Key Code-wise is at the last page

1. An equation of the **tangent line** to the curve $y = \dfrac{1}{x}$ at the point $\left(2, \dfrac{1}{2}\right)$ is

~ #36
§ 3.1

(a) $x + 4y = 4$ ________ (correct)

(b) $x - 4y = 0$

(c) $8x + 2y = 17$

(d) $3x + 4y = 8$

(e) $x + 4y = 1$

$y' = -\dfrac{1}{x^2}$

slope $= y'\big|_{x=2} = -\dfrac{1}{4}$

Eq: $y - \dfrac{1}{2} = -\dfrac{1}{4}(x-2)$

$\Rightarrow y - \dfrac{1}{2} = -\dfrac{1}{4}x + \dfrac{1}{2}$

$\xrightarrow{4} 4y - 2 = -x + 2$

$\Rightarrow x + 4y = 4$

~ #53
§ 3.4

2. If $f(x) = \sqrt{\dfrac{x+1}{x-1}}$, then $f'(3) =$

$f'(x) = \dfrac{1}{2}\left(\dfrac{x+1}{x-1}\right)^{-1/2} \cdot \dfrac{(x-1)\cdot 1 - (x+1)\cdot 1}{(x-1)^2}$

(a) $-\dfrac{\sqrt{2}}{8}$ ________ (correct)

$= \dfrac{1}{\cancel{2}}\left(\dfrac{x+1}{x-1}\right)^{-1/2} \cdot \dfrac{-\cancel{2}}{(x-1)^2}$

(b) $\dfrac{\sqrt{2}}{4}$

(c) $-\dfrac{\sqrt{2}}{2}$

$f'(3) = \left(\dfrac{4}{2}\right)^{-1/2} \cdot \dfrac{-1}{2^2}$

(d) $\sqrt{2}$

$= 2^{-1/2} \cdot \dfrac{-1}{4}$

(e) $-\dfrac{\sqrt{2}}{6}$

$= \dfrac{1}{\sqrt{2}} \cdot \dfrac{-1}{4} \cdot \dfrac{\sqrt{2}}{\sqrt{2}}$

$= \dfrac{-\sqrt{2}}{2 \cdot 4} = -\dfrac{\sqrt{2}}{8}$

~#32
§4.8

3. The **tangent line approximation** of $f(x) = \frac{\pi}{4} - \arctan(x-2)$ at the point $(3, 0)$ is given by

$f'(x) = 0 - \frac{1}{1+(x-2)^2}$

(a) $y = \frac{3-x}{2}$ ______ (correct)

(b) $y = 3 - x$

(c) $y = \frac{x-3}{2}$

(d) $y = 2x - 6$

(e) $y = 1 - \frac{x}{3}$

$f'(3) = \frac{-1}{1+1} = -\frac{1}{2}$

$y = f(3) + f'(3)(x-3)$

$= 0 - \frac{1}{2}(x-3)$

$= \frac{3-x}{2}$

~#19
§4.6

4. The **slant asymptote** for the graph of $f(x) = \frac{2x^2 - 3x + 2}{x-1}$ is

(a) $y = 2x - 1$ ______ (correct)

(b) $y = 2x + 1$

(c) $y = 2x + 2$

(d) $y = x + 2$

(e) $y = x - 2$

$$
\begin{array}{r|l}
 & 2x - 1 \\
x-1 & 2x^2 - 3x + 2 \\
 & 2x^2 - 2x \\ \hline
 & -x + 2 \\
 & -x + 1 \\ \hline
 & 1
\end{array}
$$

$y = 2x - 1$

~#126
§3.4

5. If $y = \cos^4 x$, then $y'' =$

$\cdot\, y' = 4\cos^3 x \cdot -\sin x$
$= -4\cos^3 x \cdot \sin x$

(a) $-4\cos^2 x \cdot (\cos^2 x - 3\sin^2 x)$ ______ (correct)

(b) $-2\cos^2 x \cdot (2\cos^2 x - \sin^2 x)$

(c) $3\cos^2 x \cdot (\cos^2 x - 2\sin^2 x)$

(d) $4\cos^2 x \cdot (\cos^2 x - 2\sin^2 x)$

(e) $\cos^2 x \cdot (-4\cos^2 x + 3\sin^2 x)$

$y'' = -4\left[\cos^3 x \cdot \cos x + \sin x \cdot 3\cos^2 x \cdot (-\sin x)\right]$
$= -4\left[\cos^4 x - 3\cos^2 x \cdot \sin^2 x\right]$
$= -4\cos^2 x\left[\cos^2 x - 3\sin^2 x\right]$

~#19
§5.9

6. If $\sinh x = -\frac{2}{3}$, then $\coth x =$

$\cdot\, \cosh^2 x - \sinh^2 x = 1$
$\cosh^2 x = 1 + \sinh^2 x = 1 + \left(-\frac{2}{3}\right)^2 = 1 + \frac{4}{9}$
$= \frac{13}{9}$

(a) $-\frac{\sqrt{13}}{2}$ ______ (correct)

(b) $\frac{\sqrt{13}}{2}$

(c) $2\sqrt{3}$

(d) $-\frac{\sqrt{13}}{3}$

(e) $-\frac{\sqrt{13}}{4}$

$\Rightarrow \cosh x = \frac{\sqrt{13}}{3}$

$\cdot\, \coth x = \frac{\cosh x}{\sinh x}$
$= \frac{\sqrt{13}/3}{-2/3} = -\frac{\sqrt{13}}{2}$

~#96

Review Ch5 p.384

7. If $y = \sqrt{\cosh(\sqrt{x})}$, then $y \cdot y' =$

(a) $\dfrac{\sinh(\sqrt{x})}{4\sqrt{x}}$ ________________ (correct)

(b) $\dfrac{\sinh(\sqrt{x})}{\sqrt{x}}$

(c) $\sinh(\sqrt{x})$

(d) $2\sqrt{x}\sinh(\sqrt{x})$

(e) $\dfrac{\sinh(\sqrt{x})}{2\sqrt{x}}$

$$y' = \frac{1}{2\sqrt{\cosh(\sqrt{x})}} \cdot \sinh(\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{1}{2y} \cdot \frac{\sinh(\sqrt{x})}{2\sqrt{x}}$$

$$y \cdot y' = \frac{\sinh(\sqrt{x})}{4\sqrt{x}}$$

#38

§3.7

8. In an electrical circuit, the voltage $V$, the resistance $R$, and the current $I$ are related by the equation

$$V = IR$$

If $R$ is increasing at a rate of 2 ohms per second and $V$ is increasing at a rate of 3 volts per second, then the rate at which $I$ is changing when $V = 12$ volts and $R = 4$ ohms is equal to

$\frac{dR}{dt} = 2$, $\frac{dV}{dt} = 3$, $\frac{dI}{dt} = ?$, $\underbrace{V = 12, R = 4}_{\Downarrow}$

$I = 3$

(a) $-\dfrac{3}{4}$ amperes/sec ________________ (correct)

(b) $-\dfrac{3}{2}$ amperes/sec

(c) $-\dfrac{5}{4}$ amperes/sec

(d) $-\dfrac{1}{4}$ amperes/sec

(e) $-\dfrac{1}{2}$ amperes/sec

$$V = IR$$

$$\frac{dV}{dt} = I\frac{dR}{dt} + R\frac{dI}{dt}$$

$$3 = 3 \cdot 2 + 4\frac{dI}{dt}$$

$$3 - 6 = 4\frac{dI}{dt}$$

$$\Rightarrow \frac{dI}{dt} = -\frac{3}{4}.$$

~#18137

§5.1 9. If $f'(x) = \sqrt[3]{x} + \frac{1}{2\sqrt{x}}$, $f(1) = \frac{3}{4}$, then $f(4) =$

(a) $3\sqrt[3]{4} + 1$ ________ (correct)

(b) 4

(c) $\sqrt[3]{4} + 1$

(d) $2\sqrt[3]{4} - 1$

(e) $3\sqrt[3]{4} + 2$

$f'(x) = x^{1/3} + \frac{1}{2}x^{-1/2}$

$f(x) = \frac{3}{4}x^{4/3} + \frac{1}{2}\cdot 2\, x^{1/2} + C$

$f(x) = \frac{3}{4}x^{4/3} + \sqrt{x} + C$

$f(1) = \frac{3}{4} + 1 + C \Rightarrow \frac{3}{4} = \frac{3}{4} + 1 + C \Rightarrow \boxed{C = -1}$

$\Rightarrow f(x) = \frac{3}{4}x^{4/3} + \sqrt{x} - 1$

$f(4) = \frac{3}{4}4^{4/3} + \sqrt{4} - 1$

$= \frac{3}{4}\cdot 4\sqrt[3]{4} + 2 - 1$

$= 3\sqrt[3]{4} + 1$

$4^{4/3} = \sqrt[3]{4^4}$

$= 4\sqrt[3]{4}$

~#51, 66, Review ch2

10. The set of all values of $x$ at which $f(x) = \ln(1 - \sqrt{x-1})$ is **continuous** is

(a) $[1, 2)$ ________ (correct)

(b) $[1, \infty)$

(c) $(-\infty, 1) \cup [2, \infty)$

(d) $[1, 4)$

(e) $(1, 2)$

$1 - \sqrt{x-1} > 0$ & $x - 1 \geq 0$

$\Rightarrow \sqrt{x-1} < 1$ & $x \geq 1$

$\Rightarrow x - 1 < 1$

$\Rightarrow x < 2$ & $x \geq 1$

$\Rightarrow [1, 2)$.

~ #89

§3.1 11. If

$$f(x) = \begin{cases} bx^2 & x \le 1 \\ ax - 2 & x > 1 \end{cases}$$

$$\cdot\ f'(x) = \begin{cases} 2bx & x < 1 \\ a & x > 1 \end{cases}$$

is **differentiable** at $x = 1$, then $ab =$

$\cdot$ continuity at $x = 1$ : $\lim_{x\to1^-} f(x) = \lim_{x\to1^+} f(x)$

$\lim_{x\to1^-} bx^2 = \lim_{x\to1^+} (ax-2)$

(a) 8 ______________________________ (correct)

(b) −4 $\boxed{b = a-2}$ ~ (1)

(c) −1

(d) 6 $\cdot$ diff. at $x = 1$ : $f'_-(1) = f'_+(1)$

(e) 3 $\Rightarrow \boxed{2b = a}$ ~ (2)

(2) in (1) : $b = 2b - 2 \Rightarrow b = 2$

$\overset{(2)}{\Longrightarrow} a = 4$

so $ab = (4)(2)$

$= 8$

#2, Ch4 Review, p. 278

12. If $M$ and $m$ are, respectively, the **absolute maximum value** and **absolute minimum value** of

$$f(x) = x^3 + 6x^2$$

on the interval $[-6, 1]$, then $M + m =$

$f'(x) = 3x^2 + 12x$

$= 3x(x+4)$

$f'(x) = 0 \Rightarrow x = 0,\ x = -4$

(a) 32 ______________________________ (correct)

(b) 0

(c) 39

(d) −7

(e) 22

$f(0) = 0$

$f(-4) = -64 + 96 = 32$

$f(-6) = -6^3 + 6^3 = 0$

$f(1) = 1 + 6 = 7$

$\Rightarrow M = 32,\ m = 0$

$\Rightarrow M + m = 32 + 0 = 32$

Example 8, §4.6

13. The **absolute minimum** value of $f(x) = \ln(x^2 + 2x + 3)$ is equal to

(a) $\ln 2$ ______ (correct)

(b) $\ln 3$

(c) $2 \ln 2$

(d) $\ln 6$

(e) $\frac{1}{2} \ln 3$

$f'(x) = \dfrac{2x+2}{x^2+2x+3}$

. $f'(x) = 0 \Rightarrow 2x + 2 = 0 \Rightarrow x = -1$

. $x^2 + 2x + 3 = 0 \Rightarrow$ Complex roots.

$f'$: $-$ on the left of $-1$ ($\searrow$), $+$ on the right ($\nearrow$)

. Local min

$\Rightarrow$ ~~global~~ absolute min (one critical number)

abs. min value is

$f(-1) = \ln(1 - 2 + 3)$

$= \ln 2$

#31 §4.4

14. The graph of the function $f(x) = e^{-3/x}$

Domain: $(-\infty, 0) \cup (0, \infty)$

$f'(x) = e^{-3/x} \cdot \dfrac{3}{x^2} = 3 \dfrac{e^{-3/x}}{x^2}$

(a) has one inflection point ______ (correct)

(b) has two inflection points

(c) is concave upward on $(0, \infty)$

(d) is concave upward on $\left(\frac{3}{2}, \infty\right)$

(e) is concave downward on $\left(0, \frac{3}{2}\right)$

$f''(x) = 3 \cdot \dfrac{x^2 \cdot e^{-3/x} \cdot \frac{3}{x^2} - e^{-3/x} \cdot 2x}{x^4}$

$= 3 \cdot \dfrac{3e^{-3/x} - 2x e^{-3/x}}{x^4}$

$= 3 e^{-3/x} \dfrac{3 - 2x}{x^4} \longrightarrow x = \frac{3}{2}$

$f''$: $+$ ($\cup$) on the left of $0$, $+$ ($\cup$) between $0$ and $\frac{3}{2}$, $-$ ($\cap$) on the right of $\frac{3}{2}$

$\uparrow$ Inf. point

#52 §4.3

15. The graph of the function $f(x) = \dfrac{x^3}{3} - \ln x$ is

Domain : $(0, \infty)$

(a) increasing on $(1, \infty)$ ______________________(correct)

(b) increasing on $(0, 1)$

(c) increasing on $(0, \infty)$

(d) decreasing on $(1, \infty)$

(e) decreasing on $(0, \infty)$

$f'(x) = x^2 - \frac{1}{x} = \frac{x^3 - 1}{x}$

- $x^3 - 1 = 0 \Rightarrow x = 1$
- $x = 0 \notin$ domain

$f'$: $-$ on $(0, 1)$, $+$ on $(1, \infty)$; $\searrow$ then $\nearrow$

$f$ is increasing on $(1, \infty)$

~#28 §4.3

16. If the graph of $f(x) = x^3 + 3kx^2 + 1$ has a **local minimum** at $x = 10$, then $k =$

(a) $-5$ ______________________(correct)

(b) $5$

(c) $\dfrac{3}{2}$

(d) $-\dfrac{3}{2}$

(e) $-3$

$f'(x) = 3x^2 + 6kx$

$= 3x(x + 2k)$

$f'(x) = 0 \Rightarrow x = 0, \; x = -2k$ ← possible local max/min

$= 10$ ← ~~possible~~ local min

$\Rightarrow \boxed{k = -5}$

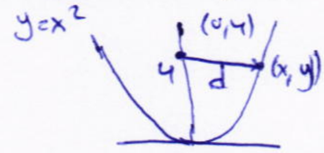
check

$f'$: $+$ before 0, $-$ between 0 and 10, $+$ after 10; $\nearrow$ $\searrow$ $\nearrow$

↑ local min — OK

~ # 15

§ 4.7 17. If $(a, b)$, where $a > 0$, is the point on the curve $y = x^2$ that is **closest** to the point $(0, 4)$, then $ab =$




$d^2 = (x-0)^2 + (y-4)^2$

$f(x) = x^2 + (x^2-4)^2 = x^2 + x^4 - 8x^2 + 16$

(a) $\frac{7\sqrt{14}}{4}$ ______ (correct)

$f(x) = x^4 - 7x^2 + 16$

(b) $\frac{7\sqrt{14}}{2}$

$f'(x) = 4x^3 - 14x = 2x(2x^2-7)$

(c) $\frac{\sqrt{14}}{2}$

$f'(x) = 0 \Rightarrow x = 0,\ x = \sqrt{\frac{7}{2}},\ x = -\sqrt{\frac{7}{2}}$

$\Rightarrow x = \sqrt{\frac{7}{2}}$ as $x > 0$ $(a>0)$

(d) $\frac{\sqrt{14}}{4}$

(e) $\frac{\sqrt{14}}{8}$

. $f''(x) = 12x^2 - 14$

$f''(\sqrt{\frac{7}{2}}) = 12 \cdot \frac{7}{2} - 14 = 42 - 14 = 28 > 0 \Rightarrow$ l. min

So $x = \sqrt{\frac{7}{2}},\ y = x^2 = \frac{7}{2}$

$\Rightarrow a = \sqrt{\frac{7}{2}},\ b = \frac{7}{2}$

$\Rightarrow ab = \frac{7}{2}\frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{7\sqrt{14}}{4}$

0.∞ Form

18. $\lim_{x\to 0^+} x^2(\ln x)^3 =$ $0 \cdot (-\infty)$

$= \lim_{x\to 0^+} \frac{(\ln x)^3}{x^{-2}}$ $\frac{-\infty}{\infty}$

(a) 0 ______ (correct)

(b) $-\frac{3}{2}$

$\overset{L'R}{=} \lim_{x\to 0^+} \frac{3(\ln x)^2 \cdot \frac{1}{x}}{-2x^{-3}} = \lim_{x\to 0^+} -\frac{3}{2} \cdot \frac{(\ln x)^2}{x^{-2}}$ $\frac{\infty}{\infty}$

(c) $\frac{3}{2}$

$\overset{L'R}{=} \lim_{x\to 0^+} -\frac{3}{2} \cdot \frac{2\ln x \cdot \frac{1}{x}}{-2x^{-3}} = \lim_{x\to 0^+} +\frac{3}{2} \frac{\ln x}{x^{-2}}$ $\frac{-\infty}{\infty}$

(d) $-\frac{3}{4}$

(e) $-\infty$

$\overset{L'R}{=} \lim_{x\to 0^+} +\frac{3}{2} \cdot \frac{\frac{1}{x}}{-2x^{-3}} = \lim_{x\to 0^+} -\frac{3}{4} \cdot x^2 = \frac{-3}{4} \cdot 0 = 0$

a special case of #117, §5.6, $a=2$

19. $\lim_{x\to\infty}\left(\frac{2^x-1}{x}\right)^{\frac{1}{x}} = \infty^0$

$\cdot \lim_{x\to\infty} \frac{2^x-1}{x} \overset{L'R}{=} \lim_{x\to\infty} \frac{2^x \cdot \ln 2}{1} = \infty$

$\cdot\ y = \left(\frac{2^x-1}{x}\right)^{1/x}$

$\ln y = \frac{1}{x}\ln\left(\frac{2^x-1}{x}\right)$

(a) 2 ________ (correct)

(b) 1

(c) $\frac{1}{2}$

(d) $2\ln 2$

(e) $\frac{\ln 2}{2}$

$\lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln\left(\frac{2^x-1}{x}\right)}{x} \quad \frac{\infty}{\infty}$

$\overset{L'R}{=} \lim_{x\to\infty} \frac{\frac{x}{2^x-1} \cdot \frac{x \cdot 2^x \cdot \ln 2 - (2^x-1)\cdot 1}{x^2}}{1}$

$= \lim_{x\to\infty} \frac{x\, 2^x \ln 2 - (2^x-1)}{x(2^x-1)}$

$= \lim_{x\to\infty} \frac{2^x \ln 2}{2^x-1} - \frac{1}{x}$

L'R

$= \lim_{x\to\infty} \frac{\cancel{2^x}(\ln 2)^2}{\cancel{2^x} \cdot \ln 2} - 0 = \ln 2$

$\Rightarrow \lim_{x\to\infty} y = 2$

#97/§3.1

20. Which one of the following statements is **TRUE** about the function

$$f(x) = \begin{cases} x\sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

$f'(0) = \lim_{x\to 0} \frac{f(x)-f(0)}{x-0}$

$= \lim_{x\to 0} \frac{x\sin(\frac{1}{x}) - 0}{x}$

$= \lim_{x\to 0} \sin(\frac{1}{x})$ DNE

(a) $f'(0)$ does not exist ________ (correct)

(b) $f'(0) = 0$

(c) $f'(0) = 1$

(d) $f$ is not continuous at $x = 0$

(e) $f$ is not continuous at $x = 1$

$\lim_{x\to 0} f(x) = \lim_{x\to 0} x\sin(\frac{1}{x}) = 0 = f(0) \Rightarrow f$ is conts at $x=0$

↓

Squeeze Thm

$\Rightarrow f$ is conts on $(-\infty,\infty)$

Math 101, 243, Final Exam

**Answer KEY**

| Q | MASTER | CODE01 | CODE02 | CODE03 | CODE04 |
|---|---|---|---|---|---|
| 1 | A | A $_{4}$ | D $_{5}$ | D $_{4}$ | C $_{1}$ |
| 2 | A | D $_{5}$ | E $_{3}$ | B $_{5}$ | B $_{2}$ |
| 3 | A | B $_{3}$ | B $_{2}$ | A $_{2}$ | A $_{4}$ |
| 4 | A | D $_{2}$ | B $_{4}$ | C $_{1}$ | D $_{5}$ |
| 5 | A | B $_{1}$ | C $_{1}$ | C $_{3}$ | C $_{3}$ |
| 6 | A | B $_{9}$ | A $_{6}$ | C $_{8}$ | C $_{6}$ |
| 7 | A | D $_{8}$ | A $_{7}$ | B $_{6}$ | A $_{7}$ |
| 8 | A | C $_{6}$ | C $_{10}$ | B $_{7}$ | A $_{8}$ |
| 9 | A | A $_{10}$ | C $_{9}$ | A $_{9}$ | E $_{10}$ |
| 10 | A | D $_{7}$ | E $_{8}$ | D $_{10}$ | B $_{9}$ |
| 11 | A | A $_{14}$ | D $_{15}$ | C $_{13}$ | B $_{15}$ |
| 12 | A | D $_{15}$ | A $_{13}$ | B $_{11}$ | E $_{12}$ |
| 13 | A | A $_{11}$ | E $_{14}$ | E $_{14}$ | B $_{13}$ |
| 14 | A | E $_{12}$ | D $_{11}$ | C $_{12}$ | D $_{11}$ |
| 15 | A | D $_{13}$ | C $_{12}$ | E $_{15}$ | E $_{14}$ |
| 16 | A | D $_{17}$ | D $_{17}$ | D $_{20}$ | C $_{19}$ |
| 17 | A | C $_{19}$ | A $_{16}$ | E $_{18}$ | E $_{17}$ |
| 18 | A | E $_{16}$ | B $_{18}$ | B $_{17}$ | E $_{18}$ |
| 19 | A | E $_{18}$ | D $_{20}$ | B $_{19}$ | B $_{20}$ |
| 20 | A | A $_{20}$ | E $_{19}$ | B $_{16}$ | A $_{16}$ |