1. If $f(x) = \begin{cases} x^2 & x \leq 2 \\ 8 - 2x & 2 < x < 4 \\ 4 & x \geq 4 \end{cases}$, then

   (a) $f'(2)$ does not exist _____ (correct)

   (b) f is continuous on $\mathbb{R}$

   (c) f is increasing on $(2, 4)$

   (d) $\lim_{x \to \infty} f(x) = \infty$

   (e) $\lim_{x \to -\infty} f(x)$ exists

2. The function $f(x) = \dfrac{\ln(x^2 + 1)}{x + 1}$

   (a) has one vertical asymptote at $x = -1$ _____ (correct)

   (b) has one vertical asymptote at $x = 1$

   (c) has two vertical asymptote at $x = 0$ and $x = -1$

   (d) has no vertical asymptote

   (e) has two vertical asymptotes at $x = 1$ and $x = -1$

3. $\lim_{x \to -\infty} 3x + \sqrt{9x^2 - x} =$

(a) $\frac{1}{6}$ _____ (correct)

(b) ∞

(c) $-\infty$

(d) $\frac{1}{3}$

(e) $\frac{1}{2}$

4. If $f(x) = xe^{2x}$, then $f^{(4)}(1) =$

(a) $48\,e^2$ _____ (correct)

(b) $36\,e^2$

(c) $20\,e^2$

(d) $28\,e^2$

(e) $24\,e^2$

5. An equation of the tangent line to the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 5$ at the point $(8, 1)$ is

(a) $2y + x - 10 = 0$ (correct)

(b) $y - x + 7 = 0$

(c) $y + 2x - 17 = 0$

(d) $2y - x + 6 = 0$

(e) $y - 2x + 15 = 0$

6. The sum of all critical numbers of the function $f(x) = \dfrac{4x}{x^2 + 1}$ is equal to

(a) 0 (correct)

(b) 1

(c) -1

(d) 2

(e) -3

7. Which of the following is/are True?
   (I) There is no function with an infinite number of critical numbers
   (II) Every polynomial has at least one critical number
   (III) A polynomial of degree $n \geq 1$ has at most $(n-1)$ critical numbers

   (a) III (correct)
   (b) II & III
   (c) II
   (d) I & III
   (e) I

8. The function $f(x) = 7x^3 - 21x + 3$ is

   (a) decreasing on $(-1, 1)$ (correct)
   (b) increasing on $(-1, 1)$
   (c) decreasing on $(-\infty, -1)$
   (d) increasing on $(0, \infty)$
   (e) increasing on $(0, 2)$

9. Consider the function $f(x) = \sqrt[3]{x}$. Which statement is **TRUE** about $f(x)$?

(a) $f(x)$ has one inflection point at $(0, 0)$ _____(correct)

(b) $f(x)$ is decreasing for all $x > 0$

(c) $f(x)$ has no inflection point

(d) $f(x)$ has two critical points

(e) $f(x)$ concave down on $(-\infty, 0)$

10. The graph of the function $f(x) = \dfrac{\cos x}{1 + \sin x}$ on $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$

(a) is decreasing and concave upward _____(correct)

(b) is decreasing and concave downward

(c) is increasing and concave upward

(d) is increasing and concave downward

(e) intersects the x-axis at $x = 0$

11. Consider the function $f(x) = \dfrac{x}{x^2+1}$. Which statement is TRUE about the graph of f?

(a) $f(x)$ has a relative minimum at $\left(-1, -\frac{1}{2}\right)$ and a relative maximum at $\left(1, \frac{1}{2}\right)$ (correct)

(b) $f(x)$ is increasing on $(-\infty, -1)$ and $(1, \infty)$

(c) $f(x)$ is concave upward on $(0, \sqrt{3})$ and concave downward on $(-\sqrt{3}, 0)$

(d) $f(x)$ has exactly one point of inflection at $(0, 0)$

(e) The line $x = 0$ is a vertical asymptote of $f(x)$

12. Let x and y be two positive numbers such that the sum of the first number cubed and the second number is 500, and the product of x and y is maximum. Then $x + y =$

(a) 380 (correct)

(b) 375

(c) 385

(d) 240

(e) 120

13. If $y = e^{-\frac{\pi}{2}} \cos 4x$, then $dy =$

(a) $e^{-\frac{\pi}{2}} \left(-4 \sin 4x - \frac{1}{2} \cos 4x \right) dx$ _____(correct)

(b) $e^{-\frac{\pi}{2}} \left(4 \sin 4x - \frac{1}{2} \cos 4x \right) dx$

(c) $e^{-\frac{\pi}{2}} \left(-4 \cos 4x - \frac{1}{2} \sin 4x \right) dx$

(d) $e^{-\frac{\pi}{2}} \left(4 \cos 4x + \frac{1}{2} \sin 4x \right) dx$

(e) $e^{-\frac{\pi}{2}} \left(4 \cos 4x - \frac{1}{2} \sin 4x \right) dx$

14. $\int (\sec y)(\tan y - \sec y)\, dy =$

(a) $\sec y - \tan y + c$ _____(correct)

(b) $\sec^2 y - \tan y + c$

(c) $\tan^2 y + \csc y + c$

(d) $\cos y + \tan y + c$

(e) $\sec^2 y + \tan y + c$

15. If f is a function such that $f''(x) = -2$, $f(-1) = 3$ and $f(0) = 1$, then $f(1) =$

(a) -3 _____(correct)

(b) -2

(c) -1

(d) 0

(e) 1

16. $\displaystyle\int \frac{x+6}{\sqrt{x}}\,dx =$

(a) $\frac{2}{3}x^{\frac{3}{2}} + 12x^{\frac{1}{2}} + c$ _____(correct)

(b) $\frac{3}{2}x^{\frac{3}{2}} + 12x^{\frac{1}{2}} + c$

(c) $\frac{3}{2}x^{\frac{3}{2}} + 6x^{\frac{1}{2}} + c$

(d) $\frac{2}{3}x^{\frac{2}{3}} + 12x^{\frac{1}{2}} + c$

(e) $\frac{3}{2}x^{\frac{2}{3}} + 12x^{\frac{1}{2}} + c$

17. $\lim_{x \to 0^+} (\sin x)^x =$

(a) 1 (correct)

(b) 0

(c) $\frac{\pi}{4}$

(d) $\frac{e}{4}$

(e) DNE

18. $\lim_{x \to 0^+} (e^x + x)^{\frac{2}{x}} =$

(a) e^4 (correct)

(b) e^2

(c) 1

(d) e

(e) ∞

19. The sum of all the relative maxima and minima of the function $f(x) = \sin x \sinh x - \cos x \cosh x$ on $(-4, 4)$ is

(a) $2\cosh \pi - 1$ _____(correct)

(b) $\cosh \pi - 1$

(c) $1 - \cosh \pi$

(d) $1 - 2\cosh \pi$

(e) -1

20. $\lim_{x \to 0} \dfrac{\sinh 2x}{x} =$

(a) 2 _____(correct)

(b) 1

(c) 0

(d) -1

(e) -2

Math 101, Term 251, Final Exam

Answer KEY

Q	MASTER	1	2	3	4	5	6	7	8
1	A	C 5	B 12	C 11	D 11	C 12	A 17	D 6	E 11
2	A	A 2	C 1	B 15	D 14	D 10	C 5	A 17	E 13
3	A	A 8	E 16	B 18	A 3	C 3	E 16	D 3	D 9
4	A	D 14	D 5	D 2	D 12	D 13	E 9	C 15	A 16
5	A	D 15	D 3	E 1	E 4	E 8	C 19	C 16	D 17
6	A	A 16	B 2	C 6	E 5	E 4	A 10	B 10	D 18
7	A	D 6	C 13	C 13	E 8	C 1	E 1	C 18	C 2
8	A	D 13	C 7	A 7	E 15	C 18	A 7	A 11	E 12
9	A	C 19	B 6	E 17	A 18	C 5	C 11	B 12	A 5
10	A	B 4	C 10	A 5	C 1	B 17	A 2	E 7	B 3
11	A	D 1	A 19	E 10	A 13	C 9	E 18	C 8	A 15
12	A	A 18	C 9	B 16	B 16	E 2	E 14	B 14	C 8
13	A	A 7	D 17	A 19	E 10	C 15	D 12	A 13	E 1
14	A	B 10	C 4	D 12	B 6	C 16	E 8	B 19	E 10
15	A	A 11	A 15	E 4	D 17	E 11	D 15	A 4	D 7
16	A	E 17	C 14	C 3	C 9	D 14	A 3	E 9	B 14
17	A	A 9	B 18	B 8	E 19	C 19	C 6	E 5	C 4
18	A	A 3	C 8	D 14	E 2	C 7	A 13	B 1	D 6
19	A	B 12	C 11	D 9	D 7	D 6	A 4	E 2	E 19
20	A	E 20	E 20	B 20	C 20	A 20	A 20	B 20	B 20