Code-wise Key is at The end.

1. If $\sqrt{x}+\dfrac{1}{x}\le h(x)\le x+e^{x-1}$ for all $x>0$, then $\lim\limits_{x\to 1} h(x) =$

The Squeeze Theorem

§2.3

(a) 2 ______________________________ (correct)

(b) 1

(c) 3

(d) 0

(e) does not exist

Since $\lim\limits_{x\to1}\left(\sqrt{x}+\frac{1}{x}\right) = 1+1 = 2$

& $\lim\limits_{x\to1}\left(x+e^{x-1}\right) = 1+1=2,$

then, by the Squeeze Theorem,

$\lim\limits_{x\to1} h(x) = 2.$

~ #37/2.5

2. $\lim\limits_{x\to 3^-}\dfrac{1-2x}{x-3} =$ $\longrightarrow \dfrac{-5}{0^-} \longrightarrow \infty$

(a) ∞ ______________________________ (correct)

(b) $-\infty$

(c) -2

(d) $-\dfrac{1}{3}$

(e) 0

~#43/2.3

3. $\lim_{x\to\frac{\pi}{4}} \dfrac{\sin^3 x - \cos^3 x}{\sin x - \cos x} = \lim_{x\to\frac{\pi}{4}} \dfrac{(\sin x - \cos x)(\sin^2 x + \sin x \cos x + \cos^2 x)}{(\sin x - \cos x)}$

$\frac{0}{0}$?

(a) $\dfrac{3}{2}$ ———————————————— (correct)

(b) 1

(c) 3

(d) $\dfrac{1}{2}$

(e) $\dfrac{1}{4}$

$= \lim_{x\to\frac{\pi}{4}} (\sin^2 x + \sin x \cos x + \cos^2 x)$

$= \left(\frac{1}{\sqrt{2}}\right)^2 + \frac{1}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}} + \left(\frac{1}{\sqrt{2}}\right)^2$

$= \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$

$= \frac{3}{2}$

~#53/2.3

4. $\lim_{x\to 2} \dfrac{x-2}{\sqrt{4x+8}-2x} = \lim_{x\to 2} \dfrac{x-2}{\sqrt{4x+8}-2x}\cdot\dfrac{\sqrt{4x+8}+2x}{\sqrt{4x+8}+2x}$

$\frac{0}{0}$?

(a) $-\dfrac{2}{3}$ ———————————————— (correct)

(b) $-\dfrac{4}{3}$

(c) $\dfrac{8}{3}$

(d) $\dfrac{1}{3}$

(e) 0

$= \lim_{x\to 2} \dfrac{(x-2)(\sqrt{4x+8}+2x)}{4x+8-4x^2}$ $\leftarrow = -4(x^2-x-2)$

$= -4(x-2)(x+1)$

$= \lim_{x\to 2} \dfrac{\sqrt{4x+8}+2x}{-4(x+1)}$

$= \dfrac{4+4}{-4(3)} = -\dfrac{8}{12} = -\dfrac{2}{3}$

~#61, 81
Sec 2.4

5. If

$$f(x) = \begin{cases} \dfrac{2x^2 - 9x + 4}{x - 4}, & x \neq 4 \\ 7a, & x = 4 \end{cases}$$

is **continuous** at $x = 4$, then $a =$

(a) 1 ______________________________ (correct)

(b) 0

(c) 2

(d) $\frac{1}{7}$

(e) $\frac{2}{7}$

f is Conts at $x = 4 \Rightarrow$

$\lim\limits_{x \to 4} f(x) = f(4)$

$\Rightarrow \lim\limits_{x \to 4} \dfrac{2x^2 - 9x + 4}{x - 4} = 7a$

$\Rightarrow \lim\limits_{x \to 4} \dfrac{(x-4)(2x-1)}{x-4} = 7a$

$\Rightarrow \lim\limits_{x \to 4} (2x - 1) = 7a$

$\Rightarrow 8 - 1 = 7a \Rightarrow 7 = 7a \Rightarrow a = 1$

~#30/4.5

6. $\lim\limits_{x \to -\infty} \dfrac{\sqrt{16x^4 + 2x^3} + 5x^2}{3x^2 - 2} =$

$\frac{\infty}{\infty}$? take x^2 as a common factor from Deno & Num.

(a) 3 ______________________________ (correct)

(b) $\frac{4}{3}$

(c) $\frac{5}{3}$

(d) 1

(e) $-\frac{1}{2}$

$\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^4(16 + \frac{2}{x})} + 5x^2}{3x^2 - 2}$

$= \lim\limits_{x \to -\infty} \dfrac{x^2\sqrt{16 + \frac{2}{x}} + 5x^2}{3x^2 - 2}$

$= \lim\limits_{x \to -\infty} \dfrac{x^2}{x^2} \cdot \dfrac{\sqrt{16 + \frac{2}{x}} + 5}{3 - \frac{2}{x^2}}$

$= \lim\limits_{x \to -\infty} \dfrac{\sqrt{16 + \frac{2}{x}} + 5}{3 - \frac{2}{x^2}}$

$= \dfrac{\sqrt{16 + 0} + 5}{3 - 0} = \dfrac{4 + 5}{3} = \dfrac{9}{3} = 3$

~#26/2.4

7. $\lim_{x\to 2^-} \dfrac{x^2 - [\![x]\!]}{2x + [\![1-x]\!]} =$ $\dfrac{4-1}{4-1} = \dfrac{3}{3} = 1$

(a) 1 ____________________ (correct)

(b) $\frac{3}{5}$

(c) $\frac{1}{2}$

(d) $\frac{2}{5}$

(e) $\frac{5}{3}$

$\cdot\ x < 2 \Rightarrow -x > -2$
$\Rightarrow 1-x > 1-2$
$\Rightarrow 1-x > -1$
$\Rightarrow \lim_{x\to 2^-} [\![1-x]\!] = -1$

~# 23/ Review Ch 2, p. 115

8. $\lim_{x\to 0} \dfrac{x\sin x}{1-\cos x} =$ $\lim_{x\to 0} \dfrac{x\sin x}{1-\cos x} \cdot \dfrac{1+\cos x}{1+\cos x}$

§2.3

$\frac{0}{0}$?

(a) 2 ____________________ (correct)

(b) 1

(c) 0

(d) ∞

(e) $\frac{1}{3}$

$= \lim_{x\to 0} \dfrac{x\sin x\,(1+\cos x)}{1-\cos^2 x} \leftarrow = \sin^2 x$

$= \lim_{x\to 0} \dfrac{x\,(1+\cos x)}{\sin x}$

$= \lim_{x\to 0} \dfrac{x}{\sin x} \cdot (1+\cos x)$

$= \lim_{x\to 0} \dfrac{1}{\left(\frac{\sin x}{x}\right)} \cdot (1+\cos x)$

$= \dfrac{1}{1} \cdot (1+1) = 2$

~#22/2.5
~#19/4.5

9. The graph of the function $f(x) = \dfrac{x^2 - 4x + 3}{2x^2 - x - 1}$ has

(a) One vertical asymptote and one horizontal asymptote. ________________(correct)

(b) Two vertical asymptotes and one horizontal asymptote.

(c) Two vertical asymptotes and two horizontal asymptotes.

(d) No vertical asymptote and one horizontal asymptote.

(e) one vertical asymptote and no horizontal asymptote.

$$f(x) = \frac{(x-1)(x-3)}{(x-1)(2x+1)}$$

$$= \frac{x-3}{2x+1}, \quad x \neq 1$$

$\lim\limits_{x \to \pm\infty} f(x) = \frac{1}{2}$; $\lim\limits_{x \to -\frac{1}{2}^+} f(x) = -\infty \Rightarrow x = -\frac{1}{2}$ is V.A.

$\Rightarrow y = \frac{1}{2}$ is H.A.

§2.5

10. Which one of the following statements is **FALSE**?
($\lim\limits_{x\to a} f(x) = \infty$ and $\lim\limits_{x\to a} g(x) = -\infty$, a is a real number)

(a) $\lim\limits_{x\to a}[f(x) + g(x)] = 0$ ____(correct)

(b) $\lim\limits_{x\to a}[f(x) - g(x)] = \infty$

(c) $\lim\limits_{x\to a}[-3f(x) + 4g(x)] = -\infty$

(d) $\lim\limits_{x\to a} \dfrac{a}{f(x)} = 0$

(e) $\lim\limits_{x\to a}[g(x)]^2 = \infty$

$a = 0$, $f(x) = \frac{2}{x^2}$, $g(x) = -\frac{1}{x^2}$

$\lim\limits_{x\to 0}[f(x) + g(x)] = \lim\limits_{x\to 0} \frac{1}{x^2} = \infty \neq 0$

11. The set of **all values** of x on which the function $f(x) = \dfrac{x}{(x-1)\sqrt{x+1}}$ is **continuous** is

~# 37, 41/2.4

(a) $(-1, 1) \cup (1, \infty)$ ——— (correct)

(b) $(1, \infty)$

(c) $(-1, 0) \cup (0, 1) \cup (1, \infty)$

(d) $(-\infty, -1) \cup (-1, \infty)$

(e) $[-1, 1)$

Deno: $(x-1)\sqrt{x+1}$

$\Rightarrow x - 1 \neq 0$ & $x + 1 > 0$

$\Rightarrow x \neq 1$ & $x > -1$

f is conts on $(-1, 1) \cup (1, \infty)$

12. The **Intermediate Value Theorem** guarantees that the function $f(x) = x^3 - 2x + 3$ has a zero in the interval

~# 91, 100/2.4

(a) $[-2, 0]$ ——— (correct) $f(-2) = -8 + 4 + 3 = -1 < 0$; $f(0) = 3 > 0$ ✓

(b) $[-1, 0]$ ⟶ $f(-1) = -1 + 2 + 3 = 4 > 0$; $f(0) = 3 > 0$ ✗

(c) $[0, 1]$ ⟶ $f(0) = 3 > 0$; $f(1) = 1 - 2 + 3 = 2 > 0$ ✗

(d) $[1, 2]$ ⟶ $f(1) = 1 - 2 + 3 = 2 > 0$; $f(2) = 8 - 4 + 3 = 7 > 0$ ✗

(e) $[-3, -2]$ $f(-3) = -27 + 6 + 3 < 0$; $f(-2) = -8 + 4 + 3 = -1 < 0$ ✗

13. If the line $y = 2x - 4$ is tangent to the graph of $f(x) = kx^3$, then $k =$

~#73/3.2

Let (x_0, y_0) be the point of tangency. Then

(a) $\frac{2}{27}$ (correct)

(b) $\frac{2}{9}$

(c) $\frac{1}{3}$

(d) $\frac{4}{3}$

(e) $\frac{4}{9}$

$k x_0^3 = 2x_0 - 4$ —— (1)

$3k x_0^2 = 2$ —— (2) : slope

• (2) $\Rightarrow k \neq 0$, $x_0 \neq 0$

(2) $\Rightarrow k = \frac{2}{3x_0^2}$ —— (3)

$\overset{(1)}{\Rightarrow} \frac{2}{3x_0^2} x_0^3 = 2x_0 - 4$

$\Rightarrow \frac{2}{3} x_0 = 2x_0 - 4$

$\Rightarrow 2x_0 = 6x_0 - 12$

$\Rightarrow 4x_0 = 12$

$\Rightarrow \boxed{x_0 = 3}$

$\overset{(3)}{\Longrightarrow} k = \frac{2}{3(3)^2} = \frac{2}{27}$

14. If $y = Ax + B$ is an equation of the tangent line to the graph of $f(x) = \frac{4}{3x+1}$ at the point $(-1, -2)$, then $AB =$

~#25/3.1

(a) 15 (correct)

(b) 6

(c) −9

(d) −12

(e) −3

• slope $= \lim_{x \to -1} \frac{f(x) - f(-1)}{x+1}$

$= \lim_{x \to -1} \frac{\frac{4}{3x+1} + 2}{x+1} = \lim_{x \to -1} \frac{\frac{4 + 6x + 2}{3x+1}}{x+1}$

$= \lim_{x \to -1} \frac{\frac{6(x+1)}{3x+1}}{x+1} = \lim_{x \to -1} \frac{6}{3x+1} = \frac{6}{-3+1} = \frac{6}{-2} = -3$

• Eq: $y + 2 = -3(x+1)$

$\Rightarrow y = -3x - 3 - 2$

$\Rightarrow y = -3x - 5$

$A = -3$; $B = -5$

$AB = 15$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	D $_{3}$	A $_{2}$	A $_{5}$	B $_{2}$
2	A	A $_{2}$	D $_{3}$	D $_{3}$	E $_{1}$
3	A	B $_{4}$	B $_{1}$	E $_{1}$	C $_{4}$
4	A	B $_{1}$	C $_{4}$	A $_{4}$	C $_{3}$
5	A	C $_{5}$	A $_{5}$	B $_{2}$	B $_{5}$
6	A	D $_{9}$	B $_{6}$	A $_{9}$	E $_{10}$
7	A	C $_{7}$	D $_{10}$	C $_{6}$	D $_{7}$
8	A	B $_{8}$	A $_{9}$	B $_{8}$	A $_{8}$
9	A	B $_{10}$	C $_{8}$	C $_{7}$	E $_{9}$
10	A	A $_{6}$	A $_{7}$	A $_{10}$	C $_{6}$
11	A	C $_{11}$	A $_{12}$	A $_{13}$	D $_{13}$
12	A	C $_{14}$	D $_{11}$	E $_{11}$	A $_{11}$
13	A	A $_{12}$	E $_{14}$	A $_{14}$	D $_{12}$
14	A	D $_{13}$	E $_{13}$	A $_{12}$	E $_{14}$