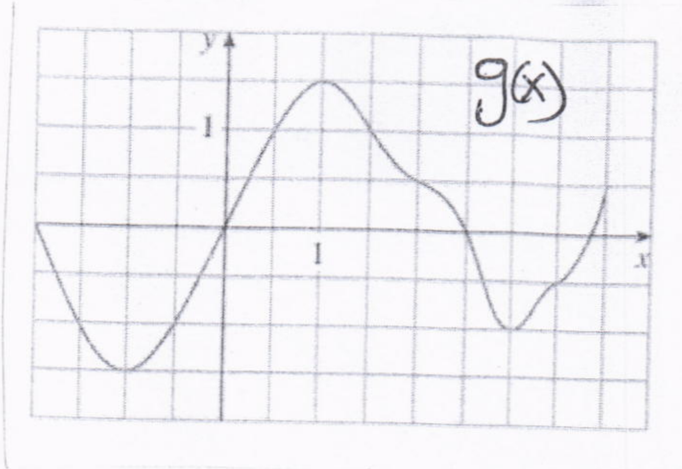


Figure 1

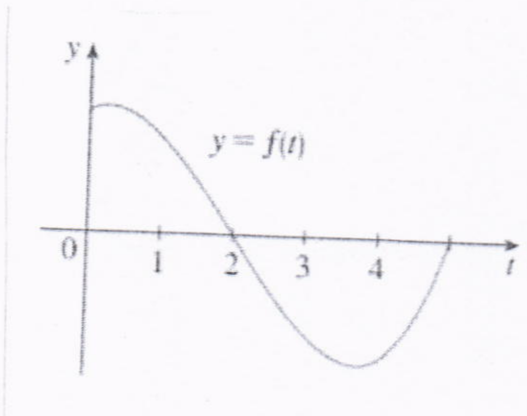


Figure 2

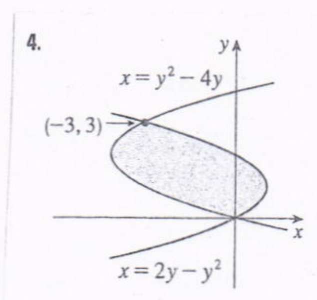


Figure 3

1. The lower sum for $f(x) = 1 - x^2$, $-1 \le x \le 1$, with $n = 4$ is

(a) $\frac{3}{4}$ (correct)

(b) $\frac{1}{4}$

(c) $\frac{1}{2}$

(d) 1

(e) $\frac{9}{8}$

$$L_4 = \left(\tfrac{1}{2}\right)(0) + \tfrac{1}{2}\left(1 - \tfrac{1}{4}\right) + \tfrac{1}{2}\left(1 - \tfrac{1}{4}\right) + \tfrac{1}{2}(0) = 3/4$$

Similar to #8/5.1

2. Using right endpoints, the area of the region that lies under the graph $f(x) = \sqrt{\cos x}$, $0 \le x \le \frac{\pi}{2}$ is

(a) $\lim_{n\to\infty} \frac{\pi}{2n} \sum_{i=1}^{n} \sqrt{\cos\left(i\frac{\pi}{2n}\right)}$ (correct)

(b) $\lim_{n\to\infty} \frac{\pi}{n} \sum_{i=1}^{n} \sqrt{\cos\left(i\frac{\pi}{n}\right)}$

(c) $\lim_{n\to\infty} \pi \sum_{i=1}^{n} \sqrt{\cos\left(i\frac{\pi}{n}\right)}$

(d) $\lim_{n\to\infty} \frac{\pi}{2n} \sum_{i=1}^{n} \sqrt{\cos\left(i\frac{\pi}{n}\right)}$

(e) $\lim_{n\to\infty} \frac{\pi}{2n} \sum_{i=1}^{n} \sqrt{\cos\left(\frac{i}{2n}\right)}$

$$\Delta x = \frac{\frac{\pi}{2} - 0}{n} = \frac{\pi}{2n}$$

$$x_1 = \frac{\pi}{2n},\ x_2 = 2\frac{\pi}{2n} \ \dots$$

$$x_i = i\frac{\pi}{2n},\ \dots\ x_n = \pi/2$$

$$A = \frac{\pi}{2n}\left[\sqrt{\cos\frac{\pi}{2n}} + \sqrt{\cos\frac{2\pi}{2}} + \cdots + \sqrt{\cos\frac{\pi}{2}}\right]$$

$$= \lim_{n\to\infty} \frac{\pi}{2n} \sum_{i=1}^{n} \sqrt{\cos\left(\frac{i\pi}{2n}\right)}$$

Similar to P/5.2

3. The graph of g is shown (figure 1). The estimate of $\int_{-2}^{4} g(x)\,dx$ with six subintervals using left endpoint is

(a) $-\frac{1}{2}$ (correct)

(b) $\frac{1}{2}$

(c) -2

(d) 0

(e) 2

$L_6 = 1\left[0 + (-1.5) + 0 + (1.5) + (-1) + (0.5)\right] = -1/2$

Similar to #6/5.2

4. $\sum\limits_{i=1}^{50} i^2 =$

(a) 42925 (correct)

(b) 25755

(c) 38965

(d) 44255

(e) 32425

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$n = 50$

$$\frac{50(50+1)(100+1)}{6}$$

$= 42925$

Formula in 5.2

5. If $F(x) = \int_2^x f(t)\,dt$, where f is the function whose graph is given (figure 2).
Which of the following values is largest?
$F(0)$, $F(1)$, $F(2)$, $F(3)$ and $F(4)$

(a) $F(2)$ (correct)
(b) $F(0)$
(c) $F(1)$
(d) $F(3)$
(e) $F(4)$

$F(0) < 0$
$F(1) < 0$
$F(3) < 0$
$F(4) < 0$
$F(2) = \int_2^2 f(t)\,dt = 0$

#52/5.2

6. For $x \in \left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, $\frac{d}{dx}\left[\int_{\tan x}^{5} (t - \tan^{-1} t)dt\right] =$

(a) $x \sec^2 x - \tan x \sec^2 x$ (correct)
(b) $(5 - \tan^{-1} 5) - \tan x \sec^2 x$
(c) $\tan x \sec^2 x - x \sec^2 x$
(d) $(\tan x - \tan^{-1} x) \sec^2 x$
(e) $\tan x - x$

$= (5)'[\ ] - (\tan x - \tan^{-1}(\tan x))\sec^2 x$

$= -\tan x \sec^2 x + x\sec^2 x$

$\approx$ 13–18/5.3

7. If $6 + \int_a^x \frac{f(t)}{t^2}\,dt = 2\sqrt{x}$, $x \geq 0$, then $f(x) + a =$

(a) $9 + \sqrt{x^3}$ (correct)

(b) $9 + \sqrt[3]{x^2}$

(c) $6 + \sqrt{x^2}$

(d) $6 + \sqrt{x^3}$

(e) $9 + \sqrt{x}$

If $x = a \Rightarrow$

$6 = 2\sqrt{a} \Rightarrow a = 9$

Diff Both sides

$0 + \frac{f(x)}{x^2} = \frac{1}{2\sqrt{x}}$

$\Rightarrow f(x) = x^{3/2}$

8. $\int_1^{64} \frac{1+\sqrt[3]{x}}{\sqrt{x}}\,dx =$

(a) $\frac{256}{5}$ (correct)

(b) $\frac{216}{5}$

(c) $\frac{212}{5}$

(d) $\frac{144}{5}$

(e) $\frac{252}{5}$

$\int_1^{64} \left(\frac{1}{\sqrt{x}} + \frac{\sqrt[3]{x}}{\sqrt{x}}\right) dx$

$= \int (x^{-1/2} + x^{-1/6})\,dx$

$= \left[2\sqrt{x} + \frac{6}{5}x^{5/6}\right]_1^{64}$

#39/5.4

$= (2)(8) + \frac{6}{5}((2^6)^{5/6}) - \left(2 + \frac{6}{5}\right)$

$= 16 + \frac{186}{5} = \boxed{\frac{256}{5}}$

9. $\int_0^{\frac{1}{\sqrt{3}}} \frac{x^2-1}{x^4-1}\,dx =$

(a) $\frac{\pi}{6}$ (correct)

(b) $\frac{\pi}{2}$

(c) $\frac{\pi}{4}$

(d) $\frac{8}{9}$

(e) 1

$$\int_0^{1/\sqrt{3}} \frac{x^2-1}{(x^2-1)(x^2+1)}\,dx$$

$$= \int_0^{1/\sqrt{3}} \frac{1}{x^2+1}\,dx = \left[\tan^{-1}(x)\right]_0^{1/\sqrt{3}}$$

#43/5.4

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) - \tan^{-1}(0)$$

$$= \pi/6 - 0$$

10. $\int_0^{\pi/2} \frac{\sin 2x}{1+\cos^2 x}\,dx =$

(a) $\ln 2$ (correct)

(b) $\tan^{-1}(\ln 2)$

(c) $\ln(1+e)$

(d) $-\ln(1+e)$

(e) $1-\ln 2$

$$= \int \frac{2\sin x\cos x}{1+\cos^2 x}\,dx$$

$\boxed{\#39/5.5}$

$$2\int\left(\frac{\cos x}{1+\cos^2 x}\right)\sin x\,dx,$$

let $u = \cos x \implies du = -\sin x\,dx$

$$-2\int \frac{u\,du}{1+u^2} = \frac{-2}{2}\int \frac{2u}{1+u^2}\,du =$$

$$-\ln(1+u^2) = \left[-\ln(1+\cos^2 x)\right]_0^{\pi/2} =$$

$$-\ln(1+0) + \ln 2 = \ln 2$$

11. $\int x^2\sqrt{2+x}\,dx =$

Similar to

$\boxed{\#46/5.5}$

(a) $\frac{2}{7}(x+2)^{7/2} - \frac{8}{5}(x+2)^{5/2} + \frac{8}{3}(x+2)^{3/2} + C$ (correct)

(b) $\frac{x^3}{3} + \frac{2}{3}(2+x)^{3/2} + C$

(c) $\frac{1}{7}(2+x)^{3/2} - \frac{5}{8}(x+2)^{5/2} + C$

(d) $\frac{2}{7}(x)^{7/2} - \frac{5}{8}(x)^{5/2} + \frac{1}{3}(x)^{3/2} + C$

(e) $\frac{2}{5}(x+2)^{5/2} - 3(x+2)^{1/2} + 8(x+2)^{3/2} + C$

$u = x+2 \Rightarrow u-2 = x \Rightarrow (u-2)^2 = x^2$

$\int (u-2)^2\sqrt{u}\,du = \int (u^2-4u+4)\sqrt{u}\,du$

$\frac{2}{7}(x+2)^{7/2} - \frac{8}{5}(x+2)^{5/2} + \frac{8}{3}(x+2)^{3/2} + C$

12. $\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x)\,dx =$

$\boxed{\#62/5.5}$

(a) $1 - \cos 1$ (correct)

(b) 1

(c) $\cos 1$

(d) $-\cos 1$

(e) $1 + \cos 1$

$u = \sin x \Rightarrow du = \cos x\,dx$

$\left.\begin{array}{l} x = 0 \Rightarrow u = 0 \\ x = \frac{\pi}{2} \Rightarrow u = 0 \end{array}\right\}$ $\int \cos x \sin(\sin x)\,dx$

$= \int_0^1 \sin u\,du = \left[-\cos u\right]_0^1$

$= -\cos 1 + \cos 0$

$= 1 - \cos 1$

13. The area of the shaded region in (figure 3) is equal to:

#4/6.1

(correct)

(a) 9
(b) 45
(c) 27
(d) 18
(e) 36

$A = \int_0^3 \left((2y - y^2) - (y^2 - 4y)\right) dy$

$\int_0^3 (-2y^2 + 6y)\, dy$

$2y - y^2 = y^2 - 4y$

$6y = 2y^2$

$3y = y^2 \Rightarrow 0, 3$

$\left(-\frac{2}{3}y^3 + 3y^2\right)\Big|_0^3 = -18 + 27 = \boxed{18}$

14. The area enclosed between $y = \cos x$ and $y = 1 - \cos x$ over $[0, \pi]$ is equal to:

#24/6.1

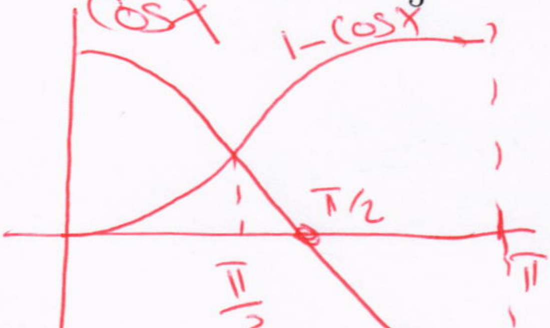
(correct)

(a) $2\sqrt{3} + \frac{\pi}{3}$
(b) $\sqrt{3} + \frac{\pi}{3}$
(c) $\frac{2}{3}\pi + \sqrt{3}$
(d) $2\sqrt{3} + 3\pi$
(e) $2 + \frac{\pi}{3}$

$1 - \cos x = \cos$

$\Rightarrow 2\cos x = 1 \Rightarrow \cos x = \frac{1}{2}$

$x = \pi/3$



$A = \int_0^{\pi/3} (\cos x) - (1 - \cos x)\, dx + \int_{\pi/3}^{\pi} \left((1 - \cos x) - (\cos x)\right) dx$

$= \int_0^{\pi/3} (2\cos x - 1)\, dx + \int_{\pi/3}^{\pi} (1 - 2\cos x)\, dx$

$\left[2\sin x - x\right]_0^{\pi/3} + \left[x - 2\sin x\right]_{\pi/3}^{\pi}$

$\left(\sqrt{3} - \frac{\pi}{3}\right) - (0) + (\pi - 0) - \left(\frac{\pi}{3} - \sqrt{3}\right) = 2\sqrt{3} + \frac{\pi}{3}$

15. The region bounded by the curve $y = x^2 + 1$ and the line $y = -x + 3$ is revolved about x-axis to generate the solid S. The volume of S is equal to:

(a) $\frac{117}{5}\pi$ (correct)

(b) $\frac{131}{3}\pi$

(c) $\frac{110}{3}\pi$

(d) $\frac{109}{5}\pi$

(e) $\frac{100}{3}\pi$

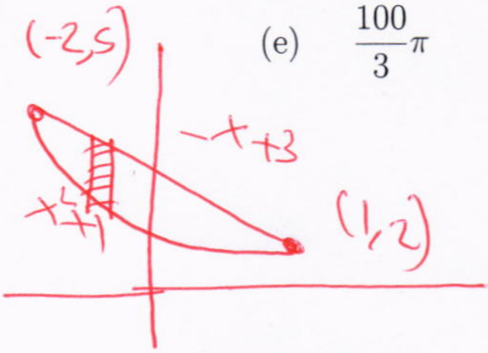
$x^2+1=-x+3$

$x^2+x-2=0 \Rightarrow x=-2, 1$

$V=\int_{-2}^{1}\left(\pi(-x+3)^2-\pi(x^2+1)^2\right)dx$

$=\pi\int_{-2}^{1}\left(x^2-6x+9-x^4-2x^2-1\right)dx$

$\pi\left[-\frac{x^5}{5}-\frac{1}{3}x^3-\frac{6}{2}x^2+8x\right]_{-2}^{1}=\frac{117}{5}\pi$



16. The volume of the solid formed by revolving the region bounded by the graphs of $f(x) = 2 - x^2$ and $g(x) = 1$ about the line $y = 1$, is given by

(a) $\pi \int_{-1}^{1} (1 - x^2)^2\, dx$ (correct)

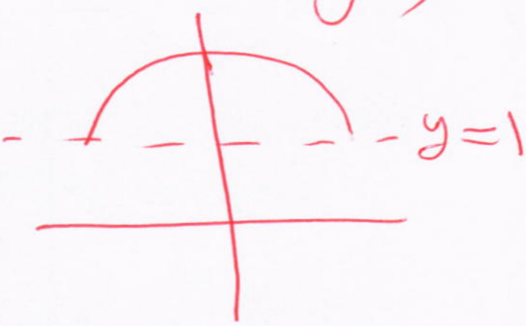
(b) $2\pi \int_{0}^{1} (1 + x^2)^2\, dx$

(c) $\pi \int_{-1}^{1} (-1 - x^2)^2\, dx$

(d) $\pi \int_{0}^{2} (-1 - x^2)^2\, dx$

(e) None of other choices

$R(x)=f(x)-g(x)$



$R(x)=(2-x^2)-(1)$

$V=\pi\int_{-1}^{1}(1-x^2)^2\,dx$

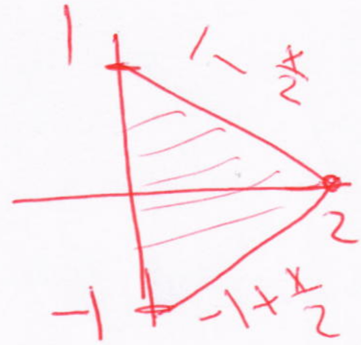
17. The volume of the solid having base the region bounded by the lines $y = 1-\frac{x}{2}$ and $y = -1+\frac{x}{2}$ and $x = 0$, where the cross sections perpendicular to the x-axis, are equilateral triangles is

(Hint: Area of an equilateral triangle of side s is $\frac{\sqrt{3}}{4}(s)^2$)

Base: $\left(1-\frac{x}{2}\right)-\left(-1+\frac{x}{2}\right)$ (correct)

$= 2 - x$

$A = \frac{\sqrt{3}}{4}(\text{Base})^2$



(a) $\frac{2}{3}\sqrt{3}$

(b) $3\sqrt{3}$

(c) $2\sqrt{3}$

(d) $\frac{3}{2}\sqrt{3}$

(e) $\sqrt{3}$

$V = \int_0^2 \frac{\sqrt{3}}{4}(2-x)^2\,dx$

$\frac{\sqrt{3}}{4}\int_0^2 (4-4x+x^2)\,dx = \dots$

18. Which one of the following statements is always True?

(a) $\int_0^3 e^{x^2}\,dx = \int_0^5 e^{x^2}\,dx + \int_5^3 e^{x^2}\,dx$ (correct)

(b) If $\int_0^1 f(x)\,dx = 0$, then $f(x) = 0$ for $x \in [0,1]$

(c) $\int_0^2 (x-x^2)\,dx$ represents the area under the curve $y = x-x^2$ from 0 to 2

(d) If $f(x)$ has a discontinuity at 0, then $\int_{-1}^1 f(x)\,dx$ does not exist

(e) If f is continuous on $[a,b]$ and $f(x) \geq 0$, then $\int_a^b \sqrt{f(x)}\,dx = \sqrt{\int_a^b f(x)\,dx}$

Propertis of Integral

End of ch#5, T or F