Code-wise Key is at the end

1. If $y = Ax + B$ is an equation of the **tangent line** to the graph of $f(x) = 2 + e^x \tan x$ at the point $(0, 2)$, then $A^2 + B^2 =$

~#23 §3.3

(a) 5 ______ (correct)

(b) 4

(c) 1

(d) 8

(e) 2

$f'(x) = 0 + e^x . \sec^2 x + \tan x . e^x$

slope $= f'(0) = 0 + 1 + 0 = 1$

Eq: $y - 2 = 1(x - 0)$

$\Rightarrow y = x + 2$

, $A = 1$, $B = 2 \Rightarrow A^2 + B^2 = 1 + 4 = 5$

~#12 §3.3

2. If $y = \dfrac{x^3+3}{x^2+2}$, then $\dfrac{dy}{dx} = \dfrac{(x^2+2) \cdot 3x^2 - (x^3+3) \cdot 2x}{(x^2+2)^2}$

$= \dfrac{3x^4 + 6x^2 - 2x^4 - 6x}{(x^2+2)^2}$

$= \dfrac{x^4 + 6x^2 - 6x}{(x^2+2)^2}$

(a) $\dfrac{x^4 + 6x^2 - 6x}{(x^2+2)^2}$ ______ (correct)

(b) $\dfrac{x^4 + 3x^2 - 2x}{(x^2+2)^2}$

(c) $\dfrac{-x^3 + x^2 - 1}{(x^2+2)^2}$

(d) $\dfrac{3x^2 - 2x}{(x^2+2)^2}$

(e) $\dfrac{x^3 - x^2 + 1}{x^2+2}$

~#15
§3.7

3. All edges of a cube are expanding at a rate of 3 cm/s. How fast is the volume changing when each edge is 2 cm?

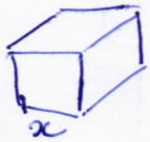
(a) $36\,cm^3/s$ ______________________________ (correct)

(b) $12\,cm^3/s$

(c) $18\,cm^3/s$

(d) $27\,cm^3/s$

(e) $9\,cm^3/s$



$\frac{dx}{dt} = 3$

$\frac{dV}{dt} = ?$ when $x = 2$.

$V = x^3 \Rightarrow \frac{dV}{dt} = 3x^2 \frac{dx}{dt}$

$= 3(2)^2 \cdot 3$

$= 9 \cdot 4$

$= 36$

Formula
§ 3.6

4. Let f be a differentiable function and have an inverse on $(-\infty, \infty)$. Assume that $f(2) = 6$, $f'(2) = 4$, $f(6) = 1$, and $f'(6) = 2$. Then $(f^{-1})'(6) =$

(a) $\frac{1}{4}$ ______________________________ (correct)

(b) $\frac{1}{2}$

(c) 1

(d) $\frac{1}{6}$

(e) $\frac{2}{3}$

$(f^{-1})'(6) = \frac{1}{f'(f^{-1}(6))}$

$= \frac{1}{f'(2)}$

$= \frac{1}{4}$

#14
§3.8

5. Using **Newton's Method** to approximate the zero(s) of the function $f(x) = x - 3 + \ln x$, we get the iterative formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad f'(x) = 1 + \frac{1}{x} = \frac{x+1}{x}$$

(a) $x_{n+1} = \dfrac{4x_n - x_n \ln x_n}{x_n + 1}$ ________________ (correct)

(b) $x_{n+1} = \dfrac{2x_n^2 - 2x_n + x_n \ln x_n}{x_n + 1}$

(c) $x_{n+1} = \dfrac{x_n^2 - x_n + x_n \ln x_n}{x_n + 1}$

(d) $x_{n+1} = \dfrac{-2x_n - x_n \ln x_n}{x_n + 1}$

(e) $x_{n+1} = \dfrac{4x_n + 2x_n \ln x_n}{x_n + 1}$

$$= x_n - \frac{x_n - 3 + \ln x_n}{\frac{x_n+1}{x_n}}$$

$$= x_n - \frac{x_n^2 - 3x_n + x_n \ln x_n}{x_n + 1}$$

$$= \frac{x_n^2 + x_n - x_n^2 + 3x_n - x_n \ln x_n}{x_n + 1}$$

$$= \frac{4x_n - x_n \ln x_n}{x_n + 1}$$

§3.3 (# 129 → 134)
+ Chain Rule

6. If $f(x) = \dfrac{6}{4x+2}$, then $f^{(100)}(1) =$

(a) $\left(\dfrac{2}{3}\right)^{100} \cdot 100!$ ________________ (correct)

(b) $-\left(\dfrac{2}{3}\right)^{100} \cdot 100!$

(c) $6 \cdot \left(\dfrac{2}{3}\right)^{100} \cdot 100!$

(d) $\dfrac{2^{100}}{3^{101}} \cdot 100!$

(e) $-\dfrac{2^{100}}{3^{99}} \cdot 99!$

$$f(x) = 6 \cdot (4x+2)^{-1}$$

$$f'(x) = 6 \cdot (-1)(4x+2)^{-2} \cdot 4$$

$$f''(x) = 6 \cdot (-1)(-2)(4x+2)^{-3} \cdot 4^2$$

$$f'''(x) = 6 \cdot (-1)(-2)(-3)(4x+2)^{-4} \cdot 4^3$$

$$\vdots$$

$$f^{(n)}(x) = 6 \cdot (-1)(-2)(-3)\cdots(-n)(4x+2)^{-(n+1)} \cdot 4^n$$

$$= 6 \cdot (-1)^n \cdot n! \cdot \frac{1}{(4x+2)^{n+1}} \cdot 4^n$$

$$f^{(100)}(1) = 6 \cdot 1 \cdot 100! \cdot \frac{1}{6^{101}} \cdot 4^{100}$$

$$= 100! \cdot \frac{1}{6^{100}} \cdot 4^{100}$$

$$\frac{4}{6} = \frac{2}{3}$$

$$= \left(\frac{2}{3}\right)^{100} \cdot 100!$$

~#153, 154
§3.4

7. Let $f(1) = -3$, $f'(1) = 6$, $g(2) = 2$, and $g'(2) = 4$. If $h(x) = f\left(\frac{x}{g(x)}\right)$, then $h'(2) =$

$$h'(x) = f'\left(\frac{x}{g(x)}\right) \cdot \frac{g(x) \cdot 1 - x\, g'(x)}{(g(x))^2}$$

(a) -9 ________________ (correct)

(b) 27

(c) $-\frac{3}{2}$

(d) $\frac{9}{2}$

(e) -3

$$h'(2) = f'\left(\frac{2}{g(2)}\right) \cdot \frac{g(2) - 2g'(2)}{(g(2))^2}$$

$$= f'(1) \cdot \frac{2 - 2 \cdot 4}{4}$$

$$= 6 \cdot \frac{-6}{4}$$

$$= -\frac{36}{4} = -9$$

~#16, 35
§3.5

8. The slope of the **tangent line** to the curve

$$2 + \tan(xy) = x^2 + y^2 + \tan(1)$$

at the point $(1, 1)$ is equal to

(a) -1 ________________ (correct)

(b) $\dfrac{2 - \sec^2 1}{1 - \sec^2 1}$

(c) 2

(d) $\dfrac{\sec^2 1}{2 - \sec^2 1}$

(e) 0

$$0 + \sec^2(xy)\,[xy' + y] = 2x + 2y \cdot y' + 0$$

$x = 1, y = 1$

$$\sec^2(1)\,[y' + 1] = 2 + 2y'$$

$$\sec^2 1 \cdot y' + \sec^2 1 = 2 + 2y'$$

$$\sec^2 1 \cdot y' - 2y' = 2 - \sec^2 1$$

$$y'(\sec^2 1 - 2) = 2 - \sec^2 1$$

$$y' = \frac{2 - \sec^2 1}{\sec^2 1 - 2} = \frac{-(\sec^2 1 - 2)}{\sec^2 1 - 2}$$

$$= -1$$

9. If $y = \dfrac{(2+\sqrt{x})(3+\sqrt[3]{x})}{(4+\sqrt[4]{x})(5+\sqrt[5]{x})}$, then the value of $\dfrac{dy}{dx}$ at $x=1$ is equal to

~ #74
§3.5

$\ln y = \ln(2+\sqrt{x}) + \ln(3+\sqrt[3]{x}) - \ln(4+\sqrt[4]{x}) - \ln(5+\sqrt[5]{x})$

(a) $\dfrac{1}{15}$ (correct)

(b) $\dfrac{2}{5}$

(c) $\dfrac{5}{3}$

(d) $\dfrac{2}{15}$

(e) $\dfrac{7}{30}$

$\frac{1}{y}\cdot y' = \frac{1}{2+\sqrt{x}}\cdot\frac{1}{2\sqrt{x}} + \frac{1}{3+\sqrt[3]{x}}\cdot\frac{1}{3}x^{-2/3} - \frac{1}{4+\sqrt[4]{x}}\cdot\frac{1}{4}x^{-3/4} - \frac{1}{5+\sqrt[5]{x}}\cdot\frac{1}{5}\cdot x^{-4/5}$

$\longrightarrow x=1 \Rightarrow y = \frac{3\cdot 4}{5\cdot 6} = \frac{2}{5}$

$\frac{5}{2}y' = \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{3} - \frac{1}{5}\cdot\frac{1}{4} - \frac{1}{6}\cdot\frac{1}{5}$

$y' = \frac{2}{5}\left(\frac{1}{6}+\frac{1}{12}-\frac{1}{20}-\frac{1}{30}\right)$

$= \frac{1}{5}\left(\frac{1}{3}+\frac{1}{6}-\frac{1}{10}-\frac{1}{15}\right)$

$= \frac{1}{5}\left(\frac{3}{6}-\frac{5}{30}\right) = \frac{1}{5}\left(\frac{3}{6}-\frac{1}{6}\right) = \frac{1}{5}\cdot\frac{2}{6} = \frac{2}{30}$

$= \frac{1}{15}$

~ #78
§3.5

10. If $f(x) = \left(1+\dfrac{1}{x}\right)^x$, then $f'(1) =$

$\ln f(x) = \ln\left(1+\frac{1}{x}\right)^x = x\ln\left(1+\frac{1}{x}\right)$

(a) $-1+2\ln 2$ (correct)

(b) $-\dfrac{1}{4}+\dfrac{1}{2}\ln 2$

(c) $1+2\ln 2$

(d) $2-\ln 2$

(e) $2+2\ln 2$

$\frac{1}{f(x)}\cdot f'(x) = x\cdot\frac{1}{1+\frac{1}{x}}\cdot\left(0-\frac{1}{x^2}\right) + \ln\left(1+\frac{1}{x}\right)\cdot 1$

$\bullet\ x=1 \Rightarrow f(1) = \left(1+\frac{1}{1}\right)^1 = 2$

$\frac{1}{2}\cdot f'(1) = 1\cdot\frac{1}{2}(-1) + \ln 2$

$\Rightarrow f'(1) = -1 + 2\ln 2$

~ #38

§3.6 11. If $f(x) = x\tan^{-1}\left(\frac{x}{2}\right) - \ln(4+x^2)$, then $f'(x) =$

$$f'(x) = x \cdot \frac{1}{1+\left(\frac{x}{2}\right)^2} \cdot \frac{1}{2} + \tan^{-1}\left(\frac{x}{2}\right) \cdot 1 - \frac{1}{4+x^2} \cdot (2x)$$

(a) $\tan^{-1}\left(\frac{x}{2}\right)$ ——— (correct)

(b) $\tan^{-1}\left(\frac{x}{2}\right) - \frac{x}{4+x^2}$

(c) $x\tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{4+x^2}$

(d) $x\tan^{-1}\left(\frac{x}{2}\right) + \frac{4x}{4+x^2}$

(e) $\tan^{-1}\left(\frac{x}{2}\right) + \frac{4x}{4+x^2}$

$$= \frac{x}{\frac{4+x^2}{4}} \cdot \frac{1}{2} + \tan^{-1}\left(\frac{x}{2}\right) - \frac{2x}{4+x^2}$$

$$= \frac{2x}{4+x^2} + \tan^{-1}\left(\frac{x}{2}\right) - \frac{2x}{4+x^2}$$

$$= \tan^{-1}\left(\frac{x}{2}\right)$$

$$\frac{3\sqrt{3}}{16} > \frac{1}{4} \iff 3\sqrt{3} > 4$$
$$\iff 9 \cdot 3 > 16$$
$$\iff 27 > 16$$

~ #33, 34

§4.1

12. The **maximum value** of $f(x) = \frac{x}{(x^2+1)^2}$ on $[-1, 1]$ is equal to

$$f'(x) = \frac{(x^2+1)^2 \cdot 1 - x \cdot 2(x^2+1) \cdot 2x}{(x^2+1)^4}$$

(a) $\frac{3\sqrt{3}}{16}$ ——— (correct)

(b) $\frac{1}{4}$

(c) 0

(d) $\frac{\sqrt{3}}{4}$

(e) $\frac{3\sqrt{3}}{8}$

$$= \frac{x^2+1-4x^2}{(x^2+1)^3} = \frac{1-3x^2}{(x^2+1)^3}$$

$$f'(x) = 0 \Rightarrow 1 - 3x^2 = 0 \Rightarrow x^2 = \frac{1}{3} \Rightarrow x = \pm\frac{1}{\sqrt{3}} \in (-1, 1)$$

Critical #'s

$$f\left(-\frac{1}{\sqrt{3}}\right) = \frac{-1/\sqrt{3}}{\left(\frac{1}{3}+1\right)^2} = \frac{-1/\sqrt{3}}{\frac{16}{9}} = -\frac{9}{16\sqrt{3}} = -\frac{3\sqrt{3}}{16}$$

$$f\left(\frac{1}{\sqrt{3}}\right) = \frac{3\sqrt{3}}{16}$$ * max value

$$f(-1) = \frac{-1}{(1+1)^2} = -\frac{1}{4}$$

$$f(1) = \frac{1}{(1+1)^2} = \frac{1}{4}$$

13. The value(s) of c satisfying the conclusion of the **Mean Value Theorem** when applied to $f(x) = \sqrt{2-x}$ on the interval $[-7, 2]$ is (are)

#50
§4.2

(a) $-\frac{1}{4}$ ________________________________ (correct)

(b) $-\frac{1}{4}$ and $\frac{1}{4}$

(c) $-\frac{1}{2}$

(d) -2 and -4

(e) $-\frac{2}{3}$

$\frac{f(2)-f(-7)}{2-(-7)} = \frac{-1}{2\sqrt{2-c}} = f'(c)$; $f'(x) = \frac{-1}{2\sqrt{2-x}}$

$\frac{0-3}{9} = \frac{-1}{2\sqrt{2-c}}$

$-\frac{1}{3} = -\frac{1}{2\sqrt{2-c}} \Longrightarrow \frac{1}{9} = \frac{1}{4(2-c)}$

$\Longrightarrow 2-c = \frac{9}{4} \Longrightarrow c = 2-\frac{9}{4}$

$\Rightarrow c = -\frac{1}{4} \in (-7, 2)$.

~#83
§3.3

14. If (a, b), where $a > 1$, is the point on the graph of $f(x) = \frac{x+2}{x-2}$ at which the tangent line is **parallel** to the line $4x + y = 1$, then $4a + b =$

• $\hookrightarrow y = -4x + 1 \Rightarrow$ Slope $= -4$

(a) 17 ________________________________ (correct)

(b) 7

(c) -8

(d) -2

(e) 1

• $f'(x) = \frac{(x-2)\cdot 1 - (x+2)\cdot 1}{(x-2)^2} = \frac{-4}{(x-2)^2}$

• $f'(x) = -4 \Longrightarrow \frac{-4}{(x-2)^2} = -4 \Rightarrow (x-2)^2 = 1$

$\Longrightarrow x-2 = -1$ or $x-2 = 1$

$\Longrightarrow x = 1$ or $x = 3$

$a = 3$ $(a>1)$

$\Rightarrow b = f(a) = f(3) = \frac{3+2}{3-2} = 5$

• $4a+b = 4(3)+5 = 12+5 = 17$

Math 101, 243, Major Exam II

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	D $_{3}$	B $_{3}$	A $_{4}$	A $_{3}$
2	A	C $_{1}$	E $_{4}$	D $_{1}$	E $_{4}$
3	A	B $_{2}$	B $_{2}$	C $_{2}$	C $_{2}$
4	A	B $_{4}$	A $_{1}$	A $_{3}$	B $_{1}$
5	A	D $_{5}$	C $_{5}$	D $_{8}$	A $_{9}$
6	A	A $_{8}$	A $_{9}$	A $_{5}$	D $_{8}$
7	A	A $_{9}$	C $_{8}$	E $_{7}$	C $_{5}$
8	A	D $_{6}$	A $_{6}$	E $_{6}$	D $_{7}$
9	A	E $_{7}$	C $_{7}$	C $_{9}$	D $_{6}$
10	A	C $_{11}$	B $_{14}$	B $_{10}$	D $_{10}$
11	A	D $_{12}$	D $_{11}$	D $_{11}$	C $_{13}$
12	A	E $_{14}$	D $_{13}$	A $_{12}$	A $_{11}$
13	A	E $_{13}$	B $_{12}$	A $_{14}$	A $_{14}$
14	A	E $_{10}$	C $_{10}$	D $_{13}$	B $_{12}$