1. The volume of the solid obtained by rotating the region bounded by

$x = \sqrt{\sin y},\ 0 \le y \le \pi,\ x = 0$

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about $y = 4$ is given by

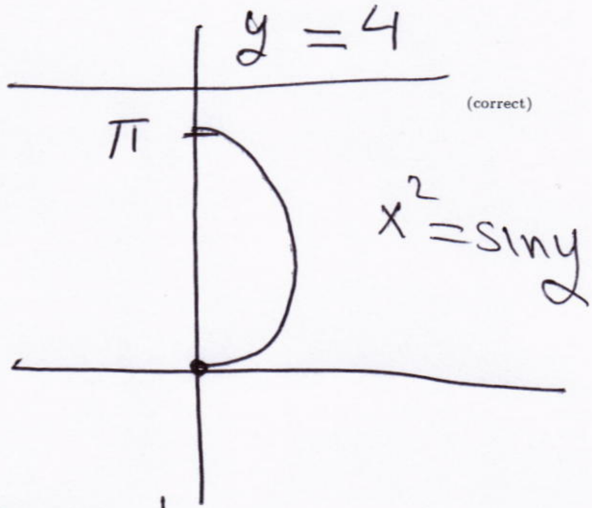
(a) $\int_0^{\pi} 2\pi(4-y)\sqrt{\sin y}\,dy$ (correct)

(b) $\int_0^{\pi} 2\pi y\sqrt{\sin y}\,dy$

(c) $\int_0^{\pi} 2\pi(y-4)^2\sqrt{\sin y}\,dy$

(d) $\int_0^{\pi} 2\pi y\,\sin y\,dy$

(e) $\int_0^{\pi} 2\pi(4-y)^2\,\sin y\,dy$



$2\pi\int_0^{\pi}(4-y)\sqrt{\sin y}\,dy$

2. The average value of $f(x) = \dfrac{\ln x}{\sqrt{x}}$ over $[4, 9]$ is equal to:

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(a) $\dfrac{12\ln 3 - 8\ln 2 - 4}{5}$ (correct)

(b) $\dfrac{\ln 9}{3} - \dfrac{\ln 4}{2}$

(c) $\dfrac{1}{5}\left[\dfrac{\ln 9}{3} - \dfrac{\ln 4}{2}\right]$

(d) $6\ln 3 - 4\ln 2 - 2$

(e) $\dfrac{6\ln 3 - 4\ln 2 - 2}{5}$

$f_{avg} = \dfrac{1}{9-4}\int_4^9 \dfrac{\ln x}{\sqrt{x}}\,dx$

by Parts:

$u = \ln x \Rightarrow du = \dfrac{dx}{x}$

$dv = \dfrac{dx}{\sqrt{x}} \Rightarrow v = 2\sqrt{x}$

$\int_4^9 \dfrac{\ln x}{\sqrt{x}}\,dx = \left[2\sqrt{x}\ln x\right]_4^9 - \left[4\sqrt{x}\right]_4^9$

$= (6\ln 9 - 4\ln 4) - (12 - 8)$

So $f_{avg} = \dfrac{6\ln 9 - 4\ln 4 - 4}{5}$

3. A number b that makes the average value of $f(x) = 2 + 6x - 3x^2$ over the interval $[0, b]$ equals 3, is equal to

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(a) $\dfrac{3-\sqrt{5}}{2}$

(b) 0

(c) 1

(d) $\dfrac{\sqrt{5}-3}{2}$

(e) $\dfrac{-3-\sqrt{5}}{2}$

$$f_{avg} = \frac{1}{b-0}\int_0^b (2+6x-3x^2)\,dx$$

(correct)

$$\int_0^b (2+6x-3x^2)\,dx = 3b$$

$$\left[2x+3x^2-x^3\right]_0^b = 3b \quad (b\neq 0)$$

$$2b+3b^2-b^3 = 3b$$

$$b^2-3b+1=0 \Rightarrow b = \frac{3\pm\sqrt{5}}{2}$$

4. Suppose that $f(1) = 2$, $f(4) = 7$, $f'(1) = 5$, $f'(4) = 3$ and f'' is continuous. Then, the value of $\int_1^4 x\, f''(x)\, dx =$

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(a) 2

(b) -10

(c) 4

(d) 12

(e) -8

By Parts

(correct)

$$u = x \Rightarrow du = dx$$

$$dv = f''(x)\,dx \Rightarrow v = f'(x)$$

$$\int_1^4 x f''(x)\,dx = \left[x f'(x)\right]_1^4 - \int_1^4 f'(x)\,dx$$

$$(4f'(4) - f'(1)) - (f(4) - f(1)) =$$

$$4f'(4) - f'(1) - f(4) + f(1) =$$

$$4(3) - 5 - 7 + 2 = \boxed{2}$$

5. $\int x \sec^2 x \, dx =$

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$u = x \Rightarrow du = dx$

(a) $x \tan x - \ln|\sec x| + c$ (correct)

(b) $\tan x - x + c$

(c) $x \tan x - \sec x + c$

(d) $\ln|\sec x + \tan x| - \tan x + c$

(e) $\tan x + \sec x - \sec^2 x + c$

$dv = \sec^2 x \, dx \Rightarrow v = \tan x$

$\int x \sec^2 x \, dx = x \tan x - \int \tan x \, dx$

$= x \tan x - \ln|\sec x| + C$

6. $\int \frac{\sin^3(\sqrt{x})}{\sqrt{x}} \, dx =$

6/7.2

(a) $\frac{2}{3}(\cos^3 \sqrt{x}) - 2\cos(\sqrt{x}) + c$ (correct)

(b) $\sqrt{3}\cos^2(\sqrt{x}) + 2\cos(\sqrt{x}) + c$

(c) $\frac{1}{3}\sin^2(x) - \sqrt{3}\sin(\sqrt{x}) + c$

(d) $\frac{2}{3}\sin^3(\sqrt{x}) - \sqrt{x}\cos(\sqrt{x}) + c$

(e) $\frac{1}{3}\cos^3(\sqrt{x}) - \sqrt{x}\cos(\sqrt{x}) + c$

$u = \sqrt{x} \Rightarrow du = \frac{1}{2\sqrt{x}} dx$

$2\int \sin^3 u \, du = 2\int \sin u (1 - \cos^2 u) \, du$

$w = \cos u \Rightarrow dw = -\sin u \, du$

$-2\int (1 - w^2) \, dw = 2\int (w^2 - 1) \, dw = \frac{2w^3}{3} - 2w + C$

$\frac{2}{3}(\cos^3 \sqrt{x}) - 2\cos(\sqrt{x}) + C$

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7. $\int_0^{\pi/6} \sqrt{1+\cos 2x}\,dx =$

(a) $\frac{\sqrt{2}}{2}$

(b) $\sqrt{3}$

(c) $\sqrt{2}$

(d) $\frac{1}{2}$

(e) $\frac{\sqrt{3}}{2}$

$\int_0^{\pi/6} \sqrt{1+(2\cos^2 x-1)}\,dx$ (correct)

$\sqrt{2}\int_0^{\pi/6} |\cos x|\,dx = \sqrt{2}\int_0^{\pi/6} \cos x\,dx$

$\sqrt{2}\left[\sin x\right]_0^{\pi/6} = \sqrt{2}\left[\frac{1}{2}-0\right]$

$= \sqrt{2}/2$

8. $\int_0^{\pi/4} \sec^4 x \tan^4 x\,dx =$

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(a) $\frac{12}{35}$

(b) $\frac{\pi^7}{7}+\frac{\pi^5}{5}$

(c) $\frac{\pi^7}{7(4)^7}+\frac{\pi^5}{5(4)^5}$

(d) $4\sqrt{2}$

(e) $\frac{2}{35}$

$\int_0^{\pi/4} (\tan^2+1)\tan^4 x \cdot \sec^2 x\,dx$ (correct)

$u=\tan x \Rightarrow du=\sec^2 x\,dx$

$\int (u^2+1)u^4\,du$ $\begin{cases} x=0 \Rightarrow u=0, \\ x=\pi/4 \Rightarrow u=1 \end{cases}$

$\int_0^1 (u^6+u^4)\,du = \frac{1}{7}+\frac{1}{5}$

$= \boxed{12/35}$

9. $\displaystyle\int \frac{dx}{x^2\sqrt{x^2-16}} =$

$\boxed{8/7.3}$

(a) $\dfrac{\sqrt{x^2-16}}{16x} + c$ (correct)

(b) $\dfrac{1}{\sqrt{x^2-16}} + c$

(c) $\dfrac{x}{\sqrt{x^2-16}} + c$

(d) $\dfrac{1}{x\sqrt{x^2-16}} + c$

(e) $\dfrac{\sqrt{x^2-16}}{16} + c$

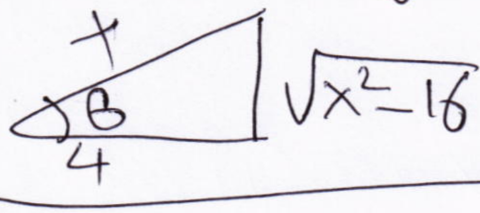
$u = 4\sec\theta \Longrightarrow$

$dx = 4\sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-16} = \sqrt{16\sec^2\theta - 16} = 4\tan\theta$

$$\int \frac{dx}{x^2\sqrt{x^2-16}} = \int \frac{4\sec\theta\tan\theta\, d\theta}{16\sec^2\theta\, 4\tan\theta}$$

$$= \frac{1}{16}\int \frac{d\theta}{\sec\theta} = \frac{1}{16}\sin\theta + C = \frac{1}{16}\frac{\sqrt{x^2-16}}{x} + C$$



10. $\displaystyle\int_0^1 \frac{dx}{(x^2+1)^2} =$

$\dfrac{14}{483}$

(a) $\dfrac{\pi}{8} + \dfrac{1}{4}$ (correct)

(b) $\dfrac{1}{4}$

(c) $\dfrac{\pi}{2} + 1$

(d) $\dfrac{1}{8}$

(e) $\dfrac{\pi}{8} + 1$

$x = \tan\theta$, $[0,1] \longrightarrow [0, \frac{\pi}{4}]$

$dx = \sec^2\theta\, d\theta$

$$\int \frac{\sec^2\theta\, d\theta}{(1+\tan^2\theta)^2} = \int \frac{\sec^2\theta\, d\theta}{(\sec^2\theta)^2}$$

$$\int \cos^2\theta\, d\theta = \frac{1}{2}\int (1+\cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/4} = \frac{1}{2}\left[\frac{\pi}{4} + \frac{\sin\frac{\pi}{2}}{2}\right] - (0)$$

$$= \boxed{\frac{\pi}{8} + \frac{1}{4}}$$

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11. $\int \frac{x}{x-5}\,dx =$

(a) $x + 5\ln|x-5| + C$ (correct)

(b) $\frac{1}{5}\ln|x-5| + C$

(c) $\frac{1}{5}x\ln|x-5| + C$

(d) $5\ln|x-5| + C$

(e) $x - \frac{x^2}{10} + C$

$$\int \frac{x-5+5}{(x-5)}\,dx =$$

$$\int \left(1 + \frac{5}{x-5}\right)dx =$$

$$x + 5\ln|x-5| + C$$

12. If $\frac{x^3-2x^2-4}{x^3-2x^2} = 1 + \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$. Then $B + C =$

(a) 1 (correct)

(b) -1

(c) 0

(d) 2

(e) -2

$$= 1 - \frac{4}{x^2(x+2)}$$

So

$$x^3-2x^2 \overline{)\,x^3-2x^2-4\,} = 1$$
$$-x^3+2x^2$$
$$-4$$

$$\frac{-4}{x^2(x+2)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$$

$$-4 = Ax(x+2) + B(x\cancel{+}2) + Cx^2$$

$$x = 0 \Rightarrow -4 = B(-2) \Rightarrow B = 2$$

$$x = 2 \Rightarrow -4 = 4C \Rightarrow C = -1$$

$$\boxed{1}$$

13. $\displaystyle\int \frac{1}{e^x+1}\,dx =$

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(Hint: Let $u = e^x$, then use partial fraction technique)

(a) $x - \ln(e^x+1) + C$ (correct)

(b) $\ln(e^x - x) + C$

(c) $1 - \ln(1+e^x) + C$

(d) $x - e^x \ln(x+1) + C$

(e) $-\ln(x + e^x) + C$

$u = e^x \Rightarrow du = e^x\,dx \Rightarrow dx = \frac{du}{u}$

$\int \frac{du}{u(u+1)} = \int \left(\frac{A}{u} + \frac{B}{u+1}\right) du$

$A = 1,\ B = -1$

$\ln|u| - \ln|u+1| = \ln e^x - \ln(e^x+1)$

$= x - \ln(e^x+1) + C$

14. $\displaystyle\int \frac{1}{1-\cos x}\,dx =$

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(a) $-\cot\left(\frac{x}{2}\right) + C$ (correct)

(b) $-\tan\left(\frac{x}{2}\right) - 1 + C$

(c) $-\sec\left(\frac{x}{2}\right) + C$

(d) $1 - \tan\left(\frac{1}{4}x\right) + C$

(e) $\cot\left(\frac{x}{4}\right) + C$

$t = \tan(x/2)$

$\int \frac{\frac{2\,dt}{1+t^2}}{1 - \frac{1-t^2}{1+t^2}} =$

$\int \frac{2}{(1+t^2) - (1-t^2)}$

$\int \frac{dt}{t^2} = -\frac{1}{t} + C$

$= -\cot(x/2) + C$

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15. The Improper Integral $\int_0^\infty \frac{dx}{x^2+3x+2}$ is

(a) Convergent to $\ln 2$ (correct)

(b) Divergent to $+\infty$

(c) Divergent to $-\infty$

(d) Convergent to $2\ln 2$

(e) Convergent to $\frac{1}{2}\ln 2$

$$\frac{1}{x^2+3x+2} = \frac{1}{(x+2)(x+1)}$$

$A = 1,\ B = -1$

$$\int_0^\infty (\quad)\,dx = \lim_{t\to\infty} \int_0^t \left(\frac{1}{x+1} - \frac{1}{x+2}\right)$$

$$= \lim_{t\to\infty} \left[\ln\left(\frac{x+1}{t+2}\right)\right]_0^t = \lim_{t\to\infty} \left[\ln\left(\frac{t+1}{t+2}\right) - \ln\left(\frac{1}{2}\right)\right]$$

$$= \ln 1 + \ln 2$$

16. $\int \frac{dx}{x^2+2x+2} =$

(a) $\tan^{-1}(x+1) + C$ (correct)

(b) $(x+1)\tan^{-1}(x) + C$

(c) $x\tan^{-1}(x+1) + C$

(d) $\tan^{-1}(x+2) + C$

(e) $2\tan^{-1}(x+1) + C$

$$\frac{1}{x^2+2x+1+1}$$

$$= \int \frac{1}{(x+1)^2+1}\,dx$$

$u = x+1 \Rightarrow$

$$\int \frac{1}{u^2+1}\,du = \tan^{-1}(u) + C$$

$$\tan^{-1}(x+1) + C$$

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17. $\int_6^8 \frac{4}{(x-6)^3}\,dx$

$$= \lim_{t\to 6^+} \int_t^8 4(x-6)^{-3}\,dx$$

(a) Diverges to $+\infty$ (correct)

(b) Diverges to $-\infty$

(c) Converges to $-\frac{2}{3}$

(d) Converges to $\frac{1}{2}$

(e) Converges to 4

$$= \lim_{t\to 6^+} \left[\frac{4(x-6)^{-2}}{-2}\right]_t^8$$

$$= \lim_{t\to 6^+} \left[\frac{-2}{2^2} + \frac{1}{(t-6)^2}\right]$$

$$= -\frac{1}{2} + \infty = \infty$$

18. The length of the curve $f(x) = \sqrt{x - x^2} + \sin^{-1}(\sqrt{x})$ over $[0, 1]$ is equal to:

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$$f'(x) = \frac{1-2x}{2\sqrt{x-x^2}} + \frac{1}{2\sqrt{x}\sqrt{1-x}}$$

(a) 2 (correct)

(b) e

(c) $e - 1$

(d) $\sqrt{2}$

(e) $e + \frac{1}{2}$

$$= \frac{1-x}{\sqrt{x-x^2}} = \frac{1-x}{\sqrt{x(1-x)}}$$

$$\frac{\sqrt{1-x}\sqrt{1-x}}{\sqrt{x}\,\sqrt{1-x}} = \sqrt{\frac{1}{x} - 1}$$

$$1 + f'(x)^2 = 1 + \frac{1}{x} - 1 = 1/x$$

$$L = \int_0^1 \frac{dx}{\sqrt{x}} = \left[2\sqrt{x}\right]_0^1 = 2 \quad (\text{Imp. Integ.})$$