1. If $H(x) = \int_x^{\sqrt{x}} \frac{e^{2t}}{t^2}\, dt$, (for $x > 0$), then $H'(1) =$

(a) $-\frac{e^2}{2}$ (correct)

(b) $\frac{e^2}{2}$

(c) $2e^2$

(d) $-2e^2$

(e) e^2

$$H'(x) = \frac{e^{2\sqrt{x}}}{(\sqrt{x})^2} \cdot \frac{d}{dx}[\sqrt{x}] - \frac{e^{2x}}{x^2} \cdot \frac{d}{dx}[x]$$

$$= \frac{e^{2\sqrt{x}}}{x} \cdot \frac{1}{2\sqrt{x}} - \frac{e^{2x}}{x^2} \cdot 1$$

$$H'(1) = \frac{e^2}{1} \cdot \frac{1}{2} - \frac{e^2}{1} \cdot 1$$

$$= \frac{e^2}{2} - e^2$$

$$= -\frac{e^2}{2}$$

2. If $\int_0^2 f(x)\, dx = A$, then $\int_0^{\frac{\pi}{4}} f(2\tan x) \cdot \sec^2 x\, dx =$

(a) $\frac{A}{2}$ (correct)

(b) 0

(c) $2A$

(d) $3A$

(e) A

Let $u = 2\tan x$. Then

$du = 2\sec^2 x\, dx$

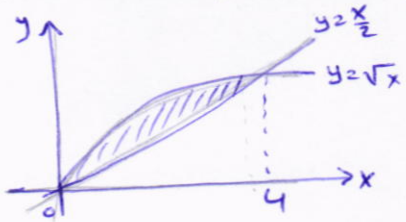
$x = 0 \Rightarrow u = 0$

$x = \frac{\pi}{4} \Rightarrow u = 2$

$$= \int_0^2 f(u) \cdot \frac{1}{2}\, du$$

$$= \frac{1}{2} \int_0^2 f(u)\, du = \frac{1}{2} \cdot A = \frac{A}{2}$$

3. The **area** of the region enclosed by the curves $y = \sqrt{x}$ and $y = \dfrac{x}{2}$ is



- pts of Int: $\sqrt{x} = \frac{x}{2} \Rightarrow x = \frac{x^2}{4}$
$\Rightarrow 4x = x^2 \Rightarrow x^2 - 4x = 0$
$\Rightarrow x(x-4) = 0$ (correct)
$\Rightarrow x = 0$ or $x = 4$

(a) $\dfrac{4}{3}$

(b) $\dfrac{2}{3}$

(c) $\dfrac{5}{3}$

(d) $\dfrac{1}{3}$

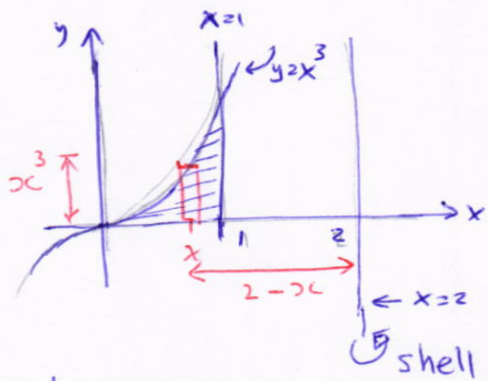
(e) $\dfrac{7}{3}$

$$A = \int_0^4 \left(\sqrt{x} - \frac{x}{2}\right) dx$$
$$= \frac{2}{3}x^{3/2} - \frac{x^2}{4}\Big]_0^4$$
$$= \frac{2}{3}4^{3/2} - \frac{16}{4} - 0$$
$$= \frac{2}{3}\cdot 8 - 4 = \frac{16}{3} - \frac{12}{3} = \frac{4}{3}$$

4. The **volume** of the solid generated by rotating the region bounded by the curves

$$y = x^3,\ y = 0,\ x = 1$$

about the line $x = 2$ is



(correct)

(a) $\dfrac{3\pi}{5}$

(b) $\dfrac{2\pi}{5}$

(c) $\dfrac{\pi}{5}$

(d) $\dfrac{7\pi}{10}$

(e) $\dfrac{3\pi}{10}$

$$V = 2\pi \int_0^1 (2-x)\cdot x^3\, dx$$
$$= 2\pi \int_0^1 (2x^3 - x^4)\, dx$$
$$= 2\pi \cdot \frac{x^4}{2} - \frac{x^5}{5}\Big]_0^1$$
$$= 2\pi \cdot \left(\frac{1}{2} - \frac{1}{5}\right)$$
$$= 2\pi \cdot \frac{3}{10}$$
$$= \frac{3\pi}{5}$$

5. $\int_1^e (\ln x)^2\,dx =$

(a) $e-2$

(b) e^3-1

(c) e^2-2e

(d) $1-2e$

(e) $2-e^2$

$u=(\ln x)^2 \qquad dv = dx$

$du = \frac{2\ln x}{x} \qquad v = x$

$\int (\ln x)^2\,dx = x(\ln x)^2 - 2\underbrace{\int \ln x\,dx}_{\text{by part again}}$ (correct)

$= x(\ln x)^2 - 2[x\ln x - x] + C$

$= x(\ln x)^2 - 2x\ln x + 2x + C$

$\Rightarrow \int_1^e (\ln x)^2\,dx = x(\ln x)^2 - 2x\ln x + 2x\Big]_1^e$

$= (e - 2e + 2e) - (0 - 0 + 2)$

$= e - 2$

6. $\int \sqrt{\cos\theta}\cdot\sin^3\theta\,d\theta =$ $\int \sqrt{\cos\theta}\,.\,\sin^2\theta\,.\,\sin\theta\,d\theta$

(a) $\frac{2}{7}(\cos\theta)^{7/2} - \frac{2}{3}(\cos\theta)^{3/2} + C$

(b) $\frac{2}{7}(\cos\theta)^{7/2} + \frac{2}{3}(\cos\theta)^{3/2} + C$

(c) $2(\cos\theta)^{1/2} + \frac{2}{5}(\cos\theta)^{5/2} + C$

(d) $\frac{2}{3}(\cos\theta)^{3/2} - \frac{2}{5}(\cos\theta)^{5/2} + C$

(e) $\frac{1}{7}(\cos\theta)^{7/2} - \frac{1}{3}(\cos\theta)^{3/2} + C$

$= \int \sqrt{\cos\theta}\,(1-\cos^2\theta)\,.\sin\theta\,d\theta$

$u = \cos\theta \Rightarrow du = -\sin\theta\,d\theta$ (correct)

$= \int \sqrt{u}\,(1-u^2)\,.\,-du$

$= \int (u^{5/2} - u^{1/2})\,du$

$= \frac{2}{7}u^{7/2} - \frac{2}{3}u^{3/2} + C$

$= \frac{2}{7}\cos^{7/2}\theta - \frac{2}{3}\cos^{3/2}\theta + C$

7. $\int_0^2 \frac{\sqrt{x}}{2x+4}\,dx =$

$u=\sqrt{x} \Rightarrow x=u^2 \Rightarrow \cdot\, dx = 2u\,du$

$\cdot\, x=0 \Rightarrow u=0$ & $x=2 \Rightarrow u=\sqrt{2}$

$$\int_0^{\sqrt{2}} \frac{u}{2u^2+4}\cdot 2u\,du = \int_0^{\sqrt{2}} \frac{u^2}{u^2+2}\,du$$

$$= \int_0^{\sqrt{2}} 1 - \frac{2}{u^2+2}\,du$$

$$= u - \frac{2}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)\Big]_0^{\sqrt{2}}$$

$$= \sqrt{2} - \sqrt{2}\cdot\frac{\pi}{4} - 0$$

$$= \sqrt{2}\left(1-\frac{\pi}{4}\right)$$

(a) $\sqrt{2}\left(1-\frac{\pi}{4}\right)$ (correct)

(b) $\sqrt{2}\left(1-\frac{\pi}{2}\right)$

(c) $\sqrt{2}+\frac{\pi}{2\sqrt{2}}$

(d) $\sqrt{2}\left(1+\frac{\pi}{4}\right)$

(e) $\sqrt{2}\left(1+\frac{\pi}{2}\right)$

8. The improper integral $\int_{-\infty}^0 2x^3\,e^{-x^4}\,dx$ is

$$= \lim_{t\to-\infty} \int_t^0 2x^3 e^{-x^4}\,dx$$

$$= \lim_{t\to-\infty} -\frac{1}{2}e^{-x^4}\Big]_t^0$$

$$= \lim_{t\to-\infty} \left(-\frac{1}{2}+\frac{1}{2}e^{-t^4}\right)$$

$$= -\frac{1}{2}+0$$

$= -\frac{1}{2}$, Conv.

(a) convergent and its value is $-\frac{1}{2}$ (correct)

(b) convergent and its value is $-\frac{1}{8}$

(c) convergent and its value is 4

(d) convergent and its value is $\frac{1}{4}$

(e) divergent

9. The **length** of the curve

$$y = \ln(\sec x),\ 0 \le x \le \frac{\pi}{3}$$

is equal to

(a) $\ln(2+\sqrt{3})$

(b) $\ln(1+\sqrt{3})$

(c) $\ln\left(\frac{1}{2}+\sqrt{3}\right)$

(d) $\ln\left(1+\frac{\sqrt{3}}{2}\right)$

(e) $\ln\left(\frac{1+\sqrt{3}}{2}\right)$

$$L = \int_0^{\pi/3} \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$$

$\cdot\ \frac{dy}{dx} = \frac{1}{\sec x} \cdot \sec x \tan x = \tan x$

$\cdot\ 1+\left(\frac{dy}{dx}\right)^2 = 1+\tan^2 x = \sec^2 x$

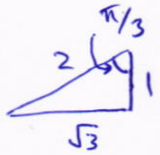
$\cdot\ \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{\sec^2 x} = |\sec x|$

$= \sec x,\ 0 \le x \le \frac{\pi}{3}$

(correct)

$$= \int_0^{\pi/3} \sec x\, dx$$

$$= \ln|\sec x + \tan x| \Big]_0^{\pi/3}$$



$$= \ln(2+\sqrt{3}) - \ln(1+0)$$

$$= \ln(2+\sqrt{3}) - \underbrace{\ln 1}_{=0}$$

$$= \ln(2+\sqrt{3})$$

10. The **surface area** of the surface obtained by rotating the curve

$$x = \frac{1}{2}\sin^{-1}(y),\ \ 0 \le x \le \frac{\pi}{8}$$

about the x-axis is given by

(a) $\displaystyle\int_0^{\sqrt{2}/2} \pi\, y\sqrt{\frac{5-4y^2}{1-y^2}}\, dy$

(b) $\displaystyle\int_0^{\sqrt{2}/2} 2\pi\, y\sqrt{\frac{5+4y^2}{1-y^2}}\, dy$

(c) $\displaystyle\int_0^{\pi/8} 2\pi\, x\sqrt{\frac{5-4x^2}{1-x^2}}\, dx$

(d) $\displaystyle\int_0^{\sqrt{2}/2} 4\pi\, y\sqrt{\frac{5-y^2}{1-y^2}}\, dy$

(e) $\displaystyle\int_0^{\sqrt{2}/2} \pi\sqrt{\frac{5-y^2}{1-y^2}}\, dy$

$2x = \sin^{-1} y \Rightarrow y = \sin(2x)$.

$x = 0 \Rightarrow y = 0$

$x = \frac{\pi}{8} \Rightarrow y = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$

$$\cdot\ S = \int_c^d 2\pi y\sqrt{1+\left(\frac{dx}{dy}\right)^2}\, dy$$

(correct)

$\cdot\ \frac{dx}{dy} = \frac{1}{2}\cdot\frac{1}{\sqrt{1-y^2}}$

$1+\left(\frac{dx}{dy}\right)^2 = 1+\frac{1}{4(1-y^2)} = \frac{5-4y^2}{4(1-y^2)}$

$\sqrt{1+\left(\frac{dx}{dy}\right)^2} = \frac{1}{2}\sqrt{\frac{5-4y^2}{1-y^2}}$

$$= \int_0^{\sqrt{2}/2} 2\pi y \cdot \frac{1}{2}\sqrt{\frac{5-4y^2}{1-y^2}}\, dy$$

$$= \int_0^{\sqrt{2}/2} \pi y \cdot \sqrt{\frac{5-4y^2}{1-y^2}}\, dy$$

11. The sequence $\left\{\dfrac{\cos^2 n}{2^n}\right\}_{n=1}^{\infty}$ is

(a) convergent and its limit is 0 (correct)

(b) convergent and its limit is 1

(c) convergent and its limit is $\dfrac{1}{2}$

(d) divergent and its limit is ∞

(e) divergent and it has no limit

$0 \le \cos^2 n \le 1$

$\Rightarrow \quad 0 \le \dfrac{\cos^2 n}{2^n} \le \dfrac{1}{2^n}$

Since $\lim_{n\to\infty} 0 = 0$ &

$\lim_{n\to\infty} \dfrac{1}{2^n} = 0$

then, by the Squeeze theorem,

$\lim_{n\to\infty} \dfrac{\cos^2 n}{2^n} = 0$ (Conv.)

12. The series $\sum_{n=1}^{\infty}\left(\cos\left(\dfrac{1}{n}\right) - \cos\left(\dfrac{1}{n+1}\right)\right)$ is

(a) convergent and its sum is $-1 + \cos 1$ (correct)

(b) convergent and its sum is $1 + \cos 1$

(c) convergent and its sum is $\cos 1$

(d) convergent and its sum is 0

(e) divergent

$$S_n = \sum_{k=1}^{n}\left[\cos\left(\tfrac{1}{k}\right) - \cos\left(\tfrac{1}{k+1}\right)\right]$$

$$= \left(\cos 1 - \cancel{\cos\tfrac{1}{2}}\right) + \left(\cancel{\cos\left(\tfrac{1}{2}\right)} - \cancel{\cos\left(\tfrac{1}{3}\right)}\right) + \left(\cancel{\cos\left(\tfrac{1}{3}\right)} - \cos\left(\tfrac{1}{4}\right)\right) + \cdots + \left(\cos\left(\tfrac{1}{n}\right) - \cos\left(\tfrac{1}{n+1}\right)\right)$$

$$= \cos 1 - \cos\left(\tfrac{1}{n+1}\right)$$

$$\lim_{n\to\infty} S_n = \cos 1 - \cos 0 = \cos 1 - 1 = -1 + \cos 1$$

13. The series $\sum_{n=0}^{\infty}(-1)^n \frac{5^{n-1}}{2^{3n+2}}$ is

$\sum_{n=0}^{\infty} (-1)^n \frac{5^n \cdot 5^{-1}}{2^{3n} \cdot 2^2} = \sum_{n=0}^{\infty} \frac{1}{20}\left(-\frac{5}{8}\right)^n$

a geometric series with

$a = \frac{1}{20}$ & $r = -\frac{5}{8}$

Since $|r| = \frac{5}{8} < 1$, then the series is convergent & its sum is

$\frac{a}{1-r} = \frac{\frac{1}{20}}{1+\frac{5}{8}} = \frac{\frac{1}{20}}{\frac{13}{8}} = \frac{1}{20} \cdot \frac{8}{13}$

$= \frac{1}{5} \cdot \frac{2}{13} = \frac{2}{65}$

(a) convergent and its sum is $\frac{2}{65}$ (correct)

(b) convergent and its sum is $\frac{1}{65}$

(c) convergent and its sum is $\frac{2}{5}$

(d) convergent and its sum is $\frac{8}{13}$

(e) divergent

14. The series $\sum_{n=1}^{\infty} \frac{2^n+3}{n \cdot 2^n+1}$ is

(a) divergent by the limit comparison test (correct)

(b) convergent by the limit comparison test

(c) convergent by the comparison test

(d) convergent by the integral test

(e) divergent by the test for divergence

$a_n = \frac{2^n+3}{n \cdot 2^n+1} \longrightarrow b_n = \frac{2^n}{n \cdot 2^n} = \frac{1}{n}$

$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{2^n+3}{n \cdot 2^n + 1} \cdot \frac{n}{1} = \lim_{n\to\infty} \frac{n \cdot 2^n + 3n}{n \cdot 2^n + 1}$

$= \lim_{n\to\infty} \frac{1+\frac{3}{2^n}}{1+\frac{1}{n \cdot 2^n}} = \frac{1+0}{1+0} = 1 > 0$

Since $\sum b_n = \sum \frac{1}{n}$ div. (the Harmonic series), then

$\sum a_n = \sum \frac{2^n+3}{n \cdot 2^n+1}$ div by the Limit Comparison Test.

15. The series $\sum_{n=1}^{\infty}(-1)^{n+1}ne^{-n}$ is

$b_n = n e^{-n}$

- $\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{n}{e^n} \overset{LR}{=} \lim_{n\to\infty} \frac{1}{e^n} = 0$
- $f(x) = x e^{-x} \Rightarrow f'(x) = -x e^{-x} + e^{-x}$

$\Rightarrow f'(x) = e^{-x}(1-x) < 0 \ \forall x > 1$

so f & hence b_n is decreasing.

Then $\sum_{n=1}^{\infty}(-1)^{n+1} n e^{-n}$ is Conv. by the alternating Series Test.

(a) convergent by the alternating series test (correct)

(b) convergent by the integral test

(c) convergent by the comparison test

(d) divergent by the comparison test

(e) divergent by test for divergence.

16. The series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt[3]{n}}$ is

Since
- $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{\sqrt[3]{n}}$ Conv. by the alternating Series test
- $\sum_{n=1}^{\infty}\left|\frac{(-1)^{n-1}}{\sqrt[3]{n}}\right| = \sum_{n=1}^{\infty}\frac{1}{\sqrt[3]{n}}$ div. (a p-series $p = 1/3 < 1$)

Then the given Series is Conditionally convergent.

(a) conditionally convergent (correct)

(b) absolutely convergent

(c) divergent

(d) divergent by the test for divergence

(e) convergent by the integral test

17. The series

$$\sum_{n=1}^{\infty} \frac{2^{n^2}}{n!}$$

is

$a_n = \frac{2^{n^2}}{n!}$, $a_{n+1} = \frac{2^{(n+1)^2}}{(n+1)!} = \frac{2^{n^2+2n+1}}{(n+1)\cdot n!}$

$\frac{a_{n+1}}{a_n} = \frac{2^{2n+1}}{n+1}$; $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{2^{2n+1}}{n+1}$ $\frac{\infty}{\infty}$

$\overset{L'R}{=} \lim_{n\to\infty} \frac{2^{2n+1}\cdot 2 \cdot \ln 2}{1} = \infty$

Div. by The ratio test.

(a) divergent by the ratio test (correct)
(b) convergent by the ratio test
(c) a series where the ratio test in inconclusive
(d) convergent by the root test
(e) divergent by the integral test

18. The **radius of convergence** of the power series $\sum_{n=0}^{\infty} (-1)^n \frac{(x-3)^n}{2^{2n+3}}$ is

$a_n = (-1)^n \frac{(x-3)^n}{2^{2n+3}}$

$a_{n+1} = (-1)^{n+1} \frac{(x-3)^{n+1}}{2^{2(n+1)+3}} = (-1)^{n+1} \frac{(x-3)^{n+1}}{2^{2n+5}}$

$\left|\frac{a_{n+1}}{a_n}\right| = \left| - \frac{(x-3)^{n+1}}{2^{2n+5}} \cdot \frac{2^{2n+3}}{(x-3)^n} \right|$

$= \frac{|x-3|}{2^2} = \frac{|x-3|}{4}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{|x-3|}{4} = \frac{|x-3|}{4} < 1 \Rightarrow$ Conv.

$\Rightarrow |x-3| < 4$

$\Rightarrow$ radius of Conv. is 4.

(a) 4 (correct)
(b) 2
(c) 1
(d) 3
(e) ∞

19. A **power series representation** for $f(x) = \dfrac{x^3}{2-x}$ is (for $|x| < 2$)

(a) $\sum_{n=0}^{\infty} \dfrac{x^{n+3}}{2^{n+1}}$

(b) $\sum_{n=0}^{\infty} (-1)^n \dfrac{x^{n+3}}{2^{n+1}}$

(c) $\sum_{n=0}^{\infty} \dfrac{x^{3n}}{2^n}$

(d) $\sum_{n=0}^{\infty} (-1)^n \dfrac{x^{n+2}}{2^{n+1}}$

(e) $\sum_{n=0}^{\infty} \dfrac{x^{n+3}}{2^n}$

$$= \frac{x^3}{2} \cdot \frac{1}{1 - \frac{x}{2}}$$

$$= \frac{x^3}{2} \cdot \sum_{n=0}^{\infty} \left(\frac{x}{2}\right)^n, \quad \left|\frac{x}{2}\right| < 1$$

$$= \frac{x^3}{2} \cdot \sum_{n=0}^{\infty} \frac{x^n}{2^n}$$

$$= \sum_{n=0}^{\infty} \frac{x^{n+3}}{2^{n+1}}, \quad |x| < 2$$

(correct)

20. The **Taylor series** of $f(x) = e^{3x}$ at $a = 2$ is

(a) $\sum_{n=0}^{\infty} \dfrac{3^n \cdot e^6}{n!}(x-2)^n$

(b) $\sum_{n=0}^{\infty} \dfrac{3^n}{e^6 \cdot n!}(x-2)^n$

(c) $\sum_{n=0}^{\infty} \dfrac{e^6}{3^n \cdot n!}(x-2)^n$

(d) $\sum_{n=0}^{\infty} \dfrac{3^n}{n!}(x-2)^n$

(e) $\sum_{n=0}^{\infty} \dfrac{3e^{6n}}{n!}(x-2)^n$

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(2)}{n!}(x-2)^n$$

$$f(x) = e^{3x}$$

$$f'(x) = 3e^{3x}$$

$$f''(x) = 3^2 e^{3x}$$

$$f'''(x) = 3^3 e^{3x}$$

$$\vdots$$

$$f^{(n)}(x) = 3^n e^{3x} \Longrightarrow f^{(n)}(2) = 3^n e^6$$

$$\sum_{n=0}^{\infty} \frac{3^n \cdot e^6}{n!}(x-2)^n$$

(correct)

21. $\int x^2 \sin(x^2)\, dx =$

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \frac{x^{4n+5}}{4n+5} + C$ (correct)

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \frac{x^{4n+3}}{4n+3} + C$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \frac{x^{4n+4}}{4n+4} + C$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \frac{x^{4n+2}}{4n+2} + C$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \frac{x^{4n+1}}{4n+1} + C$

$$\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{2n+1}$$

$$\sin(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} (x^2)^{2n+1}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{4n+2}$$

$$x^2 \sin(x^2) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^2 \cdot x^{4n+2}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} x^{4n+4}$$

$$\int x^2 \sin(x^2)\, dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \int x^{4n+4}\, dx$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)!} \cdot \frac{x^{4n+5}}{4n+5} + C$$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	D	E	E	B
2	A	A	E	A	D
3	A	D	E	D	B
4	A	B	D	E	A
5	A	A	A	B	D
6	A	D	B	C	D
7	A	A	D	A	D
8	A	E	E	D	C
9	A	A	E	B	C
10	A	C	D	D	A
11	A	D	B	E	D
12	A	B	D	A	C
13	A	D	C	D	E
14	A	D	C	A	D
15	A	E	A	A	B
16	A	C	C	B	E
17	A	D	A	A	A
18	A	D	D	A	C
19	A	C	E	D	E
20	A	E	D	B	A
21	A	C	C	C	D