e) $\dfrac{3xy}{2-3y^2}$

$$2x + 2y \cdot y' - 3 \cdot \frac{1}{y} \cdot y' = 0$$

$$\Rightarrow \quad 2y \cdot y' - \frac{3}{y} \cdot y' = -2x$$

$$\Rightarrow \quad y'\left[2y - \frac{3}{y}\right] = -2x$$

$$\Rightarrow \quad y'\left[\frac{2y^2-3}{y}\right] = -2x$$

$$\Rightarrow \quad y' = \frac{-2xy}{2y^2-3}$$

$$= \frac{2xy}{3-2y^2}$$

#12
§ 3.8

5. Using **Newton's Method** to approximate the zero of the function $f(x) = x - 2\sqrt{x+1}$, starting with $x_1 = 3$, we get $x_2 =$

(a) 5 ______________________________ (correct)

(b) 4

(c) $\frac{7}{2}$

(d) $\frac{5}{2}$

(e) $\frac{9}{2}$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 3 - \frac{f(3)}{f'(3)}$

$= 3 - \frac{-1}{\frac{1}{2}}$

$= 3 + 2$

$= 5$

: $f(3) = 3 - 2\sqrt{3+1} = 3 - 2 \cdot 2 = 3 - 4 = -1$

. $f'(x) = 1 - \frac{1}{\sqrt{x+1}}$

. $f'(3) = 1 - \frac{1}{\sqrt{3+1}} = 1 - \frac{1}{2} = \frac{1}{2}$

~#153, #154
§ 3.4

6. Let f be a differentiable function and

$$g(x) = f(x^3 - 3\sin(\pi x)),\ f(1) = 3,\ f'(1) = \frac{1}{3}.$$

Then $g'(1) =$

(a) $\pi + 1$ ______________________________ (correct)

(b) $3\pi + 3$

(c) π

(d) $3 - 3\pi$

(e) $1 - 3\pi$

$g'(x) = f'(x^3 - 3\sin(\pi x)) \cdot [3x^2 - 3\cos(\pi x) \cdot \pi]$

$g'(1) = f'(1 - 0) \cdot [3 + 3\pi]$

$= f'(1) \cdot (3 + 3\pi)$

$= \frac{1}{3} \cdot (3 + 3\pi)$

$= 1 + \pi$

~#29
§4.1

7. If m and M are respectively the absolute minimum and absolute maximum values of the function $f(x) = x^3 - \frac{3}{2}x^2 + 1$ on the interval $[-1, 2]$, then $m + M =$

$f'(x) = 3x^2 - 3x$
$= 3x(x-1) = 0 \Rightarrow x = 0, 1 \in (-1, 2)$

(a) $\frac{3}{2}$ ________________ (correct)

(b) -1

(c) $-\frac{1}{2}$

(d) $\frac{7}{2}$

(e) 0

$f(0) = 0 - 0 + 1 = 1$

$f(1) = 1 - \frac{3}{2} + 1 = \frac{1}{2}$

$f(-1) = -1 - \frac{3}{2} + 1 = -\frac{3}{2} \leftarrow m$

$f(2) = 8 - 6 + 1 = 3 \leftarrow M$

$m + M = -\frac{3}{2} + 3 = \frac{-3+6}{2} = \frac{3}{2}$

~Example 3
§3.7

8. Air is being pumped into a spherical balloon at a rate of $32\pi\, cm^3/min$. When the volume of the balloon is $\frac{32\pi}{3} cm^3$, the radius of the balloon is changing at a rate of (Hint: The volume of the sphere is $V = \frac{4}{3}\pi r^3$)

$\frac{dV}{dt} = 32\pi$, $\frac{dr}{dt} = ?$ when $V = \frac{32\pi}{3}$

(a) $2\ cm/min$ ________________ (correct)

(b) $4\ cm/min$

(c) $3\ cm/min$

(d) $\frac{3}{4}\ cm/min$

(e) $\frac{3}{2}\ cm/min$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$

$32\pi = 4\pi (2)^2 \frac{dr}{dt}$

$\frac{dr}{dt} = \frac{32\pi}{16\pi}$

$= 2$ cm/min

$V = \frac{4}{3}\pi r^3$

$\frac{32}{3}\pi = \frac{4}{3}\pi r^3$

$\Rightarrow r^3 = 8 \Rightarrow r = 2$

~#78, 80

§ 3.5

9. If $y = (1 + x^2)^{\cos x}$, then $\dfrac{y'}{y} =$

$\ln y = \cos x \cdot \ln(1+x^2)$

(a) $\dfrac{2x\cos x - \sin x \cdot (1+x^2)\ln(1+x^2)}{1+x^2}$ ______ (correct)

(b) $\dfrac{2x\cos x - \sin x \cdot \ln(1+x^2)}{1+x^2}$

(c) $\dfrac{\cos x - \sin x}{1+x^2}$

(d) $\dfrac{x\cos x + (1+x^2)\sin x}{1+x^2}$

(e) $-\dfrac{2x\sin x}{1+x^2}$

$\Rightarrow \frac{1}{y} \cdot y' = \cos x \cdot \frac{2x}{1+x^2} + \ln(1+x^2) \cdot (-\sin x)$

$\Rightarrow \frac{y'}{y} = \frac{2x\cos x - \sin x \cdot (1+x^2)\ln(1+x^2)}{1+x^2}$

~#83

§3.3

10. If (A, B) and (C, D) are the points on the graph of the function $f(x) = \dfrac{x+1}{x-1}$ at which the tangent lines are perpendicular to the line $2x - y = 1$, then $A+B+C+D =$

. $2x - y = 1 \Rightarrow y = 2x - 1$

$\Rightarrow$ Slope of the tangent lines at $x = A$ & $x = B$ is equal to $-\frac{1}{2}$.

(a) 4 ______ (correct)

(b) -1

(c) 0

(d) 3

(e) 5

$\Rightarrow f'(x) = -\frac{1}{2}$

$\Rightarrow \frac{(x-1)\cdot 1 - (x+1)\cdot 1}{(x-1)^2} = -\frac{1}{2} \Rightarrow \frac{-2}{(x-1)^2} = -\frac{1}{2}$

$\Rightarrow (x-1)^2 = 4 \Rightarrow x - 1 = 2$ or $x - 1 = -2$

$\Rightarrow x = 3$ (A) or $x = -1$ (C)

. $f(3) = \frac{3+1}{3-1} = \frac{4}{2} = 2 = B$

. $f(-1) = \frac{-1+1}{-1-1} = 0 = D$

So $A + B + C + D = 3 + 2 + (-1) + 0$

$= 4$

11. If $f(x) = \frac{1}{e^x}$, then $f^{(55)}(0) =$

~#130
§3.3

(a) -1 ____________________ (correct)

(b) 1

(c) $\frac{1}{e}$

(d) e

(e) $-\frac{1}{e}$

$f(x) = e^{-x}$

$f'(x) = -e^{-x}$

$f''(x) = (-1)^2 e^{-x}$

$f'''(x) = (-1)^3 e^{-x}$

$\vdots$

$f^{(n)}(x) = (-1)^n e^{-x} \Longrightarrow f^{(55)}(0) = (-1)^{55} e^{-0}$

$= (-1)(1)$

$= -1$

~#74
§3.5

12. If $y = \frac{(x^2+2)(x^3+3)}{(x^2-2)(x^3-3)}$, then the value of $\frac{dy}{dx}$ at $x = 1$ is equal to

(a) $\frac{59}{2}$ ____________________ (correct)

(b) $\frac{49}{2}$

(c) $\frac{39}{4}$

(d) $\frac{17}{2}$

(e) $\frac{29}{4}$

$\ln y = \ln(x^2+2) + \ln(x^3+3) - \ln(x^2-2) - \ln(x^3-3)$

$\frac{1}{y} \cdot y' = \frac{2x}{x^2+2} + \frac{3x^2}{x^3+3} - \frac{2x}{x^2-2} - \frac{3x^2}{x^3-3}$

at $x=1$, $y = \frac{(1+2)(1+3)}{(1-2)(1-3)} = \frac{3 \cdot 4}{(-1)(-2)} = 6$

$\xRightarrow{x=1} \frac{1}{6} \cdot y' = \frac{2}{3} + \frac{3}{4} - \frac{2}{-1} - \frac{3}{-2}$

$= \frac{2}{3} + \frac{3}{4} + 2 + \frac{3}{2} = \frac{8+9+24+18}{12}$

$= \frac{59}{12}$

$\Rightarrow y' = \frac{59}{2}$

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p. 202

13. If $f(x) = x\sqrt{x-3}$, then $(f^{-1})'(4) =$
(Hint: What is the value of $f(4)$?)

$\longrightarrow f(4) = 4\sqrt{4-3} = 4(1) = 4$
$\Rightarrow f^{-1}(4) = 4$

(a) $\frac{1}{3}$ (correct)

(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) 1

(e) $\frac{1}{5}$

$$(f^{-1})'(4) = \frac{1}{f'(f^{-1}(4))} = \frac{1}{f'(4)} = \frac{1}{3}$$

$\cdot\ f'(x) = x \cdot \frac{1}{2\sqrt{x-3}} + \sqrt{x-3} \cdot 1$

$f'(4) = 4 \cdot \frac{1}{2 \cdot 1} + 1 = 2 + 1 = 3$

~ # 26
§ 4.2

14. The value of c that satisfies the **Mean Value Theorem** when applied to $f(x) = 2x - \ln x$ on the interval $[1, e]$ is equal to

$c \in (1, e)$

(a) $e - 1$ (correct)

(b) $e - \frac{1}{2}$

(c) $\frac{e+1}{e}$

(d) $\frac{2e}{e-1}$

(e) $2e - 3$

$$f'(c) = \frac{f(e) - f(1)}{e-1}$$

$$2 - \frac{1}{c} = \frac{(2e-1) - 2}{e-1}$$

$$2 - \frac{1}{c} = \frac{2e-3}{e-1}$$

$$\frac{1}{c} = 2 - \frac{2e-3}{e-1}$$

$$\Rightarrow \frac{1}{c} = \frac{2(e-1) - (2e-3)}{e-1}$$

$$\Rightarrow \frac{1}{c} = \frac{1}{e-1}$$

$$\Rightarrow c = e - 1 \in (1, e).$$

$f'(x) = 2 - \frac{1}{x}$

$\cdot\ f'(c) = 2 - \frac{1}{c}$

$\cdot\ f(e) = 2e - \ln e = 2e - 1$

$f(1) = 2 - \ln 1 = 2 - 0 = 2$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	A 4	B 4	B 2	A 2
2	A	E 3	D 2	B 1	C 3
3	A	A 1	D 1	B 3	E 1
4	A	A 2	C 3	B 4	B 4
5	A	A 5	C 9	B 8	E 9
6	A	E 6	B 5	E 6	C 5
7	A	D 7	E 6	E 9	A 6
8	A	E 8	C 7	C 7	C 8
9	A	D 9	E 8	C 5	E 7
10	A	A 14	E 12	D 11	C 11
11	A	B 12	E 13	C 10	E 14
12	A	B 13	C 10	D 13	E 12
13	A	B 10	A 11	A 14	A 10
14	A	A 11	D 14	C 12	E 13