

1. If the points $P_1(2, 0)$, $P_2(0, y_1)$, and $P_3(1, y_2)$ lie on the graph of $y = f(x)$ and the slope of the secant line through P_1 and P_2 is 2 while the slope of the secant line through P_1 and P_3 is 3, then $f(0) + f(1) =$

Similar to Ex 7 & 8 §2.1

(a) -7 _____ (correct)

(b) 21

(c) 1

(d) -3

(e) -1

Let $m_1 =$ slope of the line through P_1 and P_2
 $\& \ m_2 =$ slope of the line through P_1 and P_3

$$\Rightarrow m_1 = \frac{y_1 - 0}{0 - 2} = 2 \quad \& \quad m_2 = \frac{y_2 - 0}{1 - 2} = 3$$

$$\Rightarrow y_1 = -4 \quad \& \quad y_2 = -3$$

$$\Rightarrow f(0) = y_1 = -4 \quad \& \quad f(1) = y_2 = -3$$

$$\therefore f(0) + f(1) = -7$$

Similar to

Ex. 16

§2.2

$$2. \lim_{x \rightarrow 2^-} \frac{\frac{x+1}{x} - \frac{3}{2}}{x-2} =$$

$$\lim_{x \rightarrow 2^-} \frac{2(x+1) - 3x}{2x(x-2)} = \lim_{x \rightarrow 2^-} \frac{2-x}{2x(x-2)}$$

(a) $-\frac{1}{4}$ _____ (correct)

(b) -1

(c) ∞

(d) $-\infty$

(e) $\frac{1}{2}$

$$= \lim_{x \rightarrow 2^-} \frac{-1}{2x} = -\frac{1}{4}$$

$$3. \lim_{x \rightarrow 1} \frac{|x+1| - |-x-1|}{x-1} = \lim_{x \rightarrow 1} \frac{\cancel{|x+1|} - \cancel{|x+1|}}{x-1} = \lim_{x \rightarrow 1} \frac{0}{x-1}$$

(a) 0 _____ (correct)

(b) 2

(c) -2

(d) DNE

(e) -1

$$= \lim_{x \rightarrow 1} 0 = 0$$

$$\text{Note } |-x| = |x|.$$

Ex. 73 & 74 Sec. 2.3

$$4. \lim_{x \rightarrow 0} \frac{1 - e^{-x} + 4(e^{2x} - 1)}{e^x - 1} = \lim_{x \rightarrow 0} \frac{1 - e^{-x}}{e^x - 1} + \lim_{x \rightarrow 0} \frac{4(e^{2x} - 1)}{e^x - 1}$$

(a) 9 _____ (correct)

(b) 7

(c) DNE

(d) $\frac{9}{7}$ (e) $\frac{7}{9}$

$$= \lim_{x \rightarrow 0} \frac{e^x(1 - e^{-x})}{e^x(e^x - 1)} + \lim_{x \rightarrow 0} \frac{4(\cancel{e^x - 1})(e^x + 1)}{\cancel{e^x - 1}}$$

$$= \lim_{x \rightarrow 0} \frac{(\cancel{e^x - 1})}{e^x(\cancel{e^x - 1})} + \lim_{x \rightarrow 0} \frac{4(e^x + 1)}{1}$$

$$= 1 + 4(2) = 9$$

$$5. \lim_{x \rightarrow 0} \frac{2 \cos x - 2}{x^2} = \lim_{x \rightarrow 0} \frac{2(\cos x - 1)}{x^2} \cdot \frac{(\cos x + 1)}{(\cos x + 1)}$$

Similar to Ex. 82

Sec. 2.3 (a) -1 _____ (correct)

(b) 1

(c) 0

(d) $\frac{1}{2}$

(e) DNE

$$= \lim_{x \rightarrow 0} \frac{2(\cos^2 x - 1)}{x^2 (\cos x + 1)} = \lim_{x \rightarrow 0} \frac{-2 \sin^2 x}{x^2 (\cos x + 1)}$$

$$= \lim_{x \rightarrow 0} -2 \left(\frac{\sin x}{x} \right)^2 \cdot \frac{1}{\cos x + 1}$$

$$= -2 \cdot (1)^2 \cdot \frac{1}{2} = -1$$

Similar to Ex. 26 Sec. 2.4

$$6. \lim_{x \rightarrow 2} (2x - \lfloor x \rfloor) =$$

(a) DNE _____ (correct)

(b) 2

(c) 3

(d) 1

(e) 0

$$\lim_{x \rightarrow 2^+} (2x - \lfloor x \rfloor) = 4 - 2 = 2$$

$$\lim_{x \rightarrow 2^-} (2x - \lfloor x \rfloor) = 4 - 1 = 3$$

$$\Rightarrow \lim_{x \rightarrow 2^+} (2x - \lfloor x \rfloor) \neq \lim_{x \rightarrow 2^-} (2x - \lfloor x \rfloor)$$

$\therefore$ DNE

7. The value of a for which the function

$$f(x) = \begin{cases} \frac{x^2 - a^2}{x - a}, & x \neq a \\ 8, & x = a \end{cases}$$

is continuous is equal to

Ex. 63 Sec. 2.4

(a) 4 _____ (correct)

(b) 8

(c) 2

(d) 0

(e) -2

for the function to be continuous at a
we have to have $\lim_{x \rightarrow a} f(x) = f(a)$

$$\therefore \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} \frac{(x-a)(x+a)}{(x-a)} = \lim_{x \rightarrow a} x+a = 2a$$

$$\therefore f(a) = 8 \Rightarrow 2a = 8 \Rightarrow \boxed{a = 4}$$

Ex. 49 Sec. 4.5

$$8. \lim_{x \rightarrow -\infty} (x + \sqrt{x^2 + 3}) = \lim_{x \rightarrow -\infty} x + \sqrt{x^2 + 3} \cdot \frac{x - \sqrt{x^2 + 3}}{x - \sqrt{x^2 + 3}}$$

(a) 0 _____ (correct)

(b) 3

(c) DNE

(d) $-\infty$

(e) ∞

$$= \lim_{x \rightarrow -\infty} \frac{x^2 - (x^2 + 3)}{x - \sqrt{x^2 + 3}}$$

$$= \lim_{x \rightarrow -\infty} \frac{-3}{x - \sqrt{x^2 + 3}} = 0$$

9. The graph of the function $f(x) = \frac{|x(x+1)|}{x^2-1}$ has

Similar to Ex. 43
Sec. 4.5

- (a) One horizontal and one vertical asymptotes. _____ (correct)
 (b) Two horizontal and one vertical asymptotes.
 (c) One horizontal and two vertical asymptotes.
 (d) Two horizontal and two vertical asymptotes.
 (e) No horizontal and one vertical asymptotes.

$$\therefore f(x) = \begin{cases} \frac{x(x+1)}{x^2-1} = \frac{x}{x-1} & \text{if } x < -1 \text{ or } x > 1 \\ -\frac{x(x+1)}{x^2-1} = -\frac{x}{x-1} & \text{if } -1 < x < 1 \end{cases}$$

$\therefore x=1$ is the only V.A.
 $\therefore \lim_{x \rightarrow \infty} f(x) = 1 = \lim_{x \rightarrow -\infty} f(x)$
 $y=1$ is the only H.A.

Ex 21. Sec. 2.5

10. The graph of the function $f(x) = \frac{4x^2 + 4x - 24}{x^4 - 2x^3 - 9x^2 + 18x}$ has

- (a) Two vertical asymptotes and two holes _____ (correct)
 (b) Three vertical asymptotes and one hole
 (c) Four vertical asymptotes and no holes
 (d) No vertical asymptotes and four holes
 (e) One vertical asymptote and one hole

$$\begin{aligned} f(x) &= \frac{4(x^2+x-6)}{x^3(x-2) - 9x(x-2)} = \frac{4(x+3)(x-2)}{x(x-2)(x^2-9)} \\ &= \frac{4(x+3)(x-2)}{x(x-2)(x-3)(x+3)} = \frac{4}{x(x-3)}, \quad x \neq -3, 2 \end{aligned}$$

$\therefore$ The graph of $f(x)$ has vertical asymptotes $x=0$ and $x=3$ and two holes at $x=-3$ and 2 .

11. If $f(x) = \begin{cases} x^2 + 1, & x \leq 2 \\ 4x - 2, & x > 2, \end{cases}$ Then

Similar to Ex. 89 Sec. 3.1

(a) f is not differentiable at $x = 2$ _____ (correct)

(b) $f'(2) = 4$

(c) $f'(2) = 2$

(d) $f'(2) = 1$

(e) $f'(2) = \frac{1}{2}$

$f(x)$ is not contin. at $x=2$
as $\lim_{x \rightarrow 2^+} f(x) = 2 \neq f(2) = 3$

$\Rightarrow f$ is not differentiable
at $x=2$.

Ex. 97 Sec. 3.1

12. If $f(x) = \begin{cases} x^2 \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0, \end{cases}$ Then

(a) $f'(0) = 0$ _____ (correct)

(b) f is not differentiable at $x = 0$

(c) $f'(0) = 1$

(d) $f'(0) = 2$

(e) $f'(0) = \frac{1}{2}$

$$f'(0) = \lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x} - f(0)}{x - 0}$$

$$= \lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x}}{x} = \lim_{x \rightarrow 0} \frac{\sin \frac{1}{x}}{\frac{1}{x}} = 0$$

Note: $\frac{1}{x} \rightarrow \infty$ as $x \rightarrow 0^+$

& $\frac{1}{x} \rightarrow -\infty$ as $x \rightarrow 0^-$

while $|\sin \frac{1}{x}| \leq 1$

13. The value of k for which the line $y = \frac{kx}{4} + 4$ is tangent to the graph of $f(x) = k\sqrt{x}$ is equal to

Similar to Ex. 74 sec. 3.2 $f'(x) = \frac{k}{2} x^{-1/2} = \frac{k}{2\sqrt{x}}$

(a) 4 _____ (correct)

(b) -4

(c) 2

(d) -2

(e) $\frac{1}{4}$

$\therefore$ The slope of the line = $f'(x)$ at the point of tangency $(x, f(x))$

$$\Rightarrow \frac{k}{4} = \frac{k}{2\sqrt{x}} \Rightarrow 2\sqrt{x} = 4 \Rightarrow \sqrt{x} = 2$$

$$\Rightarrow x = 4$$

$$\text{Now, at } x=4, k\sqrt{4} = \frac{k(4)}{4} + 4$$

$$\Rightarrow 2k = k + 4 \Rightarrow \boxed{k = 4}$$

Similar to Ex. 54 sec. 3.2

$$14. \frac{d}{dx} \left(e^2 + x^e + \frac{2}{\sqrt[3]{x}} + 3 \cos x \right) = 0 + e x^{e-1} + \frac{d}{dx} (2x^{-1/3}) - 3 \sin x$$

(a) $ex^{e-1} - \frac{2}{3x^{4/3}} - 3 \sin x$ _____ (correct)

(b) $ex^{e-1} + \frac{2}{3x^{4/3}} - 3 \sin x$

(c) $ex^{e-1} - \frac{2}{3x^{1/3}} + 3 \sin x$

(d) $2e + ex^{e-1} + 2x^{-2/3} - 3 \sin x$

(e) $ex^e \ln x - \frac{2}{3x^{4/3}} - 3 \cos x$

$$= e x^{e-1} - \frac{2}{3} x^{-4/3} - 3 \sin x$$