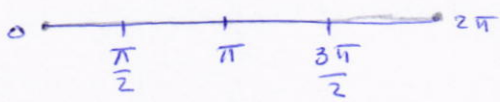
1. Using **four** approximating rectangles and **left endpoints**, the **area** under the graph of $f(x) = \sin\left(\frac{x}{2}\right)$ from $x = 0$ to $x = 2\pi$ is approximately equal to

$\cdot\ \Delta x = \frac{2\pi - 0}{4} = \frac{\pi}{2}$



(a) $\frac{\pi}{2}(1 + \sqrt{2})$ (correct)

(b) $\frac{\pi}{4}(1 + 2\sqrt{2})$

(c) $\frac{\pi}{4}\left(\frac{1}{2} + \sqrt{2}\right)$

(d) $\frac{\pi}{4}(2 + \sqrt{2})$

(e) $\frac{\pi}{3}\left(1 + \frac{\sqrt{2}}{2}\right)$

$$A \approx f(0)\Delta x + f(\tfrac{\pi}{2})\Delta x + f(\pi)\Delta x + f(\tfrac{3\pi}{2})\Delta x$$
$$= \Delta x\left[0 + \sin(\tfrac{\pi}{4}) + \sin(\tfrac{\pi}{2}) + \sin(\tfrac{3\pi}{4})\right]$$
$$= \frac{\pi}{2}\left[0 + \frac{\sqrt{2}}{2} + 1 + \frac{\sqrt{2}}{2}\right]$$
$$= \frac{\pi}{2}(1 + \sqrt{2})$$

2. $\int \frac{3t^6 + t^3 + 6}{t^4}\,dt =$

$$\int \left(3t^2 + \frac{1}{t} + 6t^{-4}\right)dt$$
$$= t^3 + \ln|t| + 6\frac{t^{-3}}{-3} + C$$
$$= t^3 + \ln|t| - \frac{2}{t^3} + C$$
$$= t^3 - \frac{2}{t^3} + \ln|t| + C$$

(a) $t^3 - \frac{2}{t^3} + \ln|t| + C$ (correct)

(b) $t^3 - \frac{2}{t^3} - \frac{1}{t^2} + C$

(c) $t^3 - \frac{1}{t^3} - \ln|t| + C$

(d) $2t^3 + \frac{3}{t^3} + \ln|t| + C$

(e) $t^3 - \frac{1}{t^3} + \ln t + C$

3. If $f(x) = \begin{cases} 4 & \text{if} \quad -3 \le x \le 0 \\ 4 - x^2 & \text{if} \quad 0 < x \le 3 \end{cases}$, then $\int_{-3}^{3} f(x)\,dx =$

(a) 15 (correct)

(b) 9

(c) 0

(d) 12

(e) 6

$$\int_{-3}^{3} f(x)\,dx = \int_{-3}^{0} f(x)\,dx + \int_{0}^{3} f(x)\,dx$$
$$= \int_{-3}^{0} 4\,dx + \int_{0}^{3} (4 - x^2)\,dx$$
$$= 4x\Big]_{-3}^{0} + 4x - \frac{x^3}{3}\Big]_{0}^{3}$$
$$= 4(0 - (-3)) + 4(3) - \frac{3^3}{3} - 0$$
$$= 12 + 12 - 9$$
$$= 15$$

4. If $G(x) = \int_{2}^{2x^2} \frac{\sqrt{4+t^2}}{4t}\,dt$, then $G'(x) =$ $\frac{\sqrt{4+(2x^2)^2}}{4 \cdot 2x^2} \cdot \frac{d}{dx}[2x^2]$

(a) $\frac{\sqrt{1+x^4}}{x}$ (correct)

(b) $\frac{\sqrt{1+x^4}}{2x}$

(c) $\frac{\sqrt{4+4x^4}}{8x^2}$

(d) $\frac{\sqrt{4+x^4}}{2x^2}$

(e) $\frac{\sqrt{1+x^4}}{4x}$

$$= \frac{\sqrt{4+4x^4}}{8x^2} \cdot 4x$$
$$= \frac{\sqrt{4(1+x^4)}}{2x}$$
$$= \frac{2\sqrt{1+x^4}}{2x} = \frac{\sqrt{1+x^4}}{x}.$$

5. If $\int_{-c}^{0} \sqrt{c^2 - x^2}\, dx = 4\pi^3$, then $c =$

(a) 4π (correct)

(b) 16π

(c) 4

(d) $16\pi^2$

(e) $4\pi^2$

$\frac{1}{4}\pi c^2 = 4\pi^3$

$\Rightarrow c^2 = 16\pi^2$

$\Rightarrow c = 4\pi$ ($c > 0$; It follows from the given equation)

6. If $F(x) = \int_{\sqrt{x}}^{x^3} t^2 \cos(\pi t)\, dt$, then $F'(1) =$

(a) $-\frac{5}{2}$ (correct)

(b) -2

(c) 0

(d) $\frac{3}{2}$

(e) 1

$F'(x) = (x^3)^2 \cos(\pi x^3) \cdot \frac{d}{dx}[x^3] - (\sqrt{x})^2 \cos(\pi\sqrt{x}) \cdot \frac{d}{dx}[\sqrt{x}]$

$= x^6 \cos(\pi x^3) \cdot 3x^2 - x \cos(\pi\sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$

$= 3x^8 \cos(\pi x^3) - \frac{1}{2}\sqrt{x} \cos(\pi\sqrt{x})$

$F'(1) = 3\cos(\pi) - \frac{1}{2}\cos(\pi)$

$= -3 + \frac{1}{2}$

$= -\frac{5}{2}$

7. $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{5i^4}{n^5} + \frac{2i}{n^2} \right) = \lim_{n\to\infty} \sum_{i=1}^{n} \left[5\left(\frac{i}{n}\right)^4 + 2\left(\frac{i}{n}\right) \right] \cdot \frac{1}{n}$

- $x_i^* = a + i\Delta x$, $\Delta x = \frac{1}{n} = \frac{b-a}{n}$, $x_i^* = \frac{i}{n}$

$\Rightarrow a = 0$, $b - a = 1$

$\Rightarrow a = 0$, $b = 1$

- $= \int_0^1 (5x^4 + 2x)\, dx$

$= x^5 + x^2 \Big]_0^1 = 1 + 1 - 0 = 2$

(a) 2 (correct)

(b) 1

(c) $\frac{7}{10}$

(d) 3

(e) $\frac{17}{10}$

8. $\int_{\frac{\pi}{2}}^{2\pi} \sqrt{1 - \sin^2 t}\; dt = \int_{\frac{\pi}{2}}^{2\pi} \sqrt{\cos^2 t}\, dt = \int_{\frac{\pi}{2}}^{2\pi} |\cos t|\, dt$

$= \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} -\cos t\, dt + \int_{\frac{3\pi}{2}}^{2\pi} \cos t\, dt$

$= -\sin t \Big]_{\frac{\pi}{2}}^{\frac{3\pi}{2}} + \sin t \Big]_{\frac{3\pi}{2}}^{2\pi}$

$= -(-1-1) + (0-(-1))$

$= 2 + 1$

$= 3$

(a) 3 (correct)

(b) 1

(c) 0

(d) -1

(e) $\frac{3}{2}$

9. $\displaystyle\int_e^{e^4} \frac{\ln x}{x\sqrt{\ln x}}\,dx =$

Let $u = \ln x$. Then $du = \frac{1}{x}\,dx$

$x = e \Rightarrow u = \ln e = 1$

$x = e^4 \Rightarrow u = \ln e^4 = 4\ln e = 4(1) = 4$

$$\int_1^4 \frac{u}{\sqrt{u}}\,du$$

$$= \int_1^4 \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big]_1^4$$

$$= \frac{2}{3}\left[4^{3/2} - 1\right]$$

$$= \frac{2}{3}\left[8 - 1\right] = \frac{2}{3}\cdot 7 = \frac{14}{3}$$

(a) $\frac{14}{3}$ (correct)

(b) $\frac{16}{3}$

(c) $\frac{13}{3}$

(d) $\frac{10}{3}$

(e) $\frac{8}{3}$

10. Let f be an **odd** and continuous function on $(-\infty, \infty)$. If $\int_3^0 f(x)\,dx = 2$ and $\int_3^5 f(x)\,dx = 9$, then $\int_{-5}^0 f(x)\,dx =$

(a) -7 (correct)

(b) 7

(c) -2

(d) 11

(e) -11

Since f is odd, then

$$\int_{-5}^{5} f(x)\,dx = 0$$

$$\Rightarrow \int_{-5}^{0} f(x)\,dx + \int_0^3 f(x)\,dx + \int_3^5 f(x)\,dx = 0$$

$$\Rightarrow \int_{-5}^{0} f(x)\,dx + (-2) + 9 = 0$$

$$\Rightarrow \int_{-5}^{0} f(x)\,dx = 2 - 9 = -7$$

11. $\int \frac{2x}{(x+1)^3}\,dx =$

Let $u = x+1$. Then $du = dx$ & $x = u-1$

$= \int \frac{2(u-1)}{u^3}\,du = 2\int (u^{-2} - u^{-3})\,du$

$= 2\left[\frac{u^{-1}}{-1} - \frac{u^{-2}}{-2}\right] + C$

$= 2\left[\frac{-1}{u} + \frac{1}{2u^2}\right] + C$

$= \frac{1}{u^2} - \frac{2}{u} + C$

$= \frac{1}{(x+1)^2} - \frac{2}{x+1} + C$

(a) $\frac{1}{(x+1)^2} - \frac{2}{x+1} + C$ (correct)

(b) $\frac{1}{2(x+1)^2} - \frac{1}{x+1} + C$

(c) $\frac{4}{x+1} + \frac{3}{(x+1)^2} + C$

(d) $\frac{1}{x+1} - \frac{2}{(x+1)^2} + \frac{3}{(x+1)^3} + C$

(e) $\frac{1}{(x+1)^3} - \frac{2}{(x+1)^2} + C$

12. A particle moves along a line so that its velocity at time t is $v(t) = t^2 - t - 2$, (m/s). The **distance** traveled during the time period $0 \le t \le 3$ is

$v(t) = (t-2)(t+1) = 0 \Rightarrow t = -1, t = 2$

$v(t)$: 0 $(-)$ 2 $(+)$ 3

$D = \int_0^3 |v(t)|\,dt$

$= \int_0^2 -v(t)\,dt + \int_2^3 v(t)\,dt$

$= \int_0^2 (-t^2 + t + 2)\,dt + \int_2^3 (t^2 - t - 2)\,dt$

$= \left. -\frac{t^3}{3} + \frac{t^2}{2} + 2t \right]_0^2 + \left. \frac{t^3}{3} - \frac{t^2}{2} - 2t \right]_2^3$

$= \left(-\frac{8}{3} + 2 + 4\right) - 0 + \left(9 - \frac{9}{2} - 6\right) - \left(\frac{8}{3} - 2 - 4\right)$

$= -\frac{8}{3} + 6 + 3 - \frac{9}{2} - \frac{8}{3} + 6$

$= 15 - \frac{9}{2} - \frac{16}{3} = \frac{90 - 27 - 32}{6} = \frac{90 - 59}{6} = \frac{31}{6}$

(a) $\frac{31}{6}\,m$ (correct)

(b) $\frac{7}{2}\,m$

(c) $5\,m$

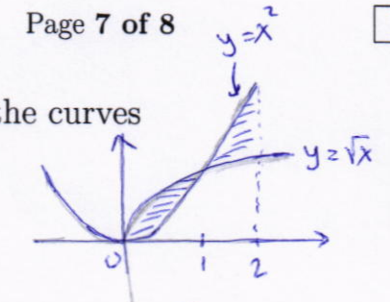
(d) $\frac{32}{5}\,m$

(e) $\frac{19}{3}\,m$

13. The **area** of the region between by the curves

$$y = \sqrt{x},\ y = x^2,$$

over the interval $[0, 2]$ is



$\circ\ x^2 = \sqrt{x} \Rightarrow x^4 = x$
$\Rightarrow x^4 - x = 0$
$\Rightarrow x(x^3 - 1) = 0$
$\Rightarrow x = 0,\ x = 1$

(a) $\dfrac{10}{3} - \dfrac{4}{3}\sqrt{2}$ (correct)

(b) $\dfrac{8}{3} - \dfrac{4}{3}\sqrt{2}$

(c) $\dfrac{8}{3} - \dfrac{2}{3}\sqrt{2}$

(d) $3 - \dfrac{2}{3}\sqrt{2}$

(e) $\dfrac{10}{3} + \dfrac{2}{3}\sqrt{2}$

$$A = \int_0^1 (\sqrt{x} - x^2)\,dx + \int_1^2 (x^2 - \sqrt{x})\,dx$$

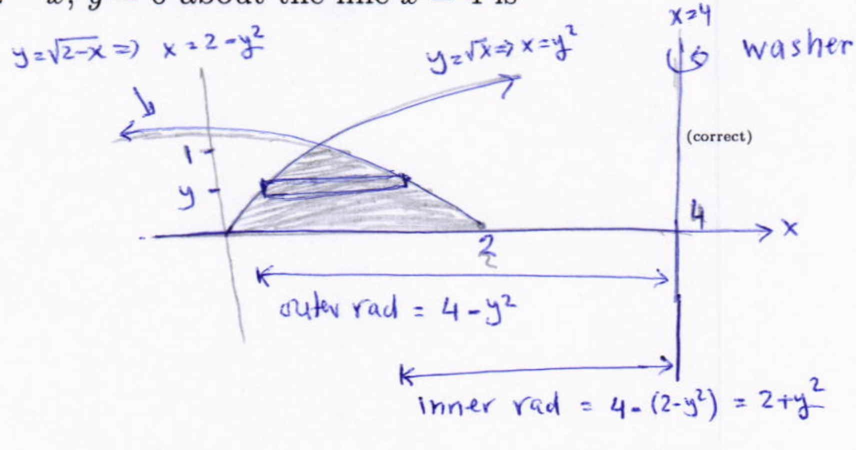
$$= \frac{2}{3}x^{3/2} - \frac{x^3}{3}\Big]_0^1 + \frac{x^3}{3} - \frac{2}{3}x^{3/2}\Big]_1^2$$

$$= \left(\frac{2}{3} - \frac{1}{3}\right) + \left(\frac{8}{3} - \frac{2}{3}\cdot 2^{3/2}\right) - \left(\frac{1}{3} - \frac{2}{3}\right)$$

$$= \frac{1}{3} + \frac{8}{3} - \frac{4}{3}\sqrt{2} + \frac{1}{3}$$

$$= \frac{10}{3} - \frac{4}{3}\sqrt{2}$$

14. The **volume** of the solid obtained by rotating the region bounded by the curves $y = \sqrt{x}$, $y = \sqrt{2 - x}$, $y = 0$ about the line $x = 4$ is



(a) $12\pi \int_0^1 (1 - y^2)\,dy$ (correct)

(b) $\pi \int_0^1 (2 + y^2)^2\,dy$

(c) $\pi \int_0^1 (4 - y^2)^2\,dy$

(d) $\pi \int_0^1 (1 - y^2)\,dy$

(e) $6\pi \int_0^1 (1 + y^2)\,dy$

pt of int: $2 - y^2 = y^2$
$\Rightarrow 2y^2 = 2$
$\Rightarrow y^2 = 1$
$\Rightarrow y = 1$

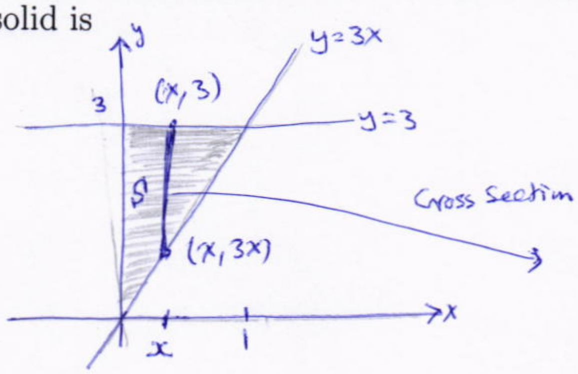
$$V = \int_0^1 \pi\left[(4 - y^2)^2 - (2 + y^2)^2\right]dy$$

$$= \pi \int_0^1 \left[16 - 8y^2 + y^4) - (4 + 4y^2 + y^4)\right]dy$$

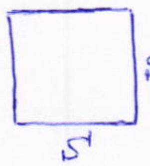
$$= \pi \int_0^1 (12 - 12y^2)\,dy$$

$$= 12\pi \int_0^1 (1 - y^2)\,dy$$

15. The **base** of a solid is the region enclosed by the lines $y = 3x$, $y = 3$, $x = 0$. If the cross-sections of the solid, perpendicular to the x-axis, are **squares**, then the **volume** of the solid is



(correct)



(a) 3

(b) 9

(c) 6

(d) 12

(e) $\frac{2}{3}$

$S' = 3-3x$

$A(x) = S'^2 = (3-3x)^2 = 9-18x+9x^2$

$V = \int_0^1 A(x)\,dx$

$= \int_0^1 (9-18x+9x^2)\,dx = 9\int_0^1 (1-2x+x^2)\,dx$

$= 9 \cdot \left(x - x^2 + \frac{x^3}{3}\right)\Big]_0^1$

$= 9 \cdot \left(1-1+\frac{1}{3}\right) = 9 \cdot \frac{1}{3} = 3$

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	E	B	C	E
2	A	C	D	A	E
3	A	C	B	D	B
4	A	D	E	A	B
5	A	E	B	D	D
6	A	A	D	A	C
7	A	E	D	D	E
8	A	E	C	D	E
9	A	A	B	C	A
10	A	A	C	E	E
11	A	B	B	A	E
12	A	B	A	B	C
13	A	E	B	B	D
14	A	B	A	B	A
15	A	E	D	B	B

Answer Counts

V	A	B	C	D	E
1	3	3	2	1	6
2	2	6	2	4	1
3	4	4	2	4	1
4	2	3	2	2	6