King Fahd University of Petroleum and Minerals
Department of Mathematics

**Math 102**
**Major Exam II**
**213**
**July 25, 2022**

# EXAM COVER

**Number of versions: 4**
**Number of questions: 18**
**Number of Answers: 5**

King Fahd University of Petroleum and Minerals
Department of Mathematics
**Math 102**
**Major Exam II**
**213**
**July 25, 2022**
**Net Time Allowed: 120 Minutes**

# MASTER VERSION

1. The average value of the function

$$f(x) = \frac{x^2}{(x^3+3)^2}$$

on the interval $[-1, 1]$ is

(a) $\frac{1}{24}$ (correct)

(b) $\frac{1}{12}$

(c) $\frac{1}{18}$

(d) $\frac{1}{6}$

(e) $\frac{1}{4}$

2. The number $b$ such that the average value of $f(x) = 3x^2 + 8x$ on the interval $[-1, b]$ is equal to 7 is

(a) 2 (correct)

(b) 3

(c) 4

(d) 1

(e) 5

3. If $\int_0^{\frac{1}{2}} \cos^{-1} x \, dx = \frac{1}{6}(\pi + B - 3\sqrt{3})$, then $B$ equals to

(a) 6 (correct)
(b) 2
(c) 4
(d) 0
(e) 8

4. $\int e^{\sqrt{x}} \, dx =$

(a) $2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$ (correct)
(b) $\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c$
(c) $2\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c$
(d) $\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$
(e) $\sqrt{x}e^{\sqrt{x}} + 3e^{\sqrt{x}} + c$

5. $\int \sqrt{\cos\theta}\,\sin^3\theta\,d\theta =$

(a) $\frac{2}{7}\sqrt{\cos^7\theta} - \frac{2}{3}\sqrt{\cos^3\theta} + c$ (correct)

(b) $\frac{1}{7}\sqrt{\cos^7\theta} - \frac{1}{3}\sqrt{\cos^3\theta} + c$

(c) $\frac{2}{7}\sqrt{\cos^7\theta} + \frac{2}{3}\sqrt{\cos^3\theta} + c$

(d) $\frac{1}{7}\sqrt{\cos^7\theta} + \frac{1}{3}\sqrt{\cos^3\theta} + c$

(e) $-\frac{2}{7}\sqrt{\cos^7\theta} + \frac{2}{3}\sqrt{\cos^3\theta} + c$

6. $\int \sin(8x)\,\cos(5x)\,dx =$

(a) $-\frac{1}{6}\cos(3x) - \frac{1}{26}\cos(13x) + c$ (correct)

(b) $-\frac{1}{6}\cos(3x) - \frac{1}{2}\cos(13x) + c$

(c) $\frac{1}{6}\cos(3x) + \frac{1}{26}\cos(13x) + c$

(d) $\frac{1}{6}\cos(3x) + \frac{1}{2}\cos(13x) + c$

(e) $-\frac{1}{3}\cos(3x) - \frac{1}{13}\cos(13x) + c$

7. $\int x \tan^2 x =$

(a) $x \tan x - \ln|\sec x| - \dfrac{x^2}{2} + c$ (correct)

(b) $x \tan x + \ln|\sec x| - \dfrac{x^2}{2} + c$

(c) $x \tan x - \ln|\sec x| + \dfrac{x^2}{2} + c$

(d) $x \tan x + \ln|\sec x| + \dfrac{x^2}{2} + c$

(e) $2x \tan x - 2\ln|\sec x| - \dfrac{x^2}{2} + c$

8. $\displaystyle\int_{\sqrt{2}}^{2} \frac{1}{x^3\sqrt{x^2-1}}\,dx =$

(a) $\dfrac{\pi}{24} + \dfrac{\sqrt{3}}{8} - \dfrac{1}{4}$ (correct)

(b) $\dfrac{\pi}{24} - \dfrac{\sqrt{3}}{8} + \dfrac{1}{4}$

(c) $\dfrac{\pi}{24} - \dfrac{\sqrt{3}}{8} - \dfrac{1}{4}$

(d) $\dfrac{\pi}{12} + \dfrac{\sqrt{3}}{8} - \dfrac{1}{4}$

(e) $\dfrac{\pi}{12} - \dfrac{\sqrt{3}}{8} + \dfrac{1}{4}$

9. $\int \sqrt{1-4x^2}\,dx =$

(a) $\frac{1}{4}[\sin^{-1}(2x) + 2x\sqrt{1-4x^2}] + c$ (correct)

(b) $\frac{1}{8}[\sin^{-1}(2x) + 2x\sqrt{1-4x^2}] + c$

(c) $\frac{1}{4}[\sin^{-1}(2x) - 2x\sqrt{1-4x^2}] + c$

(d) $\frac{1}{8}[\sin^{-1}(2x) - 2x\sqrt{1-4x^2}] + c$

(e) $\frac{1}{2}[\sin^{-1}(2x) + x\sqrt{1-4x^2}] + c$

10. $\int_0^1 \frac{dx}{(x^2+1)^2} =$

(a) $\frac{\pi}{8} + \frac{1}{4}$ (correct)

(b) $\frac{\pi}{8} - \frac{1}{4}$

(c) $\frac{\pi}{8} + \frac{1}{2}$

(d) $\frac{\pi}{8} - \frac{1}{2}$

(e) $\frac{\pi}{4} + \frac{1}{2}$

11. $\int_0^1 \frac{2}{2x^2+3x+1}\,dx =$

(a) $2\ln\frac{3}{2}$ (correct)

(b) $\ln\frac{3}{2}$

(c) $3\ln\frac{3}{2}$

(d) $4\ln\frac{3}{2}$

(e) $\ln\frac{2}{3}$

12. $\int_3^4 \frac{x^3-2x^2-4}{x^3-2x^2}\,dx =$

(a) $\frac{7}{6}+\ln\frac{2}{3}$ (correct)

(b) $\frac{7}{6}-\ln\frac{2}{3}$

(c) $\frac{7}{5}-\ln\frac{2}{3}$

(d) $\frac{7}{5}+\ln\frac{2}{3}$

(e) $\frac{7}{8}+\ln\frac{3}{2}$

13. $\displaystyle\int \frac{dx}{2\sqrt{x+3}+x} =$

(a) $\displaystyle\frac{3}{2}\ln(\sqrt{x+3}+3)+\frac{1}{2}\ln|\sqrt{x+3}-1|+c$ (correct)

(b) $\displaystyle\frac{3}{2}\ln(\sqrt{x+3}+3)-\frac{1}{2}\ln|\sqrt{x+3}-1|+c$

(c) $\displaystyle\frac{2}{3}\ln(\sqrt{x+3}+3)+\frac{1}{3}\ln|\sqrt{x+3}-1|+c$

(d) $\displaystyle\frac{2}{3}\ln(\sqrt{x+3}+3)-\frac{1}{3}\ln|\sqrt{x+3}-1|+c$

(e) $\displaystyle\frac{3}{2}\ln(\sqrt{x+3}+3)+\frac{1}{3}\ln|\sqrt{x+3}-1|+c$

14. $\displaystyle\int \frac{dx}{1+\sin x-\cos x} =$

(a) $\displaystyle\ln\left|\frac{\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)}\right|+c$ (correct)

(b) $\displaystyle\ln\left|\frac{1+\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}\right|+c$

(c) $\displaystyle\ln\left|\tan\left(\frac{x}{2}\right)\right|+c$

(d) $\displaystyle\ln\left|1+\tan\left(\frac{x}{2}\right)\right|+c$

(e) $\displaystyle\ln\left|\tan\left(\frac{x}{2}\right)\right|+x+c$

15. The improper integral $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$ is

(a) converges to $\frac{2}{e}$ (correct)

(b) converges to $\frac{4}{e}$

(c) converges to $\frac{6}{e}$

(d) converges to $\frac{1}{e}$

(e) divergent

16. The improper integral $\int_{-1}^0 \frac{e^{\frac{1}{x}}}{x^3}\,dx$ is

(a) convergent to $-\frac{2}{e}$ (correct)

(b) convergent to $\frac{3}{e}$

(c) convergent to $-\frac{3}{e}$

(d) convergent to $\frac{4}{e}$

(e) convergent to $-\frac{4}{e}$

17. The length of the curve

$$x = \frac{y^4}{8} + \frac{1}{4y^2},\ 1 \le y \le 2$$

is equal to

(a) $\frac{33}{16}$ (correct)

(b) $\frac{31}{8}$

(c) $\frac{33}{8}$

(d) $\frac{31}{16}$

(e) $\frac{5}{16}$

18. The length of the curve $y = \ln(\cos x), 0 \le x \le \frac{\pi}{3}$ is equal to

(a) $\ln(2+\sqrt{3})$ (correct)

(b) $\ln(4+\sqrt{3})$

(c) $\ln(4-\sqrt{3})$

(d) $\ln(2-\sqrt{3})$

(e) $\ln(2-\sqrt{2})$

King Fahd University of Petroleum and Minerals
Department of Mathematics

**CODE01** **CODE01**

# Math 102
# Major Exam II
# 213
# July 25, 2022
# Net Time Allowed: 120 Minutes

Name: _____

ID: _____ Sec: _____.

**Check that this exam has 18 questions.**

## Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. $\int e^{\sqrt{x}}\,dx =$

(a) $\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c$
(b) $\sqrt{x}e^{\sqrt{x}} + 3e^{\sqrt{x}} + c$
(c) $2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$
(d) $2\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c$
(e) $\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$

2. The number $b$ such that the average value of $f(x) = 3x^2 + 8x$ on the interval $[-1, b]$ is equal to 7 is

(a) 4
(b) 1
(c) 2
(d) 3
(e) 5

3. If $\int_0^{\frac{1}{2}} \cos^{-1} x \, dx = \frac{1}{6}(\pi + B - 3\sqrt{3})$, then $B$ equals to

(a) 8
(b) 6
(c) 4
(d) 0
(e) 2

4. The average value of the function

$$f(x) = \frac{x^2}{(x^3+3)^2}$$

on the interval $[-1, 1]$ is

(a) $\frac{1}{24}$
(b) $\frac{1}{18}$
(c) $\frac{1}{6}$
(d) $\frac{1}{4}$
(e) $\frac{1}{12}$

5. $\int \sqrt{\cos\theta}\,\sin^3\theta\,d\theta =$

(a) $-\frac{2}{7}\sqrt{\cos^7\theta}+\frac{2}{3}\sqrt{\cos^3\theta}+c$

(b) $\frac{1}{7}\sqrt{\cos^7\theta}+\frac{1}{3}\sqrt{\cos^3\theta}+c$

(c) $\frac{1}{7}\sqrt{\cos^7\theta}-\frac{1}{3}\sqrt{\cos^3\theta}+c$

(d) $\frac{2}{7}\sqrt{\cos^7\theta}-\frac{2}{3}\sqrt{\cos^3\theta}+c$

(e) $\frac{2}{7}\sqrt{\cos^7\theta}+\frac{2}{3}\sqrt{\cos^3\theta}+c$

6. $\int x\tan^2 x =$

(a) $x\tan x-\ln|\sec x|-\frac{x^2}{2}+c$

(b) $x\tan x+\ln|\sec x|-\frac{x^2}{2}+c$

(c) $x\tan x+\ln|\sec x|+\frac{x^2}{2}+c$

(d) $2x\tan x-2\ln|\sec x|-\frac{x^2}{2}+c$

(e) $x\tan x-\ln|\sec x|+\frac{x^2}{2}+c$

7. $\displaystyle\int_{\sqrt{2}}^{2} \frac{1}{x^3\sqrt{x^2-1}}\,dx =$

(a) $\dfrac{\pi}{24} - \dfrac{\sqrt{3}}{8} + \dfrac{1}{4}$

(b) $\dfrac{\pi}{24} - \dfrac{\sqrt{3}}{8} - \dfrac{1}{4}$

(c) $\dfrac{\pi}{12} - \dfrac{\sqrt{3}}{8} + \dfrac{1}{4}$

(d) $\dfrac{\pi}{12} + \dfrac{\sqrt{3}}{8} - \dfrac{1}{4}$

(e) $\dfrac{\pi}{24} + \dfrac{\sqrt{3}}{8} - \dfrac{1}{4}$

8. $\displaystyle\int \sin(8x)\,\cos(5x)\,dx =$

(a) $-\dfrac{1}{6}\cos(3x) - \dfrac{1}{26}\cos(13x) + c$

(b) $-\dfrac{1}{6}\cos(3x) - \dfrac{1}{2}\cos(13x) + c$

(c) $\dfrac{1}{6}\cos(3x) + \dfrac{1}{26}\cos(13x) + c$

(d) $-\dfrac{1}{3}\cos(3x) - \dfrac{1}{13}\cos(13x) + c$

(e) $\dfrac{1}{6}\cos(3x) + \dfrac{1}{2}\cos(13x) + c$

9. $\int \sqrt{1-4x^2}\,dx =$

(a) $\frac{1}{4}[\sin^{-1}(2x) + 2x\sqrt{1-4x^2}] + c$

(b) $\frac{1}{8}[\sin^{-1}(2x) + 2x\sqrt{1-4x^2}] + c$

(c) $\frac{1}{2}[\sin^{-1}(2x) + x\sqrt{1-4x^2}] + c$

(d) $\frac{1}{4}[\sin^{-1}(2x) - 2x\sqrt{1-4x^2}] + c$

(e) $\frac{1}{8}[\sin^{-1}(2x) - 2x\sqrt{1-4x^2}] + c$

10. $\int_0^1 \frac{dx}{(x^2+1)^2} =$

(a) $\frac{\pi}{8} - \frac{1}{2}$

(b) $\frac{\pi}{8} + \frac{1}{2}$

(c) $\frac{\pi}{8} + \frac{1}{4}$

(d) $\frac{\pi}{8} - \frac{1}{4}$

(e) $\frac{\pi}{4} + \frac{1}{2}$

11. $\displaystyle\int \frac{dx}{1+\sin x-\cos x} =$

(a) $\ln|\tan\left(\frac{x}{2}\right)|+x+c$

(b) $\ln\left|\frac{\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)}\right|+c$

(c) $\ln\left|1+\tan\left(\frac{x}{2}\right)\right|+c$

(d) $\ln\left|\frac{1+\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}\right|+c$

(e) $\ln\left|\tan\left(\frac{x}{2}\right)\right|+c$

12. $\displaystyle\int \frac{dx}{2\sqrt{x+3}+x} =$

(a) $\frac{3}{2}\ln(\sqrt{x+3}+3)+\frac{1}{2}\ln|\sqrt{x+3}-1|+c$

(b) $\frac{3}{2}\ln(\sqrt{x+3}+3)-\frac{1}{2}\ln|\sqrt{x+3}-1|+c$

(c) $\frac{3}{2}\ln(\sqrt{x+3}+3)+\frac{1}{3}\ln|\sqrt{x+3}-1|+c$

(d) $\frac{2}{3}\ln(\sqrt{x+3}+3)-\frac{1}{3}\ln|\sqrt{x+3}-1|+c$

(e) $\frac{2}{3}\ln(\sqrt{x+3}+3)+\frac{1}{3}\ln|\sqrt{x+3}-1|+c$

13. $\int_0^1 \frac{2}{2x^2+3x+1}\,dx =$

(a) $\ln\frac{2}{3}$

(b) $3\ln\frac{3}{2}$

(c) $2\ln\frac{3}{2}$

(d) $4\ln\frac{3}{2}$

(e) $\ln\frac{3}{2}$

14. $\int_3^4 \frac{x^3-2x^2-4}{x^3-2x^2}\,dx =$

(a) $\frac{7}{5}-\ln\frac{2}{3}$

(b) $\frac{7}{6}-\ln\frac{2}{3}$

(c) $\frac{7}{6}+\ln\frac{2}{3}$

(d) $\frac{7}{5}+\ln\frac{2}{3}$

(e) $\frac{7}{8}+\ln\frac{3}{2}$

15. The length of the curve $y = \ln(\cos x), 0 \leq x \leq \dfrac{\pi}{3}$ is equal to

(a) $\ln(2 - \sqrt{3})$
(b) $\ln(4 + \sqrt{3})$
(c) $\ln(4 - \sqrt{3})$
(d) $\ln(2 + \sqrt{3})$
(e) $\ln(2 - \sqrt{2})$

16. The length of the curve

$$x = \frac{y^4}{8} + \frac{1}{4y^2}, 1 \leq y \leq 2$$

is equal to

(a) $\dfrac{31}{16}$
(b) $\dfrac{33}{16}$
(c) $\dfrac{31}{8}$
(d) $\dfrac{5}{16}$
(e) $\dfrac{33}{8}$

17. The improper integral $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$ is

(a) converges to $\frac{2}{e}$

(b) converges to $\frac{4}{e}$

(c) divergent

(d) converges to $\frac{6}{e}$

(e) converges to $\frac{1}{e}$

18. The improper integral $\int_{-1}^0 \frac{e^{\frac{1}{x}}}{x^3}\,dx$ is

(a) convergent to $-\frac{4}{e}$

(b) convergent to $-\frac{3}{e}$

(c) convergent to $\frac{4}{e}$

(d) convergent to $\frac{3}{e}$

(e) convergent to $-\frac{2}{e}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

**CODE02** **CODE02**

# Math 102
# Major Exam II
# 213
# July 25, 2022
# Net Time Allowed: 120 Minutes

Name: _____

ID: _____ Sec: _____.

**Check that this exam has 18 questions.**

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1. The average value of the function

$$f(x) = \frac{x^2}{(x^3+3)^2}$$

on the interval $[-1, 1]$ is

(a) $\frac{1}{6}$

(b) $\frac{1}{4}$

(c) $\frac{1}{24}$

(d) $\frac{1}{18}$

(e) $\frac{1}{12}$

2. If $\int_0^{\frac{1}{2}} \cos^{-1} x \, dx = \frac{1}{6}(\pi + B - 3\sqrt{3})$, then $B$ equals to

(a) 2

(b) 6

(c) 0

(d) 8

(e) 4

3. $\int e^{\sqrt{x}}\,dx =$

(a) $\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$
(b) $2\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c$
(c) $2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$
(d) $\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c$
(e) $\sqrt{x}e^{\sqrt{x}} + 3e^{\sqrt{x}} + c$

4. The number $b$ such that the average value of $f(x) = 3x^2 + 8x$ on the interval $[-1, b]$ is equal to 7 is

(a) 1
(b) 4
(c) 5
(d) 3
(e) 2

5. $\displaystyle\int_{\sqrt{2}}^{2} \frac{1}{x^3\sqrt{x^2-1}}\,dx =$

(a) $\dfrac{\pi}{12}+\dfrac{\sqrt{3}}{8}-\dfrac{1}{4}$

(b) $\dfrac{\pi}{24}+\dfrac{\sqrt{3}}{8}-\dfrac{1}{4}$

(c) $\dfrac{\pi}{24}-\dfrac{\sqrt{3}}{8}+\dfrac{1}{4}$

(d) $\dfrac{\pi}{24}-\dfrac{\sqrt{3}}{8}-\dfrac{1}{4}$

(e) $\dfrac{\pi}{12}-\dfrac{\sqrt{3}}{8}+\dfrac{1}{4}$

6. $\displaystyle\int \sin(8x)\,\cos(5x)\,dx =$

(a) $-\dfrac{1}{6}\cos(3x)-\dfrac{1}{26}\cos(13x)+c$

(b) $\dfrac{1}{6}\cos(3x)+\dfrac{1}{2}\cos(13x)+c$

(c) $\dfrac{1}{6}\cos(3x)+\dfrac{1}{26}\cos(13x)+c$

(d) $-\dfrac{1}{3}\cos(3x)-\dfrac{1}{13}\cos(13x)+c$

(e) $-\dfrac{1}{6}\cos(3x)-\dfrac{1}{2}\cos(13x)+c$

7. $\int \sqrt{\cos\theta}\,\sin^3\theta\,d\theta =$

(a) $\frac{2}{7}\sqrt{\cos^7\theta} - \frac{2}{3}\sqrt{\cos^3\theta} + c$

(b) $\frac{1}{7}\sqrt{\cos^7\theta} + \frac{1}{3}\sqrt{\cos^3\theta} + c$

(c) $\frac{1}{7}\sqrt{\cos^7\theta} - \frac{1}{3}\sqrt{\cos^3\theta} + c$

(d) $\frac{2}{7}\sqrt{\cos^7\theta} + \frac{2}{3}\sqrt{\cos^3\theta} + c$

(e) $-\frac{2}{7}\sqrt{\cos^7\theta} + \frac{2}{3}\sqrt{\cos^3\theta} + c$

8. $\int \sqrt{1-4x^2}\,dx =$

(a) $\frac{1}{4}[\sin^{-1}(2x) + 2x\sqrt{1-4x^2}] + c$

(b) $\frac{1}{4}[\sin^{-1}(2x) - 2x\sqrt{1-4x^2}] + c$

(c) $\frac{1}{2}[\sin^{-1}(2x) + x\sqrt{1-4x^2}] + c$

(d) $\frac{1}{8}[\sin^{-1}(2x) - 2x\sqrt{1-4x^2}] + c$

(e) $\frac{1}{8}[\sin^{-1}(2x) + 2x\sqrt{1-4x^2}] + c$

9. $\int x \tan^2 x =$

(a) $x \tan x - \ln|\sec x| - \dfrac{x^2}{2} + c$

(b) $x \tan x + \ln|\sec x| - \dfrac{x^2}{2} + c$

(c) $2x \tan x - 2\ln|\sec x| - \dfrac{x^2}{2} + c$

(d) $x \tan x + \ln|\sec x| + \dfrac{x^2}{2} + c$

(e) $x \tan x - \ln|\sec x| + \dfrac{x^2}{2} + c$

10. $\int \dfrac{dx}{1 + \sin x - \cos x} =$

(a) $\ln\left|1 + \tan\left(\dfrac{x}{2}\right)\right| + c$

(b) $\ln\left|\tan\left(\dfrac{x}{2}\right)\right| + c$

(c) $\ln|\tan\left(\dfrac{x}{2}\right)| + x + c$

(d) $\ln\left|\dfrac{\tan\left(\frac{x}{2}\right)}{1 + \tan\left(\frac{x}{2}\right)}\right| + c$

(e) $\ln\left|\dfrac{1 + \tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}\right| + c$

11. $\int_0^1 \frac{2}{2x^2+3x+1}\,dx =$

(a) $\ln\frac{3}{2}$

(b) $3\ln\frac{3}{2}$

(c) $\ln\frac{2}{3}$

(d) $2\ln\frac{3}{2}$

(e) $4\ln\frac{3}{2}$

12. $\int \frac{dx}{2\sqrt{x+3}+x} =$

(a) $\frac{3}{2}\ln(\sqrt{x+3}+3)+\frac{1}{3}\ln|\sqrt{x+3}-1|+c$

(b) $\frac{2}{3}\ln(\sqrt{x+3}+3)-\frac{1}{3}\ln|\sqrt{x+3}-1|+c$

(c) $\frac{3}{2}\ln(\sqrt{x+3}+3)-\frac{1}{2}\ln|\sqrt{x+3}-1|+c$

(d) $\frac{3}{2}\ln(\sqrt{x+3}+3)+\frac{1}{2}\ln|\sqrt{x+3}-1|+c$

(e) $\frac{2}{3}\ln(\sqrt{x+3}+3)+\frac{1}{3}\ln|\sqrt{x+3}-1|+c$

13. $\int_0^1 \frac{dx}{(x^2+1)^2} =$

(a) $\frac{\pi}{8} - \frac{1}{4}$

(b) $\frac{\pi}{8} + \frac{1}{4}$

(c) $\frac{\pi}{8} + \frac{1}{2}$

(d) $\frac{\pi}{8} - \frac{1}{2}$

(e) $\frac{\pi}{4} + \frac{1}{2}$

14. $\int_3^4 \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2}\, dx =$

(a) $\frac{7}{8} + \ln\frac{3}{2}$

(b) $\frac{7}{6} - \ln\frac{2}{3}$

(c) $\frac{7}{6} + \ln\frac{2}{3}$

(d) $\frac{7}{5} + \ln\frac{2}{3}$

(e) $\frac{7}{5} - \ln\frac{2}{3}$

15. The improper integral $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$ is

(a) converges to $\frac{1}{e}$

(b) converges to $\frac{2}{e}$

(c) converges to $\frac{4}{e}$

(d) divergent

(e) converges to $\frac{6}{e}$

16. The length of the curve

$$x = \frac{y^4}{8} + \frac{1}{4y^2},\ 1 \le y \le 2$$

is equal to

(a) $\frac{31}{8}$

(b) $\frac{5}{16}$

(c) $\frac{31}{16}$

(d) $\frac{33}{16}$

(e) $\frac{33}{8}$

17. The length of the curve $y = \ln(\cos x), 0 \leq x \leq \dfrac{\pi}{3}$ is equal to

(a) $\ln(2 - \sqrt{3})$
(b) $\ln(4 - \sqrt{3})$
(c) $\ln(2 + \sqrt{3})$
(d) $\ln(4 + \sqrt{3})$
(e) $\ln(2 - \sqrt{2})$

18. The improper integral $\displaystyle\int_{-1}^{0} \frac{e^{\frac{1}{x}}}{x^3}\, dx$ is

(a) convergent to $\dfrac{3}{e}$
(b) convergent to $-\dfrac{4}{e}$
(c) convergent to $-\dfrac{2}{e}$
(d) convergent to $\dfrac{4}{e}$
(e) convergent to $-\dfrac{3}{e}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

**CODE03** **CODE03**

# Math 102
# Major Exam II
# 213
# July 25, 2022
# Net Time Allowed: 120 Minutes

Name: _____

ID: _____ Sec: _____.

**Check that this exam has 18 questions.**

## Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The number $b$ such that the average value of $f(x) = 3x^2 + 8x$ on the interval $[-1, b]$ is equal to 7 is

(a) 3
(b) 4
(c) 2
(d) 5
(e) 1

2. $\int e^{\sqrt{x}}\, dx =$

(a) $\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$
(b) $2\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c$
(c) $\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c$
(d) $\sqrt{x}e^{\sqrt{x}} + 3e^{\sqrt{x}} + c$
(e) $2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$

3. If $\int_0^{\frac{1}{2}} \cos^{-1} x \, dx = \frac{1}{6}(\pi + B - 3\sqrt{3})$, then $B$ equals to

(a) 6
(b) 8
(c) 2
(d) 4
(e) 0

4. The average value of the function

$$f(x) = \frac{x^2}{(x^3+3)^2}$$

on the interval $[-1, 1]$ is

(a) $\frac{1}{18}$
(b) $\frac{1}{12}$
(c) $\frac{1}{4}$
(d) $\frac{1}{6}$
(e) $\frac{1}{24}$

5. $\int x \tan^2 x =$

(a) $x \tan x - \ln|\sec x| + \dfrac{x^2}{2} + c$

(b) $x \tan x + \ln|\sec x| - \dfrac{x^2}{2} + c$

(c) $x \tan x + \ln|\sec x| + \dfrac{x^2}{2} + c$

(d) $x \tan x - \ln|\sec x| - \dfrac{x^2}{2} + c$

(e) $2x \tan x - 2\ln|\sec x| - \dfrac{x^2}{2} + c$

6. $\int \sin(8x)\, \cos(5x)\, dx =$

(a) $\dfrac{1}{6}\cos(3x) + \dfrac{1}{2}\cos(13x) + c$

(b) $\dfrac{1}{6}\cos(3x) + \dfrac{1}{26}\cos(13x) + c$

(c) $-\dfrac{1}{3}\cos(3x) - \dfrac{1}{13}\cos(13x) + c$

(d) $-\dfrac{1}{6}\cos(3x) - \dfrac{1}{2}\cos(13x) + c$

(e) $-\dfrac{1}{6}\cos(3x) - \dfrac{1}{26}\cos(13x) + c$

7. $\int \sqrt{\cos\theta}\,\sin^3\theta\,d\theta =$

(a) $\frac{2}{7}\sqrt{\cos^7\theta}+\frac{2}{3}\sqrt{\cos^3\theta}+c$

(b) $-\frac{2}{7}\sqrt{\cos^7\theta}+\frac{2}{3}\sqrt{\cos^3\theta}+c$

(c) $\frac{1}{7}\sqrt{\cos^7\theta}+\frac{1}{3}\sqrt{\cos^3\theta}+c$

(d) $\frac{1}{7}\sqrt{\cos^7\theta}-\frac{1}{3}\sqrt{\cos^3\theta}+c$

(e) $\frac{2}{7}\sqrt{\cos^7\theta}-\frac{2}{3}\sqrt{\cos^3\theta}+c$

8. $\int_{\sqrt{2}}^{2}\frac{1}{x^3\sqrt{x^2-1}}\,dx =$

(a) $\frac{\pi}{12}-\frac{\sqrt{3}}{8}+\frac{1}{4}$

(b) $\frac{\pi}{24}-\frac{\sqrt{3}}{8}+\frac{1}{4}$

(c) $\frac{\pi}{12}+\frac{\sqrt{3}}{8}-\frac{1}{4}$

(d) $\frac{\pi}{24}+\frac{\sqrt{3}}{8}-\frac{1}{4}$

(e) $\frac{\pi}{24}-\frac{\sqrt{3}}{8}-\frac{1}{4}$

9. $\int \sqrt{1-4x^2}\,dx =$

(a) $\frac{1}{8}[\sin^{-1}(2x) + 2x\sqrt{1-4x^2}] + c$

(b) $\frac{1}{4}[\sin^{-1}(2x) + 2x\sqrt{1-4x^2}] + c$

(c) $\frac{1}{8}[\sin^{-1}(2x) - 2x\sqrt{1-4x^2}] + c$

(d) $\frac{1}{4}[\sin^{-1}(2x) - 2x\sqrt{1-4x^2}] + c$

(e) $\frac{1}{2}[\sin^{-1}(2x) + x\sqrt{1-4x^2}] + c$

10. $\int \frac{dx}{2\sqrt{x+3}+x} =$

(a) $\frac{2}{3}\ln(\sqrt{x+3}+3) + \frac{1}{3}\ln|\sqrt{x+3}-1| + c$

(b) $\frac{2}{3}\ln(\sqrt{x+3}+3) - \frac{1}{3}\ln|\sqrt{x+3}-1| + c$

(c) $\frac{3}{2}\ln(\sqrt{x+3}+3) + \frac{1}{3}\ln|\sqrt{x+3}-1| + c$

(d) $\frac{3}{2}\ln(\sqrt{x+3}+3) + \frac{1}{2}\ln|\sqrt{x+3}-1| + c$

(e) $\frac{3}{2}\ln(\sqrt{x+3}+3) - \frac{1}{2}\ln|\sqrt{x+3}-1| + c$

11. $\int_0^1 \frac{dx}{(x^2+1)^2} =$

(a) $\frac{\pi}{8} - \frac{1}{2}$

(b) $\frac{\pi}{8} + \frac{1}{2}$

(c) $\frac{\pi}{8} - \frac{1}{4}$

(d) $\frac{\pi}{8} + \frac{1}{4}$

(e) $\frac{\pi}{4} + \frac{1}{2}$

12. $\int_3^4 \frac{x^3 - 2x^2 - 4}{x^3 - 2x^2}\, dx =$

(a) $\frac{7}{6} + \ln\frac{2}{3}$

(b) $\frac{7}{5} + \ln\frac{2}{3}$

(c) $\frac{7}{8} + \ln\frac{3}{2}$

(d) $\frac{7}{5} - \ln\frac{2}{3}$

(e) $\frac{7}{6} - \ln\frac{2}{3}$

13. $\displaystyle\int_0^1 \frac{2}{2x^2+3x+1}\,dx =$

(a) $\ln\dfrac{2}{3}$

(b) $2\ln\dfrac{3}{2}$

(c) $4\ln\dfrac{3}{2}$

(d) $3\ln\dfrac{3}{2}$

(e) $\ln\dfrac{3}{2}$

14. $\displaystyle\int \frac{dx}{1+\sin x-\cos x} =$

(a) $\ln\left|\dfrac{1+\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}\right|+c$

(b) $\ln\left|\tan\left(\dfrac{x}{2}\right)\right|+c$

(c) $\ln\left|\dfrac{\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)}\right|+c$

(d) $\ln|\tan\left(\dfrac{x}{2}\right)|+x+c$

(e) $\ln\left|1+\tan\left(\dfrac{x}{2}\right)\right|+c$

15. The improper integral $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$ is

(a) converges to $\frac{6}{e}$

(b) converges to $\frac{4}{e}$

(c) converges to $\frac{2}{e}$

(d) converges to $\frac{1}{e}$

(e) divergent

16. The length of the curve

$$x = \frac{y^4}{8} + \frac{1}{4y^2},\ 1 \le y \le 2$$

is equal to

(a) $\frac{5}{16}$

(b) $\frac{33}{8}$

(c) $\frac{31}{8}$

(d) $\frac{31}{16}$

(e) $\frac{33}{16}$

17. The length of the curve $y = \ln(\cos x), 0 \leq x \leq \dfrac{\pi}{3}$ is equal to

(a) $\ln(4 - \sqrt{3})$
(b) $\ln(2 - \sqrt{3})$
(c) $\ln(2 - \sqrt{2})$
(d) $\ln(2 + \sqrt{3})$
(e) $\ln(4 + \sqrt{3})$

18. The improper integral $\displaystyle\int_{-1}^{0} \frac{e^{\frac{1}{x}}}{x^3}\, dx$ is

(a) convergent to $\dfrac{4}{e}$
(b) convergent to $-\dfrac{4}{e}$
(c) convergent to $-\dfrac{3}{e}$
(d) convergent to $\dfrac{3}{e}$
(e) convergent to $-\dfrac{2}{e}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

**CODE04** **CODE04**

# Math 102
# Major Exam II
# 213
# July 25, 2022
# Net Time Allowed: 120 Minutes

Name: _____

ID: _____ Sec: _____.

**Check that this exam has 18 questions.**

## Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The average value of the function

$$f(x) = \frac{x^2}{(x^3+3)^2}$$

on the interval $[-1, 1]$ is

(a) $\frac{1}{12}$
(b) $\frac{1}{18}$
(c) $\frac{1}{24}$
(d) $\frac{1}{4}$
(e) $\frac{1}{6}$

2. $\int e^{\sqrt{x}}\,dx =$

(a) $\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$
(b) $2\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c$
(c) $2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + c$
(d) $\sqrt{x}e^{\sqrt{x}} - e^{\sqrt{x}} + c$
(e) $\sqrt{x}e^{\sqrt{x}} + 3e^{\sqrt{x}} + c$

3. If $\int_0^{\frac{1}{2}} \cos^{-1} x \, dx = \frac{1}{6}(\pi + B - 3\sqrt{3})$, then $B$ equals to

(a) 6
(b) 0
(c) 8
(d) 2
(e) 4

4. The number $b$ such that the average value of $f(x) = 3x^2 + 8x$ on the interval $[-1, b]$ is equal to 7 is

(a) 1
(b) 5
(c) 4
(d) 3
(e) 2

5. $\int \sqrt{\cos\theta}\,\sin^3\theta\,d\theta =$

(a) $\frac{1}{7}\sqrt{\cos^7\theta}+\frac{1}{3}\sqrt{\cos^3\theta}+c$

(b) $\frac{2}{7}\sqrt{\cos^7\theta}-\frac{2}{3}\sqrt{\cos^3\theta}+c$

(c) $-\frac{2}{7}\sqrt{\cos^7\theta}+\frac{2}{3}\sqrt{\cos^3\theta}+c$

(d) $\frac{2}{7}\sqrt{\cos^7\theta}+\frac{2}{3}\sqrt{\cos^3\theta}+c$

(e) $\frac{1}{7}\sqrt{\cos^7\theta}-\frac{1}{3}\sqrt{\cos^3\theta}+c$

6. $\int_{\sqrt{2}}^{2}\frac{1}{x^3\sqrt{x^2-1}}\,dx =$

(a) $\frac{\pi}{24}-\frac{\sqrt{3}}{8}+\frac{1}{4}$

(b) $\frac{\pi}{12}+\frac{\sqrt{3}}{8}-\frac{1}{4}$

(c) $\frac{\pi}{24}+\frac{\sqrt{3}}{8}-\frac{1}{4}$

(d) $\frac{\pi}{24}-\frac{\sqrt{3}}{8}-\frac{1}{4}$

(e) $\frac{\pi}{12}-\frac{\sqrt{3}}{8}+\frac{1}{4}$

7. $\int \sin(8x)\,\cos(5x)\,dx =$

(a) $-\frac{1}{6}\cos(3x) - \frac{1}{26}\cos(13x) + c$

(b) $\frac{1}{6}\cos(3x) + \frac{1}{26}\cos(13x) + c$

(c) $-\frac{1}{6}\cos(3x) - \frac{1}{2}\cos(13x) + c$

(d) $\frac{1}{6}\cos(3x) + \frac{1}{2}\cos(13x) + c$

(e) $-\frac{1}{3}\cos(3x) - \frac{1}{13}\cos(13x) + c$

8. $\int x\tan^2 x =$

(a) $x\tan x - \ln|\sec x| - \frac{x^2}{2} + c$

(b) $x\tan x - \ln|\sec x| + \frac{x^2}{2} + c$

(c) $2x\tan x - 2\ln|\sec x| - \frac{x^2}{2} + c$

(d) $x\tan x + \ln|\sec x| - \frac{x^2}{2} + c$

(e) $x\tan x + \ln|\sec x| + \frac{x^2}{2} + c$

9. $\int \sqrt{1-4x^2}\,dx =$

(a) $\frac{1}{4}[\sin^{-1}(2x) - 2x\sqrt{1-4x^2}] + c$

(b) $\frac{1}{4}[\sin^{-1}(2x) + 2x\sqrt{1-4x^2}] + c$

(c) $\frac{1}{8}[\sin^{-1}(2x) - 2x\sqrt{1-4x^2}] + c$

(d) $\frac{1}{2}[\sin^{-1}(2x) + x\sqrt{1-4x^2}] + c$

(e) $\frac{1}{8}[\sin^{-1}(2x) + 2x\sqrt{1-4x^2}] + c$

10. $\int_3^4 \frac{x^3-2x^2-4}{x^3-2x^2}\,dx =$

(a) $\frac{7}{5} + \ln\frac{2}{3}$

(b) $\frac{7}{6} + \ln\frac{2}{3}$

(c) $\frac{7}{5} - \ln\frac{2}{3}$

(d) $\frac{7}{8} + \ln\frac{3}{2}$

(e) $\frac{7}{6} - \ln\frac{2}{3}$

11. $\displaystyle\int \frac{dx}{2\sqrt{x+3}+x} =$

(a) $\displaystyle\frac{3}{2}\ln(\sqrt{x+3}+3) - \frac{1}{2}\ln|\sqrt{x+3}-1| + c$

(b) $\displaystyle\frac{2}{3}\ln(\sqrt{x+3}+3) + \frac{1}{3}\ln|\sqrt{x+3}-1| + c$

(c) $\displaystyle\frac{2}{3}\ln(\sqrt{x+3}+3) - \frac{1}{3}\ln|\sqrt{x+3}-1| + c$

(d) $\displaystyle\frac{3}{2}\ln(\sqrt{x+3}+3) + \frac{1}{3}\ln|\sqrt{x+3}-1| + c$

(e) $\displaystyle\frac{3}{2}\ln(\sqrt{x+3}+3) + \frac{1}{2}\ln|\sqrt{x+3}-1| + c$

12. $\displaystyle\int_0^1 \frac{2}{2x^2+3x+1}\,dx =$

(a) $\displaystyle\ln\frac{3}{2}$

(b) $\displaystyle 3\ln\frac{3}{2}$

(c) $\displaystyle 4\ln\frac{3}{2}$

(d) $\displaystyle\ln\frac{2}{3}$

(e) $\displaystyle 2\ln\frac{3}{2}$

13. $\displaystyle\int_0^1 \frac{dx}{(x^2+1)^2} =$

(a) $\dfrac{\pi}{8} - \dfrac{1}{4}$

(b) $\dfrac{\pi}{8} - \dfrac{1}{2}$

(c) $\dfrac{\pi}{4} + \dfrac{1}{2}$

(d) $\dfrac{\pi}{8} + \dfrac{1}{2}$

(e) $\dfrac{\pi}{8} + \dfrac{1}{4}$

14. $\displaystyle\int \frac{dx}{1+\sin x - \cos x} =$

(a) $\ln\left|\tan\left(\dfrac{x}{2}\right)\right| + c$

(b) $\ln\left|\dfrac{\tan\left(\frac{x}{2}\right)}{1+\tan\left(\frac{x}{2}\right)}\right| + c$

(c) $\ln\left|\dfrac{1+\tan\left(\frac{x}{2}\right)}{\tan\left(\frac{x}{2}\right)}\right| + c$

(d) $\ln\left|\tan\left(\dfrac{x}{2}\right)\right| + x + c$

(e) $\ln\left|1+\tan\left(\dfrac{x}{2}\right)\right| + c$

15. The length of the curve $y = \ln(\cos x), 0 \leq x \leq \dfrac{\pi}{3}$ is equal to

(a) $\ln(2+\sqrt{3})$
(b) $\ln(4-\sqrt{3})$
(c) $\ln(2-\sqrt{3})$
(d) $\ln(2-\sqrt{2})$
(e) $\ln(4+\sqrt{3})$

16. The length of the curve

$$x = \frac{y^4}{8} + \frac{1}{4y^2},\ 1 \leq y \leq 2$$

is equal to

(a) $\dfrac{31}{16}$
(b) $\dfrac{5}{16}$
(c) $\dfrac{33}{16}$
(d) $\dfrac{33}{8}$
(e) $\dfrac{31}{8}$

17. The improper integral $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$ is

(a) converges to $\frac{1}{e}$

(b) converges to $\frac{4}{e}$

(c) converges to $\frac{6}{e}$

(d) divergent

(e) converges to $\frac{2}{e}$

18. The improper integral $\int_{-1}^0 \frac{e^{\frac{1}{x}}}{x^3}\,dx$ is

(a) convergent to $\frac{4}{e}$

(b) convergent to $-\frac{2}{e}$

(c) convergent to $\frac{3}{e}$

(d) convergent to $-\frac{3}{e}$

(e) convergent to $-\frac{4}{e}$

| Q | MASTER | CODE01 | CODE02 | CODE03 | CODE04 |
|---|---|---|---|---|---|
| 1 | A | C | C | C | C |
| 2 | A | C | B | E | C |
| 3 | A | B | C | A | A |
| 4 | A | A | E | E | E |
| 5 | A | D | B | D | B |
| 6 | A | A | A | E | C |
| 7 | A | E | A | E | A |
| 8 | A | A | A | D | A |
| 9 | A | A | A | B | B |
| 10 | A | C | D | D | B |
| 11 | A | B | D | D | E |
| 12 | A | A | D | A | E |
| 13 | A | C | B | B | E |
| 14 | A | C | C | C | B |
| 15 | A | D | B | C | A |
| 16 | A | B | D | E | C |
| 17 | A | A | C | D | E |
| 18 | A | E | C | E | B |

# Answer Counts

| V | A | B | C | D | E |
|---|---|---|---|---|---|
| 1 | 6 | 3 | 5 | 2 | 2 |
| 2 | 4 | 4 | 5 | 4 | 1 |
| 3 | 2 | 2 | 3 | 5 | 6 |
| 4 | 4 | 5 | 4 | 0 | 5 |