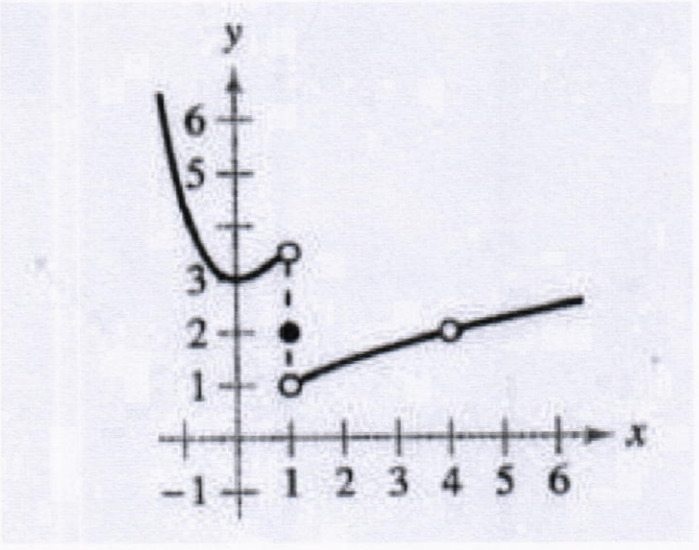
1. One statement is **TRUE** about the given graph




(a) $\lim_{x\to 4} f(x) = 2$ (correct)

(b) $\lim_{x\to 4} f(x)$ does not exist

(c) $\lim_{x\to 1} f(x) = 2$

(d) $\lim_{x\to 1} f(x) = 1$

(e) $3 < \lim_{x\to 1} f(x) < 4$

2. $\lim_{x\to 4} \dfrac{\sqrt{x+5}-3}{x-4} =$

$\dfrac{\sqrt{x+5}-3}{x-4} \cdot \dfrac{\sqrt{x+5}+3}{\sqrt{x+5}+3}$

(a) $\frac{1}{6}$ (correct)

(b) $\frac{1}{4}$

(c) $\frac{1}{5}$

(d) $\frac{1}{3}$

(e) $\frac{1}{7}$

$\dfrac{(x-4)}{(x-4)(\sqrt{x+5}+3)}$

$\dfrac{1}{\sqrt{x+5}+3}$, $\lim_{x\to 4} \dfrac{1}{3+3}$

$= 1/6$

3. $\lim_{x\to 0} \dfrac{e^{2x}-1}{e^x-1}$

(a) equals 2 ——— (correct)

(b) does not exist

(c) goes to $-\infty$

(d) equals 1

(e) equals zero

$$\lim_{x\to 0} \frac{(e^x)^2-1}{(e^x)-1}$$

$$\lim_{x\to 0} \frac{(e^x-1)(e^x+1)}{(e^x-1)}$$

$$\lim_{x\to 0} (e^x+1) = e^0+1$$

$$= 2$$

4. The domain of $f(x) = \dfrac{\sqrt{x}}{|x|-4}$ is

(a) $[0,4)\cup(4,\infty)$ ——— (correct)

(b) $(-\infty,\infty)-\{\pm 4\}$

(c) $(0,\infty)$

(d) $(-4,0)\cup(0,4)$

(e) $(0,\infty)-\{4\}$

$\sqrt{x} : [0,\infty)$

$|x| = 4 \Rightarrow x \neq \pm 4$

$[0,\infty)-\{4\}$

5. The function $f(x) = \begin{cases} \ln(x+1), & x \geq 0 \\ 1 - x^2 & x < 0 \end{cases}$

(a) has a non removable discontinuity at $x = 0$ (Jump) (correct)

(b) is continuous on $(-\infty, \infty)$

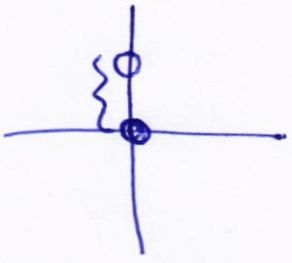
(c) has a removable discontinuity at $x = 0$

(d) has a non removable discontinuity at $x = 1$

(e) has a removable discontinuity at $x = 1$

$\lim_{x \to 0^+} \ln(x+1) = \ln(1) = 0$

$\lim_{x \to 0^-} (1 - x^2) = 1$



6. $\lim_{x \to -2^+} \dfrac{1}{x+2}$

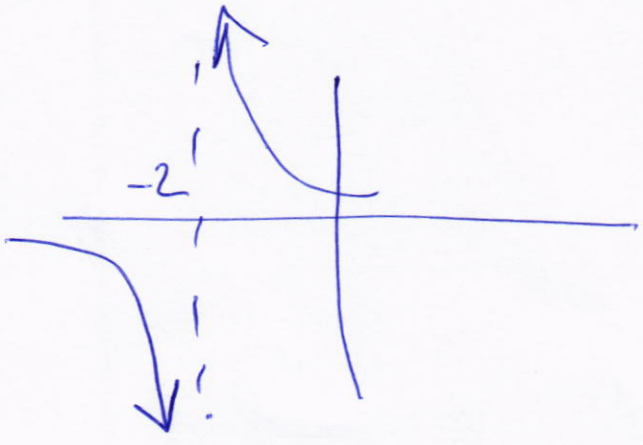
(a) goes to $+\infty$ (correct)

(b) goes to $-\infty$

(c) goes to $-\frac{1}{4}$

(d) goes to zero

(e) goes to $\frac{1}{4}$




$+\infty$

7. The function $f(x) = \dfrac{x}{x^2 - 9}$

(a) has 2 vertical asymptotes, moreover $\lim_{x \to 3^-} f(x) = -\infty$ (correct)

(b) has 2 vertical asymptotes, moreover $\lim_{x \to 3^-} f(x) = +\infty$

(c) has no vertical asymptote

(d) has one vertical asymptote, moreover $\lim_{x \to 3} f(x) = +\infty$

(e) has one vertical asymptote, moreover $\lim_{x \to 3^+} f(x) = -\infty$

8. $\lim_{x \to \infty} \dfrac{5x^3 + 1}{10x^3 - 3x^2 + 7} =$

(a) $\dfrac{1}{2}$ (correct)

(b) $2$

(c) $\dfrac{1}{10}$

(d) $\dfrac{1}{7}$

(e) $\dfrac{5}{7}$

9. $\lim_{x\to\infty} x\sin\left(\frac{1}{x}\right) =$

(a) 1 (correct)

(b) 0

(c) $\infty$

(d) $-\infty$

(e) $-1$

$$\lim_{x\to\infty} \frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$$

$x\to\infty \equiv \frac{1}{x}\to 0$

$$\lim_{\frac{1}{x}\to 0} \frac{\sin\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)} = 1, \quad \lim_{x\to 0}\frac{\sin x}{x} = 1$$

10. If $f(x) = \frac{3}{(2x)^3} + 2\sin x$, then $f'\left(\frac{\pi}{2}\right)$ is equal to

(a) $\frac{-18}{\pi^4}$ (correct)

(b) $\frac{-36}{\pi^4}$

(c) $\frac{-9}{\pi^4}$

(d) $\frac{-72}{\pi^4}$

(e) $\frac{-27}{\pi^4}$

$$f(x) = \frac{3}{8x^3} + 2\sin x$$

$$= \frac{3}{8}x^{-3} + 2\sin x$$

$$f'(x) = \frac{3}{8}(-3)x^{-4} + 2\cos x$$

$$= \frac{-9}{8}\frac{1}{x^4} + 2\sin x$$

$$f'\left(\frac{\pi}{2}\right) = -\frac{9}{8}\left(\frac{2}{\pi}\right)^4 + 0 = \frac{-18}{\pi^4}$$

11. One statement is False about $f(x) = \begin{cases} \frac{1}{2}x + 2, & x < 2 \\ \sqrt{2x}, & x \geq 2 \end{cases}$

(a) $f'(2)$ exists (correct)

(b) $f(x)$ is differentiable over $(0, 2)$

(c) $\lim_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \frac{1}{2}$

(d) $f'(0)$ exisits

(e) $\lim_{x \to 0} \frac{f(0+h) - f(0)}{h} = \frac{1}{2}$

omitted

12. The equation of the tangent line to the curve $f(x) = x^2 + 2x - 1$ at the point $(1, 2)$ is:

(a) $y = 4x - 2$ (correct)

(b) $y = 4x + 2$

(c) $y = 2x - 4$

(d) $y = 2x + 4$

(e) $y = 2x - 1$

$y - 2 = f'(1)(x - 1)$

$f'(x) = 2x + 2$

$f'(1) = 4$

13. $\lim_{x\to\infty} e^x + e^{-x} + e =$

(a) $\infty$ (correct)

(b) $-\infty$

(c) $e$

(d) $2+e$

(e) $e+\frac{1}{e}$

$e^{\infty} + e^{-\infty} + e$

$\infty + 0 + e = \infty$

14. One statement is false about $f(x) = \frac{\sin x}{x}$

(a) $x = 0$ is a vertical asymptote (correct)

(b) $\lim_{x\to 0^+} f(x) = 1$

(c) $f(x)$ is an even function

(d) Domain of $f(x)$ is $(-\infty, 0) \cup (0, \infty)$

(e) $\lim_{x\to-\infty} f(x) = 0$

$\lim_{x\to 0} \frac{\sin x}{x} = 1$

$x = 0$ is Not