a) is a Solution, Master Code

Math 102 Final Exam Term 212

1. Using the method of cylindrical shell, the volume of the solid generated by revolving about the x-axis the region bounded by $x = 1 + (y-2)^2$; $x = 2$, is equal to:

a) $\frac{16}{3}\pi$
b) $\frac{2}{3}\pi$
c) $\frac{4}{3}\pi + 2$
d) $5\pi + \frac{1}{4}$
e) $2\pi + 3$

2. The average value of $f(x) = x \sin(x^2)$ over $[0, 10]$ is equal to

a) $\frac{1}{20}(1 - \cos 100)$
b) $\frac{1}{10}(\cos 10 - 1)$
c) $\frac{1}{10}(\cos 1 - 1)$
d) $\frac{1}{20}(\sin 100 - 1)$
e) $\frac{1}{10}(\sin 10 - 1)$

3. $\displaystyle\int_0^{1/2} \cos^{-1} x \, dx =$

a) $\frac{\pi}{6} + 1 - \frac{\sqrt{3}}{2}$

b) $\frac{\pi}{6} + \frac{\sqrt{3}}{2}$

c) $\frac{\pi}{3} - 1$

d) $\frac{\pi}{3} + 1$

e) $\frac{1}{2}\left(\frac{\pi}{3} - 1\right)$

4. $\displaystyle\int \frac{dx}{\sqrt{x^2 - 6x + 13}} =$

a) $\ln|\sqrt{x^2 - 6x + 13} + x - 3| + c$

b) $\frac{1}{2}\ln|x^2 - 5x + 10| + c$

c) $\ln|\sqrt{x^2 - x} + x + 13| + c$

d) $\ln|(x-3)^2 + 4| + c$

e) $\ln|(x-3)^2 + 4x| + c$

5. $\displaystyle\int \frac{x^3+4}{x^2+4}\,dx =$

a) $\frac{1}{2}x^2 - 2\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + c$

b) $\frac{1}{2}x^2 + c$

c) $\frac{1}{4}x^4 + 4x + \ln(x^2+4) + c$

d) $\frac{1}{4}x^4 + 4x + \ln(x^2+4) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

e) $x^2 - 2\ln(x^2+4) + \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + c$

6. If $f(x) = \sin(x^3)$, then $f^{(15)}(0) =$

a) $\frac{15!}{5!}$

b) $\frac{15!}{3!}$

c) $15!$

d) $10!$

e) $5!$

7. If the sequence $\left\{ n \sin \left(\frac{1}{n} \right) \right\}$ has the limit L, and the sequence $\left\{ \frac{e^n + e^{-n}}{e^{2n} - 1} \right\}$ converges to K, then $K + L =$

a) 1
b) 0
c) $1 + e$
d) $e - 1$
e) $e^2 - e$

8. In the sequence defined by $a_1 = 2$, $a_2 = 1$, $a_{n+1} = a_n - a_{n-1}$, $a_6 =$

a) 1
b) 0
c) -1
d) 2
e) -2

9. Considering the Integral test for the series $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$, $f(x) = \frac{x}{x^4+1}$, one statement is True only:

a) $\int_1^{\infty} \frac{x}{x^4+1}\, dx = \frac{\pi}{8}$

b) $\sum_{n=1}^{\infty} \frac{n}{n^4+1} = \int_1^{\infty} \frac{x}{x^4+1}\, dx$

c) The integral test fails for $\sum_{n=1}^{\infty} \frac{n}{n^4+1}$

d) $f(x)$ is increasing on $[\sqrt[3]{8}, \infty)$

e) $\sum_{n=1}^{\infty} \frac{n}{n^4+1} = +\infty$

10. The Improper Integral $\int_0^1 \frac{dx}{x^2 - 2x}$

a) Diverges to $-\infty$

b) Convergent to $\ln 2$

c) Diverges to $+\infty$

d) Converges to $1 - \ln 2$

e) Converges to $e - \ln 2$

11. One statement is **True** about the series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\ln(n+4)}$:

a) Its a convergent alternating series

b) Its a divergent alternating series

c) The limit of the positive n-th term is 1

d) The positive n-th term form an increasing sequence

e) $S_4 < 0$

12. The Taylor Series of $f(x) = x^4 - 3x^2 + 1$ centered at $a = 1$ is given by

a) $-1 - 2(x-1) + 3(x-1)^2 + 4(x-1)^3 + (x-1)^4$

b) $-1 - (x-1) + (x-1)^2 - (x-1)^3 + (x-1)^4$

c) $-1 - x - 2x^2 - 3(x-1)^3 + 4(x-1)^4$

d) $(x-1) + (x-1)^2 + (x-1)^3 + (x-1)^4$

e) $-1 + 3(x-1) + 4(x-1)^2 + 5(x-1)^4$

13. When evaluate, as an infinite series, $\int x \cos(x^2)\, dx =$

a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \frac{x^{4n+2}}{(4n+2)} + c$

b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \frac{x^{4n+2}}{(4n+2)!} + c$

c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \frac{x^{2n+1}}{(2n+1)} + c$

d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \frac{x^{2n+3}}{(2n+3)} + c$

e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} \frac{x^{2n+2}}{(2n+2)} + c$

14. $3 + \frac{9}{2!} + \frac{27}{3!} + \frac{81}{4!} + \ldots =$

a) $e^3 - 1$

b) $3e^3$

c) $3(e^3) - 3$

d) $e^3 - 3$

e) $3 + e^3$

15. Let $g(x) = \int_0^x f(t)\,dt$, where f is the function whose graph shown,

then $g(1) + g'(2) =$

a) $-\dfrac{1}{2}$

b) -1

c) 0

d) 1

e) $\dfrac{1}{2}$

16. $\displaystyle\int_0^{\ln 2} \frac{e^x}{1+e^{2x}}\,dx =$

a) $\tan^{-1} 2 - \dfrac{\pi}{4}$

b) $\tan^{-1} 2$

c) $-\dfrac{\pi}{4}$

d) $2 - \dfrac{\pi}{4}$

e) $\tan^{-1} \ln 2$

17. $1 + 0.4 + 0.16 + 0.064 + \ldots =$

a) $\frac{5}{3}$
b) $\frac{3}{2}$
c) $+\infty$
d) $\sqrt{2}$
e) $\frac{5}{2}$

18. The area of the region enclosed by the curves $y = \frac{1}{x}$, $y = x$, $x > 0$ and $y = \frac{1}{4}x$ is equal to

a) $\ln 2$
b) $\ln 2 - \frac{3}{4}$
c) $\frac{3}{4} - \ln 2$
d) $2 \ln 2$
e) $\frac{3}{4}$

19. The region enclosed by the curves $y = x$, $y = x^2$ is rotated about the line $x = 2$, then the volume as the resolution solid is

a) $\frac{\pi}{2}$

b) $\pi\left(\frac{2}{15}\right)$

c) π

d) $\frac{8\pi}{15}$

e) $\frac{\pi}{15}$

20. $\int \csc^3 x\, dx =$

(Hint: $\int \csc x dx = -\ln|\csc x + \cot x| + c$)

a) $\frac{-1}{2}\csc x \cot x - \frac{1}{2}\ln|\csc x + \cot x| + c$

b) $\frac{1}{2}[\csc x \cot x + \ln|\csc x + \cot x|] + c$

c) $[\csc x \cot x + \ln|\csc x + \cot x|] + c$

d) $\frac{\csc^4 x}{4} + c$

e) $-\csc x \cot x - \ln|\csc x + \cot x| + c$

21. The area of the surface obtained by rotating the curve about x-axis, $y = \sqrt{1+e^x}$, $0 \leq x \leq 1$

a) $\pi(e+1)$

b) $\pi(e+3)$

c) $\frac{3}{2}\pi$

d) $\frac{\pi}{2}(e^2+3)$

e) $\frac{\pi}{2}(e-3)$

22. The series $\sum_{n=0}^{\infty} \frac{\sin nx}{3^n}$ converges for

a) all values of x

b) $x \in [0, \pi]$ only

c) $x \in [-\pi, \pi]$ only

d) $x \in [0, 2\pi]$ only

e) $x \in [-2\pi, 2\pi]$ only

23. By the limit Comparison test, the series $\sum_{n=1}^{\infty} \frac{2n^2+3n}{\sqrt{5+n^5}}$

a) diverges by comparing with $\sum_{n=1}^{\infty} \frac{2}{n^{1/2}}$

b) converges by comparing with $\sum_{n=1}^{\infty} \frac{2}{n^{1/2}}$

c) diverges by comparing with $\sum_{n=1}^{\infty} \frac{2}{n^{3/2}}$

d) converges by comparing with $\sum_{n=1}^{\infty} \frac{2}{n^{3/2}}$

e) converges by comparing with $\sum_{n=1}^{\infty} \frac{2}{n^{5/2}}$

24. Let $\{b_n\}$ be a sequence of positive numbers that converges to $\frac{1}{2}$ and $a_n = \frac{(-1)^n n!}{n^n\, b_1 b_2 \ldots b_n}$, then the series $\sum_{n=1}^{\infty} a_n$ is absolutely convergent by the ratio test since

a) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{2}{e}$

b) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{2}$

c) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = 1$

d) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{1}{e}$

e) $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \frac{3}{e}$

25. The power series representation for the function $f(x) = \dfrac{x}{(1+4x)^2}$ is

a) $\sum_{n=0}^{\infty}(n+1)4^n(-1)^n x^{n+1}$

b) $\sum_{n=0}^{\infty} 4^n(-1)^n x^{n+1}$

c) $\sum_{n=0}^{\infty}(n+1)4^n x^{n+1}$

d) $\sum_{n=0}^{\infty}(-1)^n 4^n x^{n+1} x^n$

e) $\sum_{n=0}^{\infty}(n+1)4^n x^n$

26. The volume of the solid obtained by rotating the region bounded by the curves

$$y = e^{-x^2},\ y = 0,\ x = -1,\ x = 1$$

about $y = -1$ is

a) $2\pi \int_0^1 (e^{-2x^2} + 2e^{-x^2})\,dx$

b) $\pi \int_0^1 (e^{-x^2} + 2e^{-x^2})\,dx$

c) $\pi \int_0^1 (e^{-2x^2} - 2e^{-x^2})\,dx$

d) $2\pi \int_0^1 (e^{-2x^2} - 2e^{-x})\,dx$

e) $\int_0^1 (e^{-2x^2} - 2e^{-x^2})\,dx$

27. $\int \cos x \cos^3(\sin x)\, dx =$

a) $\sin(\sin x) - \frac{1}{3}\sin^3(\sin x) + c$

b) $\frac{1}{4}\cos^4(\sin x) + c$

c) $\cos(\sin x) - \frac{1}{3}\cos^3(\sin x) + c$

d) $\sin(\sin x) + \frac{2}{3}\sin^3(\cos x) + c$

e) $\cos(\sin x) + \cos^3(\sin x) + c$

28. The interval of convergence of the series

$$\sum_{n=2}^{\infty} \frac{x^n(-1)^n}{4^n \ln n}$$

is

a) $(-4, 4]$

b) $(-4, 5)$

c) $[-4, 4]$

d) $[-4, 4)$

e) $(-\infty, \infty)$