1. If $f(t) = 2t^3 + 3\cos t$, then $f'(t) =$ $6t^2 - 3\sin t$

Sol. Key Code-Wise at the end

$\sim$ #54 §3.2

(a) $6t^2 - 3\sin t$ ______________________________ (correct)

(b) $6t^2 + 3\sin t$

(c) $2t^2 - 3\cos t$

(d) $3t^2 - 3\sin t$

(e) $2t^3 + 3\cos t$

$\sim$ #38 §5.4

2. If $y = Ax + B$ is an equation of the **tangent line** to the curve $y = e^{\sinh(2x)}$ at the point $(0, 1)$, then $AB =$

$y' = e^{\sinh(2x)} \cdot \cosh(2x) \cdot 2$

(a) 2 ______________________________ (correct)

(b) 3

(c) -4

(d) -3

(e) 1

slope $= y'\big|_{x=0} = 1 \cdot 1 \cdot 2 = 2$

Eq: $y - 1 = 2(x - 0)$

$\Rightarrow y = 2x + 1$

$A = 2,\ B = 1$

$AB = 2$

3. If $f(x) = (2x-1)^3(3-2x)^5$, then $f'(x) =$ $(2x-1)^3 \cdot 5(3-2x)^4 \cdot (-2) + (3-2x)^5 \cdot 3(2x-1)^2 \cdot (2)$

(a) $4(7-8x)(2x-1)^2(3-2x)^4$ ______ (correct)

(b) $2(9-10x)(2x-1)^2(3-2x)^4$

(c) $4(7-2x)(2x-1)^2(3-2x)^4$

(d) $2(4-2x)(2x-1)^2(3-2x)^4$

(e) $(14-15x)(2x-1)^2(3-2x)^4$

$= (2x-1)^2(3-2x)^4[-10(2x-1)+6(3-2x)]$

$= (2x-1)^2(3-2x)^4(-20x+10+18-12x)$

$= (2x-1)^2(3-2x)^4(-32x+28)$

$= 4(7-8x)(2x-1)^2(3-2x)^4$

~#25
§3.4

§2.4

4. The function $f(x) = \dfrac{\sqrt{x-2}}{x^2-3x-4}$ is **continuous** on

$x-2 \geq 0$

$x^2-3x-4 \neq 0$

$(x-4)(x+1) \neq 0$

(a) $[2,4) \cup (4,\infty)$ ______ (correct)

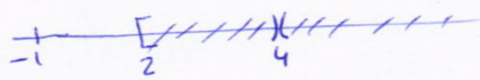
(b) $[2,\infty)$

(c) $(-\infty,-1) \cup (-1,4) \cup (4,\infty)$

(d) $(-\infty,-1) \cup (-1,2]$

(e) $(-1,4)$

$\Rightarrow x \geq 2, \; x \neq 4, \; x \neq -1$



$[2,4) \cup (4,\infty)$

5. If $y = e^x \sin x$, then $y'' - 2y' + 2y =$

~#58
§3.3

$y' = e^x \cos x + e^x \sin x$

(a) 0 ________________ (correct)

(b) $e^x \cos x$

(c) $3e^x \sin x$

(d) $e^x (\cos x - 2\sin x)$

(e) $-2e^x \sin x$

$y'' = -e^x \sin x + e^x \cos x + e^x \cos x + e^x \sin x$

$= 2e^x \cos x$

$\cdot\ y'' - 2y' + 2y$

$= \underbrace{2e^x \cos x - 2e^x \cos x}_{0} \underbrace{- 2e^x \sin x + 2e^x \sin x}_{0}$

$= 0$

~#82
§3.4

6. If (a, b) and (c, d) are the points at which the graph of $f(x) = \ln \left| \dfrac{x^2+5}{x+2} \right|$ has a **horizontal tangent line**, then $2ac =$

$f(x) = \ln|x^2+5| - \ln|x+2|$

(a) -10 ________________ (correct)

(b) -6

(c) 12

(d) 15

(e) 18

$f'(x) = \dfrac{2x}{x^2+5} - \dfrac{1}{x+2}$

$= \dfrac{2x(x+2) - (x^2+5)}{(x^2+5)(x+2)} = \dfrac{2x^2+4x-x^2-5}{(x^2+5)(x+2)}$

$= \dfrac{x^2+4x-5}{(x^2+5)(x+2)} = \dfrac{(x+5)(x-1)}{(x^2+5)(x+2)}$

H.T. $\Rightarrow$ $f'(x) = 0 \Rightarrow x = -5, x = 1$

$a = -5, c = 1$

$2ac = 2(-5)(1)$

$= -10$

9. The graph of the function $f(x) = x^4 - 4x^3 + 2$ has

#39
§4.4

(a) two inflection points ________ (correct)

(b) one inflection point

(c) three inflection points

(d) four inflection points

(e) no inflection points

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$= 12x(x-2)$

$f''(x) = 0 \Rightarrow x = 0, \ x = 2$

f'': $+$ | 0 | $-$ | 2 | $+$

$\cup$ 0 $\cap$ 2 $\cup$

↑ inf ↑ inf

Two inf. pts.

~ #20
§5.9

10. If $\tanh x = \dfrac{-2}{3}$, then $\sinh x =$

• $\operatorname{sech}^2 x = 1 - \tanh^2 x = 1 - \frac{4}{9} = \frac{5}{9}$

$\Rightarrow \operatorname{sech} x = \frac{\sqrt{5}}{3} \Rightarrow \cosh x = \frac{3}{\sqrt{5}}$ $\cosh x > 0$ for x

(a) $-\dfrac{2}{\sqrt{5}}$ ________ (correct)

(b) $-\dfrac{3}{\sqrt{5}}$

(c) $\dfrac{1}{\sqrt{5}}$

(d) $\dfrac{2}{\sqrt{5}}$

(e) $\dfrac{5}{\sqrt{5}}$

• $\tanh x = \dfrac{\sinh x}{\cosh x} \Rightarrow \sinh x = \tanh x \cdot \cosh x$

$= -\frac{2}{3} \cdot \frac{3}{\sqrt{5}}$

$= -\frac{2}{\sqrt{5}}$

11. If $f'(x) = e^x + \dfrac{1}{\sqrt{x}}$, $f(1) = e$, then $f(4) =$

(a) $e^4 + 2$ ________________________________(correct)

(b) $e^4 + 6$

(c) $e^4 - 2$

(d) $e^4 - 3$

(e) $e^4 - 4$

$f'(x) = e^x + x^{-1/2}$

$f(x) = e^x + \dfrac{x^{1/2}}{1/2} + C$

$f(x) = e^x + 2\sqrt{x} + C$

$f(1) = e + 2 + C \Rightarrow e = e + 2 + C \Rightarrow \boxed{C = -2}$

~ Examples 5, 8
§ 5.1

$\Rightarrow f(x) = e^x + 2\sqrt{x} - 2$

$\Rightarrow f(4) = e^4 + 2\sqrt{4} - 2$

$= e^4 + 4 - 2$

$= e^4 + 2$

~ #43
§ 4.4

12. The graph of the function $f(x) = x^2 - \dfrac{1}{x}$ is

$f'(x) = 2x + \dfrac{1}{x^2}$

(a) concave up on $(1, \infty)$ ________________________________(correct)

(b) concave up on $(0, \infty)$

(c) concave up on $(0, 1)$

(d) concave down on $(1, \infty)$

(e) concave down on $(-\infty, 0)$

$f''(x) = 2 - \dfrac{2}{x^3} = 2 \cdot \dfrac{x^3 - 1}{x^3}$

$f''(x) = 0 \Rightarrow x^3 - 1 = 0 \Rightarrow x = 1$

$f''(x)$ DNE when $x = 0$ ($\notin$ domain)

f''	+	0	−	1	+
	$\cup$		$\cap$		$\cup$

13. The graph of the function $g(x) = x^4 e^{-2x}$ is

$g'(x) = x^4(-2e^{-2x}) + e^{-2x}(4x^3)$

(a) increasing on $(0, 2)$ ______________________________ (correct)

(b) increasing on $(2, \infty)$

(c) increasing on $(0, \infty)$

(d) decreasing on $(-\infty, 2)$

(e) decreasing on $(0, 2)$

$= x^3 e^{-2x}(-2x+4)$

$= 2x^3 e^{-2x}(-x+2)$

$g'(x) = 0 \Rightarrow x = 0, \; x = 2$

$g'(x)$ $-$ 0 $+$ 2 $-$

~ #20 §4.3

~ Example 4 § 4.6

14. The graph of the function $f(x) = 8x^{\frac{1}{3}} - x^{\frac{4}{3}}$ has

$f'(x) = 8 \cdot \frac{1}{3} x^{-2/3} - \frac{4}{3} x^{1/3}$

(a) one local maximum only ______________________________ (correct)

(b) one local minimum only

(c) two local minima only

(d) one local minimum and one local maximum

(e) neither local minimum nor local maximum

$= \frac{4}{3} x^{-2/3}(2 - x)$

$= \frac{4}{3} \cdot \frac{2-x}{x^{2/3}}$

$f'(x) = 0 \Rightarrow x = 2$

$f'(x)$ DNE $\Rightarrow x = 0$

$f'(x)$ $+$ 0 $+$ 2 $-$

local max at $x = 2$

15. Which one of the following statements is **TRUE**?
(V.A: Vertical asymptote(s): H.A: Horizontal asymptote(s))

(a) The graph of $f(x) = \dfrac{\sin(2x)}{x}$ has no V.A. $\lim_{x\to 0} f(x) = 2 \neq \pm\infty$ (correct)

(b) The graph of $f(x) = \dfrac{\sin(2x)}{x}$ has no H.A. $\lim_{x\to\pm\infty} f(x) = 0 \Rightarrow y=0$ is H.A.

(c) Every rational function has at least one V.A.

(d) Every rational function has two H.A.

(e) Every rational function has a slant asymptote

$f(x) = \dfrac{1}{x^2+1}$

1^∞-form

16. $\displaystyle\lim_{x\to\infty}\left[\cos\left(\frac{3}{x}\right)\right]^{2x^2} =$ 1^∞

$y = \left[\cos\left(\frac{3}{x}\right)\right]^{2x^2}$

$\ln y = 2x^2 \ln\left(\cos\left(\frac{3}{x}\right)\right)$

(a) e^{-9} (correct)

(b) e^{-3}

(c) e^{-6}

(d) 1

(e) ∞

$\displaystyle\lim_{x\to\infty} \ln y = \lim_{x\to\infty} 2 \cdot \frac{\ln\left(\cos\left(\frac{3}{x}\right)\right)}{x^{-2}}$ $\quad \frac{0}{0}$

$\displaystyle\overset{L'R}{=} \lim_{x\to\infty} 2 \cdot \frac{\frac{1}{\cos\left(\frac{3}{x}\right)} \cdot -\sin\left(\frac{3}{x}\right) \cdot -\frac{3}{x^2}}{-2x^{-3}}$

$\displaystyle= \lim_{x\to\infty} -3 \frac{\tan\left(\frac{3}{x}\right)}{x^{-1}}$ $\quad \frac{0}{0}$

$\displaystyle\overset{L'R}{=} \lim_{x\to\infty} -3 \frac{\sec^2\left(\frac{3}{x}\right) \cdot -\frac{3}{x^2}}{-x^{-2}}$

$\displaystyle= \lim_{x\to\infty} -9\sec^2\left(\frac{3}{x}\right) = -9(1) = -9$

So $\displaystyle\lim_{x\to\infty} y = e^{-9}$

17. $\lim\limits_{x\to 0^+} \dfrac{2-3\ln x}{e^{2/x}} =$ $\dfrac{\infty}{\infty}$

$\frac{\infty}{\infty}$ form

$\overset{L.R}{=} \lim\limits_{x\to 0^+} \dfrac{0-\frac{3}{x}}{e^{2/x}\cdot -\frac{2}{x^2}}$

(a) 0 ____(correct)

(b) $\dfrac{3}{2}$

$= \lim\limits_{x\to 0^+} \dfrac{3}{2}\cdot\dfrac{x}{e^{2/x}}$

(c) 1

(d) ∞

$= \dfrac{3}{2}\cdot\dfrac{0}{\infty} = \dfrac{3}{2}\cdot 0 = 0$

(e) 2

~ #47 § 4.6

18. The **absolute minimum value** of $f(x) = x\ln x$ on $(0,\infty)$ is equal to

$f'(x) = x\cdot\frac{1}{x} + \ln x \cdot 1$
$= 1 + \ln x$

(a) $-\dfrac{1}{e}$ ____(correct)

$f'(x) = 0 \Rightarrow 1 + \ln x = 0 \Rightarrow \ln x = -1$
$\Rightarrow x = e^{-1}$

(b) $-\dfrac{2}{e}$

(c) $\dfrac{3}{e^2}$

(d) $-\dfrac{2}{e^2}$

(e) $\dfrac{1}{e^2}$

f': on $(0, e^{-1})$: $-$ (decreasing); on (e^{-1}, ∞): $+$ (increasing)

$\boxed{\begin{aligned} f'(e^{-2}) &= 1+\ln e^{-2} \\ &= 1-2 = -1 < 0 \\ f'(e) &= 1+\ln e \\ &= 1+1 = 2 > 0 \end{aligned}}$

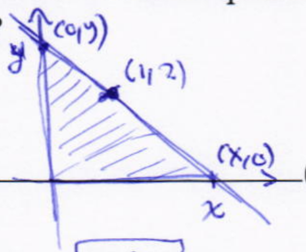
one local min at $x = e^{-1}$

$\Rightarrow$ abs. min at $x = e^{-1}$

$\Rightarrow$ abs. min. value of f is

$f(e^{-1}) = e^{-1}\ln(e^{-1})$
$= -e^{-1}$
$= -\dfrac{1}{e}$

19. What is the **minimum area** (cm^2) of the triangle formed in the first quadrant by the x-axis, the y-axis and a line through the point $(1, 2)$?



#23
§ 4.7

(a) 4 ______ (correct)

(b) 2

(c) 6

(d) 8

(e) 10

$A = \frac{1}{2} x y$

$A(x) = \frac{1}{2} x \cdot \frac{2x}{x-1} = \frac{x^2}{x-1}$

$A'(x) = \frac{(x-1)\cdot(2x) - x^2}{(x-1)^2} = \frac{x^2-2x}{(x-1)^2}$

$A'(x) = 0 \Rightarrow x = 2$ $\left[\begin{array}{l} x>0 \\ x=1 \Rightarrow \text{no triangle} \end{array}\right]$

$\boxed{x > 0}$

• slope $= \frac{y-0}{0-x} = \frac{2-0}{1-x}$

$\Rightarrow y = \frac{-2x}{1-x} = \frac{2x}{x-1}$

$\Rightarrow y = \frac{2(2)}{2-1} = 4$

$\Rightarrow A = \frac{1}{2} x \cdot y$

$= \frac{1}{2} \cdot 2 \cdot 4$

$= 4$

• Note:

$A''(x) = \frac{(x-1)^2(2x-2) - (x^2-2x)\cdot 2(x-1)}{(x-1)^4}$

$A''(2) = 2 > 0$

$\Rightarrow A$ is min at $x = 2$

~ #61
§ 3.1

20. Two lines are tangent to the curve $y = 4x - x^2$ and both pass through the point $\left(\frac{5}{2}, 6\right)$. The **product of the slopes** of these two lines is equal to

Let (x,y) be a point on the curve $y = 4x - x^2$.

(a) -8 ______ (correct)

(b) -10

(c) $-\frac{3}{4}$

(d) $-\frac{5}{8}$

(e) -12

slope $= 4 - 2x = \frac{y-6}{x-\frac{5}{2}} = \frac{4x-x^2-6}{x-\frac{5}{2}}$

$\Rightarrow (4-2x)(x-\frac{5}{2}) = 4x - x^2 - 6$

$\Rightarrow 4x - 10 - 2x^2 + 5x = 4x - x^2 - 6$

$\Rightarrow x^2 - 5x + 4 = 0$

$\Rightarrow (x-1)(x-4) = 0$

$\Rightarrow x = 1, \; x = 4$

• $m_1 = 4 - 2x \big|_{x=1} = 4 - 2 = 2$

$m_2 = 4 - 2x \big|_{x=4} = 4 - 8 = -4$

• $m_1 m_2 = (2)(-4) = -8$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	C $_{1}$	D $_{3}$	E $_{1}$	A $_{2}$
2	A	D $_{4}$	D $_{5}$	D $_{2}$	B $_{4}$
3	A	A $_{3}$	B $_{1}$	A $_{5}$	A $_{3}$
4	A	D $_{2}$	B $_{2}$	C $_{4}$	E $_{1}$
5	A	A $_{5}$	D $_{4}$	A $_{3}$	C $_{5}$
6	A	B $_{6}$	E $_{8}$	B $_{10}$	A $_{6}$
7	A	C $_{7}$	C $_{10}$	B $_{9}$	C $_{10}$
8	A	B $_{9}$	B $_{7}$	D $_{6}$	B $_{7}$
9	A	A $_{10}$	D $_{9}$	D $_{8}$	C $_{8}$
10	A	D $_{8}$	E $_{6}$	D $_{7}$	D $_{9}$
11	A	D $_{13}$	C $_{15}$	E $_{13}$	E $_{12}$
12	A	C $_{15}$	C $_{11}$	C $_{11}$	E $_{15}$
13	A	B $_{12}$	B $_{13}$	D $_{12}$	D $_{11}$
14	A	B $_{11}$	E $_{12}$	C $_{14}$	B $_{13}$
15	A	C $_{14}$	B $_{14}$	E $_{15}$	D $_{14}$
16	A	B $_{20}$	A $_{18}$	E $_{20}$	A $_{17}$
17	A	E $_{16}$	C $_{20}$	E $_{19}$	C $_{16}$
18	A	A $_{18}$	A $_{17}$	E $_{17}$	B $_{19}$
19	A	A $_{19}$	C $_{19}$	D $_{18}$	A $_{18}$
20	A	B $_{17}$	A $_{16}$	C $_{16}$	A $_{20}$