King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 101

Major Exam II

Term 251

11 November 2025

Net Time Allowed: 90 Minutes

MASTER VERSION

1. If $f(x)=\sqrt[3]{x}(\sqrt{x}+3)$, then $f'(x)=$

Ex # 36 Page 150

(a) $\dfrac{5x^{\frac{2}{3}}+6x^{\frac{1}{6}}}{6x^{\frac{5}{6}}}$ ________ (correct)

(b) $\dfrac{5x^{\frac{1}{6}}+6x^{\frac{2}{3}}}{6x^{\frac{5}{6}}}$

(c) $\dfrac{5x^{\frac{2}{3}}+x^{\frac{1}{6}}}{6x^{\frac{5}{6}}}$

(d) $\dfrac{x^{\frac{2}{3}}+x^{\frac{1}{6}}}{6x^{\frac{5}{6}}}$

(e) $\dfrac{x^{\frac{2}{3}}+6x^{\frac{2}{3}}}{6x^{\frac{5}{6}}}$

$\because f(x)=x^{1/3}\cdot x^{1/2}+3x^{1/3}=x^{5/6}+3x^{1/3}$

$\therefore f'(x)=\dfrac{5}{6x^{1/6}}+\dfrac{1}{x^{2/3}}$

$=\dfrac{5x^{2/3}+6x^{1/6}}{6x^{5/6}}$

2. If $y=\cos\sqrt{\sin(\tan \pi x)}$, then $y'\left(\dfrac{1}{4}\right)=$

Ex. # 50 P. 164

(a) $\dfrac{-\pi\sin(\sqrt{\sin 1})\,\cos 1}{\sqrt{\sin 1}}$ ________ (correct)

(b) $\dfrac{-\pi\sin(\sqrt{\sin 1})\,\cos 1}{\cos\sqrt{\sin 1}}$

(c) $\dfrac{-\pi\sin(\sqrt{\sin 1})\,\sin 1}{\sqrt{\cos 1}}$

(d) $\dfrac{-\pi\csc(\sqrt{\sin 1/2})\,\cos 1}{\sqrt{\sin 1}}$

(e) $\dfrac{-\pi\sin(\sqrt{\sin 2})\,\cos 1}{\sqrt{2}\cos\sqrt{\sin 1}}$

$y'(x)=-\sin\sqrt{\sin(\tan\pi x)}\cdot\left(\sqrt{\sin(\tan\pi x)}\right)'$

$=\dfrac{-\sin\sqrt{\sin(\tan\pi x)}\cdot\cos(\tan\pi x)\sec^2\pi x\cdot\pi}{2\sqrt{\sin(\tan\pi x)}}$

$\therefore y'\left(\frac{1}{4}\right)=\dfrac{-\cancel{2}\pi\sin\sqrt{\sin 1}\cdot\cos 1}{\cancel{2}\sqrt{\sin 1}}$

$=\dfrac{-\pi\sin\sqrt{\sin 1}\cdot\cos 1}{\sqrt{\sin 1}}$

3. The slope of the graph of $(\sin \pi x + \cos \pi y)^2 = 1$ at the point $\left(\frac{1}{6}, \frac{1}{3}\right)$ is equal to

(a) 1 ______________________________ (correct)

(b) 0

(c) $\frac{\sqrt{3}}{2}$

(d) $\frac{1}{2}$

(e) The graph has no slope at this point

By differentiating w.r.t. x,

$2(\sin \pi x + \cos \pi y)(\cos \pi x - \sin \pi y \cdot y') \cdot \pi = 0$

at $\left(\frac{1}{6}, \frac{1}{3}\right)$ we have:

$2\pi\left(\sin\frac{\pi}{6} + \cos\frac{\pi}{3}\right)\left(\cos\frac{\pi}{6} - \sin\frac{\pi}{3} \cdot y'\right) = 0$

$\Rightarrow 2\pi\left(\frac{1}{2}+\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} y'\right) = 0$

$\Rightarrow \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} y' = 0 \Rightarrow y' = \frac{\sqrt{3}/2}{\sqrt{3}/2} = 1$

4. If $y - \cos(xy) = x$, then $y''(0) =$

(a) -1 ______________________________ (correct)

(b) 2

(c) -2

(d) 0

(e) 2π

Differentiate to get

$y' + \sin(xy)(y + xy') = 1$

Now, at $x=0$, $y = 0 + \cos 0 = 1$ $\therefore$ $(0,1)$ is on the graph of the eq.

Differentiate again,

$y'' + \cos(xy)(y+xy')^2 + \cancel{\sin(xy)}(y'+y'+xy'') = 0$

0

at $(0,1)$, $y'' + 1(1+0)^2 = 0$

$\Rightarrow y'' = -1$

5. The graph of the equation $4x^2 + y^2 - 8x + 4y + 4 = 0$ has a horizontal tangent line at the point

Ex. #68 page 176

(a) $(1, -4)$ ________________ (correct)

(b) $(0, -2)$

(c) $(2, -2)$

(d) $(1, 1)$

(e) $(-2, 0)$

Differentiate the eq.

$8x + 2yy' - 8 + 4y' = 0$

$\Rightarrow y' = \dfrac{8-8x}{2y+4} = \dfrac{4(1-x)}{y+2}$

$\therefore y' = 0 \Rightarrow x = 1$

Substitute in the eq. to get

$4(1)^2 + y^2 - 8(1) + 4y + 4 = 0$

$\Rightarrow y^2 + 4y = 0 \Rightarrow y(y+4) = 0 \Rightarrow y = 0$ or $y = -4$

$\therefore$ A point at which the graph of the eq. has a tangent line is $(1,0)$ and $(1,-4)$

6. If $y = \arctan x - \dfrac{x}{1+x^2}$, then $y'(x) =$

Similar to Ex 41 page 182

(a) $\dfrac{2x^2}{(1+x^2)^2}$ ________________ (correct)

(b) $\dfrac{2}{(1+x^2)^2}$

(c) $\dfrac{1-x^2}{(1+x^2)^2}$

(d) $\dfrac{1-x^2}{1+x^2}$

(e) $\dfrac{2x}{(1+x^2)^2}$

$y'(x) = \dfrac{1}{1+x^2} - \dfrac{(1+x^2) - x(2x)}{(1+x^2)^2}$

$= \dfrac{(1+x^2) - (1-x^2)}{(1+x^2)^2} = \dfrac{2x^2}{(1+x^2)^2}$

7. The equation of the tangent line to the graph of $x \arctan y + x^2 = y - 1$ at $(0, 1)$ is

Ex. 63 page 183

(a) $4y - \pi x = 4$ ______ (correct)

(b) $y + \frac{\pi}{4}x = 1$

(c) $y - 4\pi x = 4$

(d) $4y + \pi x = 2$

(e) $y = \frac{\pi}{4}x + 4$

Differentiate the eq.

$$\arctan y + x \cdot \frac{y'}{1+y^2} + 2x = y'$$

at $(0,1)$, $\Rightarrow y' = \frac{\pi}{4} + 0 + 0 \Rightarrow y' = \frac{\pi}{4}$

$\therefore$ The eq. of the tangent line is

$$y - 1 = \frac{\pi}{4}(x - 0) \Rightarrow$$

$$4y - 4 = \pi x$$

or $4y - \pi x = 4$

8. If all edges of a cube are expanding at a rate of $6\,cm/sec$, then the surface area is changing when each edge is $2\,cm$ at the rate of

Ex. # 16 page 190

(a) $144\,cm^2/sec$ ______ (correct)

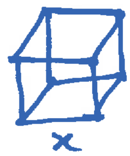
(b) $72\,cm^2/sec$

(c) $24\,cm^2/sec$

(d) $36\,cm^2/sec$

(e) $96\,cm^2/sec$

$\because \frac{dx}{dt} = 6\ cm/s$



& $S = 6x^2$

$$\Rightarrow \frac{dS}{dt} = 12x\frac{dx}{dt}$$

$\therefore$ at $x = 2$, $\frac{dS}{dt} = 12(2)(6)$

$$= 144\ cm^2/s.$$

9. If an airplane flies at an altitude of 5 kms toward a point directly over an observer (see figure below) and the speed of the plane is 400 kms per hour, then the rate at which the angle of evaluation θ is changing when the angle is 30° is equal to

Similar to Ex. # 42 page 193

(a) 20 rad/hr ______________________________ (correct)

(b) 10 rad/hr

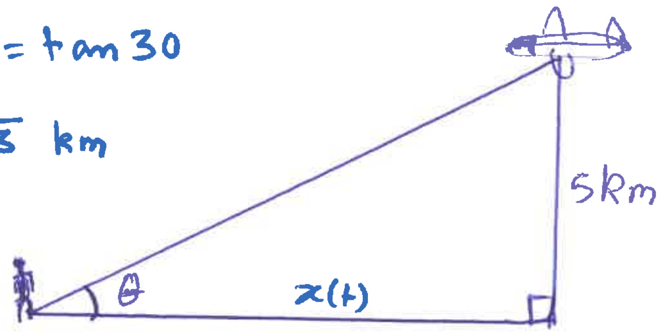
(c) 40 rad/hr

(d) 80 rad/hr

(e) 5 rad/hr

at $\theta = 30^\circ$, $\frac{5}{x(t)} = \tan 30$

$\Rightarrow x(t) = \frac{5}{1/\sqrt{3}} = 5\sqrt{3}$ km



$\because \tan\theta = \frac{5}{x(t)}$

$\Rightarrow \sec^2\theta \frac{d\theta}{dt} = -\frac{5x'(t)}{(x(t))^2}$. at $\theta = 30^\circ$, $\frac{4}{3} \cdot \frac{d\theta}{dt} = \frac{-5(-400)}{(5\sqrt{3})^2}$

$\Rightarrow \frac{d\theta}{dt} = \frac{\cancel{2000}^{500}}{\cancel{75}_{25}} \cdot \frac{\cancel{3}}{\cancel{4}} = \frac{500}{25} = 20$ rad/hr.

10. Using Newton's method to approximate the zero of the function $f(x) = x - e^{-x}$ with initial value $x_1 = 0$, $x_2 =$

Ex. # 13 page 196

(a) $\frac{1}{2}$ ______________________________ (correct)

(b) $-\frac{1}{2}$

(c) 0

(d) $-\frac{1}{4}$

(e) 1

$\because f'(x) = 1 + e^{-x}$

& $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

& $x_1 = 0$

$\therefore x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0 - \frac{f(0)}{f'(0)} = -\frac{(-1)}{2}$

$\therefore x_2 = \frac{1}{2}$

11. Let M and m be the absolute Maximum and absolute Minimum, respectively, for the function $f(x) = e^x \cos x$ on the interval $[0, \pi]$. Then, $\dfrac{M^4}{m} =$

Similar to Ex. # 45 page 212

(a) $\dfrac{-1}{4}$ ______________________________ (correct)

(b) $\dfrac{-e^{\pi}}{4}$

(c) $\dfrac{-e^{\frac{\pi}{2}}}{4}$

(d) $\dfrac{-1}{2}$

(e) $\dfrac{-e^{\frac{\pi}{4}}}{4}$

$\because f'(x) = e^x \cos x - e^x \sin x$
$= e^x(\cos x - \sin x)$

$f'(x) = 0 \Rightarrow \cos x = \sin x$ on $[0, \pi]$

$\Rightarrow x = \frac{\pi}{4}$

$f'(x)$: $0 \quad + \quad \frac{\pi}{4} \quad - \quad \pi$

$f(x)$: $\nearrow \quad \searrow$

& $\because f(0) = 1$ & $f(\pi) = -e^{\pi}$

and $f(\frac{\pi}{4}) = \dfrac{e^{\pi/4}}{\sqrt{2}}$ $\quad \therefore M = \dfrac{e^{\pi/4}}{\sqrt{2}}$ & $m = -e^{\pi}$

$\therefore \dfrac{M^4}{m} = \dfrac{e^{\pi}/4}{-e^{\pi}} = -\dfrac{1}{4}$

12. If $f(x) = x - \frac{3}{2}x^{\frac{2}{3}}$, then the number of critical numbers is

Similar to Example 3 page 210

(a) 2 ______________________________ (correct)

(b) 1

(c) 0

(d) 3

(e) 4

$\because f'(x) = 1 - \frac{3}{2} \cdot \frac{2}{3} x^{-1/3} = 1 - \dfrac{1}{x^{1/3}} = \dfrac{x^{1/3} - 1}{x^{1/3}}$

& $\because f'(x) = 0 \Rightarrow x^{1/3} - 1 = 0 \Rightarrow \boxed{x = 1}$

& $f'(x)$ D.N.E. $\Rightarrow \boxed{x = 0}$

$\therefore$ # of critical numbers is 2.

13. The value of c that satisfies Rolle's Theorem when applied to $f(x) = (x-4)(x+2)^2$ on $[-2, 4]$ is equal to

Ex. #14 page 218

(a) 2 ______________________________ (correct)

(b) 0

(c) -2

(d) 1

(e) -1

$\because f(-2)=0$, $f(4)=0$, f is continuous on $[-2,4]$ and f is differentiable on $(-2,4)$

$\therefore$ By Rolle's Th., $\exists\ c\in(-2,4)$ such that $f'(c)=0$

$\Rightarrow f'(x) = (x+2)^2 + 2(x+2)(x-4) = (x+2)(x+2+2x-8)$

$= (x+2)(3x-6) = 3(x+2)(x-2)$

$\therefore f'(x)=0 \Rightarrow x = \pm 2 \Rightarrow \boxed{c=2} \in (-2,4)$ as $-2 \notin (-2,4)$.

14. The value of c that satisfies the Mean Value Theorem when applied to $f(x) = \dfrac{x}{x+1}$ on $\left[-\dfrac{1}{2}, 2\right]$ is equal to

Ex. # 57 page 219

(a) $-1 + \dfrac{\sqrt{6}}{2}$ ______________________________ (correct)

(b) $-1 - \dfrac{\sqrt{6}}{2}$

(c) $-1 + \dfrac{\sqrt{3}}{2}$

(d) $-1 - \dfrac{\sqrt{3}}{2}$

(e) $\dfrac{2}{3}$

$\because f$ is continuous on $[-\frac{1}{2},2]$ and differentiable on $(-\frac{1}{2},2)$, by M.V.T., there exist $c\in(-\frac{1}{2},2)$ such that

$$f'(c) = \frac{f(2)-f(-\frac{1}{2})}{2+\frac{1}{2}} = \frac{\frac{2}{3} - \frac{-1/2}{1/2}}{5/2} = \frac{5}{3}\cdot\frac{2}{5} = \frac{2}{3}$$

$\because f'(x) = \dfrac{x+1-x}{(x+1)^2} = \dfrac{1}{(x+1)^2}$

$\Rightarrow \dfrac{1}{(c+1)^2} = \dfrac{2}{3} \Rightarrow (c+1)^2 = \dfrac{3}{2} \Rightarrow c+1 = \pm\dfrac{\sqrt{6}}{2}$

$\Rightarrow c = -1 - \dfrac{\sqrt{6}}{2}$ X $\notin (-\frac{1}{2},2)$, $c = -1 + \dfrac{\sqrt{6}}{2}$ ✓ $\in (-\frac{1}{2},2)$

Answer KEY

Q	MASTER	1	2	3	4
1	A	D 13	A 11	B 2	B 6
2	A	B 3	B 14	A 8	C 5
3	A	B 6	B 13	E 11	C 14
4	A	D 11	A 8	B 5	D 2
5	A	C 4	C 6	B 13	B 9
6	A	C 14	D 9	C 14	C 12
7	A	B 9	E 10	A 3	D 4
8	A	A 5	D 2	A 9	D 13
9	A	C 2	A 4	B 10	C 8
10	A	C 10	B 12	E 6	D 10
11	A	A 8	C 7	E 12	D 7
12	A	D 12	A 3	C 7	D 11
13	A	D 7	D 5	D 4	D 3
14	A	E 1	B 1	A 1	C 1