Sol. Key
Code-Wise
at the end

~ #54
§3.2

1. If $f(t) = 2t^3 + 3\cos t$, then $f'(t) =$ $6t^2 - 3\sin t$

(a) $6t^2 - 3\sin t$ ______________________________ (correct)

(b) $6t^2 + 3\sin t$

(c) $2t^2 - 3\cos t$

(d) $3t^2 - 3\sin t$

(e) $2t^3 + 3\cos t$

~ #38
§5.4

2. If $y = Ax + B$ is an equation of the **tangent line** to the curve $y = e^{\sinh(2x)}$ at the point $(0, 1)$, then $AB =$

$y' = e^{\sinh(2x)} \cdot \cosh(2x) \cdot 2$

(a) 2 ______________________________ (correct)

(b) 3

(c) -4

(d) -3

(e) 1

slope $= y'\big|_{x=0} = 1 \cdot 1 \cdot 2 = 2$

Eq: $y - 1 = 2(x - 0)$

$\Rightarrow y = 2x + 1$

$A = 2,\ B = 1$

$AB = 2$

3. If $f(x) = (2x-1)^3(3-2x)^5$, then $f'(x) =$ $(2x-1)^3 \cdot 5(3-2x)^4 \cdot (-2)$
$+ (3-2x)^5 \cdot 3(2x-1)^2 \cdot (2)$

(a) $4(7-8x)(2x-1)^2(3-2x)^4$ ____________________________________(correct)

(b) $2(9-10x)(2x-1)^2(3-2x)^4$

(c) $4(7-2x)(2x-1)^2(3-2x)^4$

(d) $2(4-2x)(2x-1)^2(3-2x)^4$

(e) $(14-15x)(2x-1)^2(3-2x)^4$

$= (2x-1)^2(3-2x)^4[-10(2x-1)+6(3-2x)]$

$= (2x-1)^2(3-2x)^4(-20x+10+18-12x)$

$= (2x-1)^2(3-2x)^4(-32x+28)$

$= 4(7-8x)(2x-1)^2(3-2x)^4$

~#25
§3.4

§2.4

4. The function $f(x) = \dfrac{\sqrt{x-2}}{x^2-3x-4}$ is **continuous** on

$x-2 \geq 0$

$x^2-3x-4 \neq 0$

$(x-4)(x+1) \neq 0$

(a) $[2,4) \cup (4,\infty)$ ____(correct)

(b) $[2,\infty)$

(c) $(-\infty,-1) \cup (-1,4) \cup (4,\infty)$

(d) $(-\infty,-1) \cup (-1,2]$

(e) $(-1,4)$

$\Rightarrow x \geq 2, \; x \neq 4, \; x \neq -1$

$-1 \quad 2 \quad 4$

$[2,4) \cup (4,\infty)$

5. If $y = e^x \sin x$, then $y'' - 2y' + 2y =$

$\sim \# 58$
§3.3

$y' = e^x \cos x + e^x \sin x$

(a) 0 (correct)

(b) $e^x \cos x$

(c) $3e^x \sin x$

(d) $e^x (\cos x - 2\sin x)$

(e) $-2e^x \sin x$

$y'' = -e^x \sin x + e^x \cos x + e^x \cos x + e^x \sin x$

$= 2e^x \cos x$

$\bullet\ y'' - 2y' + 2y$

$= \underbrace{2e^x \cos x - 2e^x \cos x}_{0} \underbrace{- 2e^x \sin x + 2e^x \sin x}_{0}$

$= 0$

$\sim \# 82$
§3.4

6. If (a, b) and (c, d) are the points at which the graph of $f(x) = \ln \left| \dfrac{x^2+5}{x+2} \right|$ has a **horizontal tangent line**, then $2ac =$

$f(x) = \ln|x^2+5| - \ln|x+2|$

(a) -10 (correct)

(b) -6

(c) 12

(d) 15

(e) 18

$f'(x) = \dfrac{2x}{x^2+5} - \dfrac{1}{x+2}$

$= \dfrac{2x(x+2) - (x^2+5)}{(x^2+5)(x+2)} = \dfrac{2x^2+4x-x^2-5}{(x^2+5)(x+2)}$

$= \dfrac{x^2+4x-5}{(x^2+5)(x+2)} = \dfrac{(x+5)(x-1)}{(x^2+5)(x+2)}$

H.T. $\Rightarrow$ $f'(x) = 0 \Rightarrow x = -5,\ x = 1$

$a = -5,\ c = 1$

$2ac = 2(-5)(1)$

$= -10$

7. The **tangent line approximation** of $f(x) = \sqrt{x^3}$ at $x = 1$ is

$f(x) = x^{3/2}$

$f'(x) = \frac{3}{2} x^{1/2}$

(a) $y = \frac{3}{2}x - \frac{1}{2}$ ______________ (correct)

(b) $y = 3x - 2$

(c) $y = \frac{5}{2}x - \frac{3}{2}$

(d) $y = \frac{3}{2}x + \frac{1}{2}$

(e) $y = x - \frac{1}{2}$

$y = f(1) + f'(1)(x-1)$

$= 1 + \frac{3}{2}(x-1)$

$= \frac{3}{2}x - \frac{1}{2}$

~#8
§4.8

8. The volume of oil in a cylindrical container is increasing at a rate of 150 cubic centimeter per second. The height of the cylinder is ten times the radius. At what rate is the height of the oil changing when the oil is 35 centimeters high?
(Hint: The volume of a cylinder is $V = \pi r^2 h$)

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§3.7

$\frac{dV}{dt} = 150$; $h = 10r$; $\frac{dh}{dt} = ?$ when $h = 35$

(a) $\frac{200}{49\pi}\, cm/s$ ______________ (correct)

(b) $\frac{100}{21\pi}\, cm/s$

(c) $\frac{500}{147\pi}\, cm/s$

(d) $\frac{3}{35\pi}\, cm/s$

(e) $\frac{200}{7\pi}\, cm/s$

$V = \pi r^2 h = \pi \left(\frac{h}{10}\right)^2 h = \frac{\pi}{100} h^3$

$\frac{dV}{dt} = \frac{\pi}{100} \cdot 3h^2 \frac{dh}{dt}$

$150 = \frac{3\pi}{100} (35)^2 \frac{dh}{dt}$

$\Rightarrow \frac{dh}{dt} = \frac{(150)(100)}{\cancel{3}\pi (35)^2} = \frac{(50)(100)}{\pi \cdot 25 \cdot 49} = \frac{2(100)}{\pi \cdot 49}$

$= \frac{200}{49\pi}$

9. The graph of the function $f(x) = x^4 - 4x^3 + 2$ has

#39
§4.4

(a) two inflection points ________________ (correct)

(b) one inflection point

(c) three inflection points

(d) four inflection points

(e) no inflection points

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$= 12x(x-2)$

$f''(x) = 0 \Rightarrow x = 0, \; x = 2$

f'': $+$ at left of 0, $-$ between 0 and 2, $+$ at right of 2

∪ 0 ∩ 2 ∪

inf inf

Two inf. pts.

~ #20
§5.9

10. If $\tanh x = \dfrac{-2}{3}$, then $\sinh x =$

• $\operatorname{sech}^2 x = 1 - \tanh^2 x = 1 - \frac{4}{9} = \frac{5}{9}$

$\Rightarrow \operatorname{sech} x = \frac{\sqrt{5}}{3} \Rightarrow \cosh x = \frac{3}{\sqrt{5}}$ $\quad$ $\cosh x > 0$ for x

(a) $-\dfrac{2}{\sqrt{5}}$ ________________ (correct)

• $\tanh x = \dfrac{\sinh x}{\cosh x} \Rightarrow \sinh x = \tanh x \cdot \cosh x$

$= -\frac{2}{3} \cdot \frac{3}{\sqrt{5}}$

$= -\frac{2}{\sqrt{5}}$

(b) $-\dfrac{3}{\sqrt{5}}$

(c) $\dfrac{1}{\sqrt{5}}$

(d) $\dfrac{2}{\sqrt{5}}$

(e) $\dfrac{5}{\sqrt{5}}$

11. If $f'(x) = e^x + \dfrac{1}{\sqrt{x}}$, $f(1) = e$, then $f(4) =$

(a) $e^4 + 2$ ________________ (correct)

(b) $e^4 + 6$

(c) $e^4 - 2$

(d) $e^4 - 3$

(e) $e^4 - 4$

~ Examples 5, 8
§ 5.1

$$f'(x) = e^x + x^{-1/2}$$
$$f(x) = e^x + \frac{x^{1/2}}{1/2} + C$$
$$f(x) = e^x + 2\sqrt{x} + C$$
$$f(1) = e + 2 + C \Rightarrow e = e + 2 + C \Rightarrow \boxed{C = -2}$$
$$\Rightarrow f(x) = e^x + 2\sqrt{x} - 2$$
$$\Rightarrow f(4) = e^4 + 2\sqrt{4} - 2$$
$$= e^4 + 4 - 2$$
$$= e^4 + 2$$

~ #43
§ 4.4

12. The graph of the function $f(x) = x^2 - \dfrac{1}{x}$ is

$$f'(x) = 2x + \frac{1}{x^2}$$

(a) concave up on $(1, \infty)$ ________________ (correct)

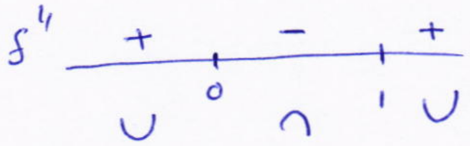
(b) concave up on $(0, \infty)$

(c) concave up on $(0, 1)$

(d) concave down on $(1, \infty)$

(e) concave down on $(-\infty, 0)$

$$f''(x) = 2 - \frac{2}{x^3} = 2 \cdot \frac{x^3 - 1}{x^3}$$
$$f''(x) = 0 \Rightarrow x^3 - 1 = 0 \Rightarrow x = 1$$
$f''(x)$ DNE when $x = 0$ ($\notin$ domain)



13. The graph of the function $g(x) = x^4 e^{-2x}$ is

$g'(x) = x^4(-2e^{-2x}) + e^{-2x}(4x^3)$

(a) increasing on $(0, 2)$ ______ (correct)

(b) increasing on $(2, \infty)$

(c) increasing on $(0, \infty)$

(d) decreasing on $(-\infty, 2)$

(e) decreasing on $(0, 2)$

$= x^3 e^{-2x}(-2x+4)$

$= 2x^3 e^{-2x}(-x+2)$

$g'(x) = 0 \Rightarrow x = 0, \ x = 2$

$g'(x)$ $-$ | $+$ | $-$

0 2

$\searrow$ $\nearrow$ $\searrow$

~ #20
§ 4.3

~ Exaple 4
§ 4.6

14. The graph of the function $f(x) = 8x^{\frac{1}{3}} - x^{\frac{4}{3}}$ has

$f'(x) = 8 \cdot \frac{1}{3} x^{-2/3} - \frac{4}{3} x^{1/3}$

(a) one local maximum only ______ (correct)

(b) one local minimum only

(c) two local minima only

(d) one local minimum and one local maximum

(e) neither local minimum nor local maximum

$= \frac{4}{3} x^{-2/3}(2 - x)$

$= \frac{4}{3} \cdot \frac{2-x}{x^{2/3}}$

$f'(x) = 0 \Rightarrow x = 2$

$f'(x)$ DNE $\Rightarrow x = 0$

$f'(x)$ $+$. $+$ | $-$

0 2

$\nearrow$ $\nearrow$ $\searrow$

$\uparrow$
local max
at $x = 2$

15. Which one of the following statements is **TRUE**?
(V.A: Vertical asymptote(s): H.A: Horizontal asymptote(s))

(a) The graph of $f(x) = \dfrac{\sin(2x)}{x}$ has no V.A. $\lim_{x \to 0} f(x) = 2 \neq \pm\infty$ ______ (correct)

(b) The graph of $f(x) = \dfrac{\sin(2x)}{x}$ has no H.A. $\lim_{x \to \pm\infty} f(x) = 0 \Rightarrow y = 0$ is H.A.

(c) Every rational function has at least one V.A.

(d) Every rational function has two H.A.

(e) Every rational function has a slant asymptote

$\left.\right\} f(x) = \frac{1}{x^2+1}$

1^{∞}-form

16. $\lim\limits_{x\to\infty} \left[\cos\left(\dfrac{3}{x}\right)\right]^{2x^2} =$ $\quad 1^{\infty}$

$y = \left[\cos\left(\frac{3}{x}\right)\right]^{2x^2}$

$\ln y = 2x^2 \ln\left(\cos\left(\frac{3}{x}\right)\right)$

(a) e^{-9} ______ (correct)

(b) e^{-3}

(c) e^{-6}

(d) 1

(e) ∞

$$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} 2 \cdot \frac{\ln\left(\cos\left(\frac{3}{x}\right)\right)}{x^{-2}} \qquad \frac{0}{0}$$

$$\overset{L'R}{=} \lim_{x\to\infty} 2 \cdot \frac{\frac{1}{\cos\left(\frac{3}{x}\right)} \cdot -\sin\left(\frac{3}{x}\right) \cdot -\frac{3}{x^2}}{-2x^{-3}}$$

$$= \lim_{x\to\infty} -3 \frac{\tan\left(\frac{3}{x}\right)}{x^{-1}} \qquad \frac{0}{0}$$

$$\overset{L'R}{=} \lim_{x\to\infty} -3 \frac{\sec^2\left(\frac{3}{x}\right) \cdot -\frac{3}{x^2}}{-x^{-2}}$$

$$= \lim_{x\to\infty} -9\sec^2\left(\frac{3}{x}\right) = -9(1) = -9$$

So $\lim\limits_{x\to\infty} y = e^{-9}$

17. $\lim_{x\to 0^+} \dfrac{2-3\ln x}{e^{2/x}} =$ $\dfrac{\infty}{\infty}$

$\frac{\infty}{\infty}$ form

$$\overset{L.R}{=} \lim_{x\to 0^+} \frac{0-\frac{3}{x}}{e^{2/x}\cdot -\frac{2}{x^2}}$$

(a) 0 (correct)

(b) $\dfrac{3}{2}$

$$= \lim_{x\to 0^+} \frac{3}{2}\cdot\frac{x}{e^{2/x}}$$

(c) 1

(d) ∞

$$= \frac{3}{2}\cdot\frac{0}{\infty} = \frac{3}{2}\cdot 0 = 0$$

(e) 2

$\sim$ #47
§ 4.6

18. The **absolute minimum value** of $f(x) = x\ln x$ on $(0,\infty)$ is equal to

$$f'(x) = x\cdot\frac{1}{x} + \ln x \cdot 1 = 1 + \ln x$$

(a) $-\dfrac{1}{e}$ (correct)

$$f'(x) = 0 \Rightarrow 1+\ln x = 0 \Rightarrow \ln x = -1 \Rightarrow x = e^{-1}$$

(b) $-\dfrac{2}{e}$

(c) $\dfrac{3}{e^2}$

(d) $-\dfrac{2}{e^2}$

(e) $\dfrac{1}{e^2}$

f': 0 — ($-$) ↘ e^{-1} ↗ ($+$)

↑ one local min at $x = e^{-1}$

$\Rightarrow$ abs. min at $x = e^{-1}$

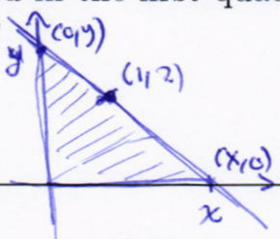
$\Rightarrow$ abs. min. value of f is

$$f(e^{-1}) = e^{-1}\ln(e^{-1}) = -e^{-1} = -\frac{1}{e}$$

$$f'(e^{-2}) = 1 + \ln e^{-2} = 1-2 = -1 < 0$$

$$f'(e) = 1 + \ln e = 1+1 = 2 > 0$$

19. What is the **minimum area** (cm^2) of the triangle formed in the first quadrant by the x-axis, the y-axis and a line through the point $(1, 2)$?



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§ 4.7

(a) 4 ______ (correct)

(b) 2

(c) 6

(d) 8

(e) 10

$A = \frac{1}{2} x y$

$A(x) = \frac{1}{2} x \cdot \frac{2x}{x-1} = \frac{x^2}{x-1}$

$A'(x) = \frac{(x-1)\cdot(2x) - x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2}$

$A'(x) = 0 \Rightarrow x = 2 \left[\begin{array}{l} x > 0 \\ x = 1 \Rightarrow \text{no triangle} \end{array} \right]$

$\boxed{x > 0}$

$\cdot$ slope $= \frac{y-0}{0-x} = \frac{2-0}{1-x}$

$\Rightarrow y = \frac{-2x}{1-x} = \frac{2x}{x-1}$

$\Rightarrow y = \frac{2(2)}{2-1} = 4$

$\Rightarrow A = \frac{1}{2} x \cdot y$

$= \frac{1}{2} \cdot 2 \cdot 4$

$= 4$

$\cdot$ Note:

$A''(x) = \frac{(x-1)^2 (2x-2) - (x^2-2x) \cdot 2(x-1)}{(x-1)^4}$

$A''(2) = 2 > 0$

$\Rightarrow A$ is min at $x = 2$

~ #61
§ 3.1

20. Two lines are tangent to the curve $y = 4x - x^2$ and both pass through the point $\left(\frac{5}{2}, 6\right)$. The **product of the slopes** of these two lines is equal to

Let (x,y) be a point on the curve $y = 4x - x^2$.

(a) -8 ______ (correct)

(b) -10

(c) $-\frac{3}{4}$

(d) $-\frac{5}{8}$

(e) -12

slope $= 4 - 2x = \frac{y-6}{x-\frac{5}{2}} = \frac{4x-x^2-6}{x-\frac{5}{2}}$

$\Rightarrow (4-2x)(x-\frac{5}{2}) = 4x - x^2 - 6$

$\Rightarrow 4x - 10 - 2x^2 + 5x = 4x - x^2 - 6$

$\Rightarrow x^2 - 5x + 4 = 0$

$\Rightarrow (x-1)(x-4) = 0$

$\Rightarrow x = 1, \; x = 4$

$\cdot \; m_1 = 4 - 2x \Big|_{x=1} = 4 - 2 = 2$

$m_2 = 4 - 2x \Big|_{x=4} = 4 - 8 = -4$

$\cdot \; m_1 m_2 = (2)(-4) = -8$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	C 1	D 3	E 1	A 2
2	A	D 4	D 5	D 2	B 4
3	A	A 3	B 1	A 5	A 3
4	A	D 2	B 2	C 4	E 1
5	A	A 5	D 4	A 3	C 5
6	A	B 6	E 8	B 10	A 6
7	A	C 7	C 10	B 9	C 10
8	A	B 9	B 7	D 6	B 7
9	A	A 10	D 9	D 8	C 8
10	A	D 8	E 6	D 7	D 9
11	A	D 13	C 15	E 13	E 12
12	A	C 15	C 11	C 11	E 15
13	A	B 12	B 13	D 12	D 11
14	A	B 11	E 12	C 14	B 13
15	A	C 14	B 14	E 15	D 14
16	A	B 20	A 18	E 20	A 17
17	A	E 16	C 20	E 19	C 16
18	A	A 18	A 17	E 17	B 19
19	A	A 19	C 19	D 18	A 18
20	A	B 17	A 16	C 16	A 20