King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 102
Final Exam
213
August 09, 2022

EXAM COVER

Number of versions: 4
Number of questions: 28

King Fahd University of Petroleum and Minerals
Department of Mathematics

Math 102

Final Exam

213

August 09, 2022

Net Time Allowed: 180 Minutes

MASTER VERSION

1. The average value of $f(x) = \sec^2\left(\frac{x}{2}\right)$ over $\left[0, \frac{\pi}{2}\right]$ is equal to

(a) $\frac{4}{\pi}$ ____ (correct)

(b) $\frac{6}{\pi}$

(c) $\frac{8}{\pi}$

(d) $\frac{10}{\pi}$

(e) $\frac{12}{\pi}$

2. $\int \tan^{-1}(4x) =$

(a) $x\tan^{-1}(4x) - \frac{1}{8}\ln\left(1 + 16x^2\right) + c$ ________________________ (correct)

(b) $2\tan^{-1}(4x) - \frac{1}{8}\ln\left(1 + 16x^2\right) + c$

(c) $3x\tan^{-1}(4x) + \frac{7}{8}\ln\left(1 + 16x^2\right) + c$

(d) $3\tan^{-1}(4x) + \frac{1}{8}\ln\left(1 + 16x^2\right) + c$

(e) $2x\tan^{-1}(4x) - \frac{7}{8}\ln\left(1 + 16x^2\right) + c$

3. If $y = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du$, then $\frac{dy}{dx}$ at $x = 0$ is equal to

(a) -1 ______________________________ (correct)
(b) 2
(c) -2
(d) -3
(e) 4

4. $\int \frac{x^3+4}{x^2+4}\,dx =$

(a) $\frac{1}{2}x^2 - 2\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + c$ ______________________________ (correct)
(b) $x^2 - 2\ln(x^2+4) - \tan^{-1}\left(\frac{x}{2}\right) + c$
(c) $4x^2 - 2\ln(x^2+4) - 2\tan^{-1}\left(\frac{x}{2}\right) + c$
(d) $3x^2 + 2\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + c$
(e) $\frac{1}{2}x^2 - 2\ln(x^2+4) - 4\tan^{-1}\left(\frac{x}{2}\right) + c$

5. $\displaystyle\int x\sqrt{1-x^4}\,dx =$

(a) $\frac{1}{4}\sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1-x^4} + c$ ________________________________ (correct)

(b) $\frac{1}{8}\sin^{-1}(x^2) + \frac{1}{8}x^2\sqrt{1-x^4} + c$

(c) $\frac{1}{4}\sin^{-1}(x^2) + \frac{1}{8}x^2\sqrt{1-x^4} + c$

(d) $\frac{1}{4}\sin^{-1}(x^2) - \frac{1}{8}x^2\sqrt{1-x^4} + c$

(e) $\frac{1}{8}\sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1-x^4} + c$

6. $\displaystyle\int \frac{\cos^5 x}{\sqrt{\sin x}}\,dx =$

(a) $2\sqrt{\sin x} - \frac{4}{5}\sin^{5/2} x + \frac{2}{9}\sin^{9/2} x + c$ ________________________________ (correct)

(b) $\sqrt{\sin x} + \frac{4}{5}\sin^{5/2} x + \sin^{9/2} x + c$

(c) $4\sqrt{\sin x} - \frac{1}{5}\sin^{4/5} x - \frac{2}{9}\sin^{9/2} x + c$

(d) $2\sqrt{\sin x} + \sin^{5/2} x + \frac{1}{9}\sin^{9/2} x + c$

(e) $\sqrt{\sin x} - \frac{1}{5}\sin^{4/5} x + \frac{2}{9}\sin^{9/2} x + c$

7. The improper integral $\int_{-2}^{3} \frac{1}{x^4}\, dx$ is

(a) divergent ______________________________ (correct)

(b) convergent to $\frac{1}{2}$

(c) convergent to $\frac{1}{4}$

(d) convergent to $-\frac{1}{2}$

(e) convergent to $-\frac{1}{4}$

8. The length of the curve $f(x) = 3 + \frac{1}{2}\cosh 2x$ over $[0, 1]$ is equal to

(a) $\frac{1}{2}\sinh 2$ ______________________________ (correct)

(b) $\sinh 2$

(c) $\frac{1}{3}\sinh 3$

(d) $\sinh 3$

(e) $\frac{1}{2}\sinh 4$

9. The area of the surface obtained by rotating the curve $y = \sqrt{1+e^x}$, $0 \le x \le 1$ about x-axis is equal to

(a) $\pi(e+1)$ ______________________________ (correct)

(b) $2\pi(e-1)$

(c) $3\pi(e+1)$

(d) $\pi(e-1)$

(e) $2\pi(e+4)$

10. Let $a_n = \dfrac{(-1)^n n^3}{n^3+2n^2+1}$, then $\lim_{n\to\infty} a_n$

(a) does not exist ______________________________ (correct)

(b) converges to 1

(c) converges to -1

(d) converges to 0

(e) converges to 2

11. The series

$$\sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$$

is

(a) convergent and its sum is $\dfrac{3}{2}$ ———————————— (correct)

(b) convergent and its sum is $\dfrac{5}{2}$

(c) convergent and its sum is $\dfrac{7}{2}$

(d) convergent and its sum is $\dfrac{9}{2}$

(e) divergent

12. The series $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$ is

(a) convergent with sum $= \dfrac{3e}{3-e}$ ———————————— (correct)

(b) convergent with sum $= \dfrac{e}{3-e}$

(c) convergent with sum $= \dfrac{e}{3-2e}$

(d) convergent with sum $= \dfrac{3e}{3+e}$

(e) divergent

13. The series $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ is

(a) convergent by integral test ______ (correct)

(b) divergent by integral test

(c) convergent to $\frac{1}{3e}$

(d) divergent by divergence test

(e) a series where the integral test is not applicable

14. The series

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3+4n+3}}$$

(a) converges by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$ ______ (correct)

(b) converges by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^{1/6}}$

(c) diverges by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^{1/6}}$

(d) diverges by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$

(e) diverges by limit comparison test

15. Using the Integral Test Remainder Estimate for the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, we find that the smallest number of terms need to be added such that $|\text{error}| < 10^{-2}$ is

(a) 101 ________________________________ (correct)
(b) 99
(c) 103
(d) 105
(e) 97

16. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is

(a) a convergent alternating series ________________________________ (correct)
(b) a convergent geometric series
(c) a divergent p-series
(d) a convergent p-series
(e) a divergent geometric series

17. The series $\displaystyle\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

(a) diverges by the ratio test ________________ (correct)

(b) converges by the ratio test

(c) converges by the root test

(d) converges by integral test

(e) converges conditionally

18. The series

$$\sum_{n=1}^{\infty} (\tan^{-1} n)^n$$

(a) diverges by the root test ________________ (correct)

(b) converges by the root test

(c) converges by the ratio test

(d) a series where the root test is inconclusive

(e) a series where the ratio test is inconclusive

19. The interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$$

is

(a) $[1,3]$ ——— (correct)

(b) $(1,3]$

(c) $[1,3)$

(d) $(1,3)$

(e) $(-\infty,\infty)$

20. The power series representation for the function $f(x) = \dfrac{x^4}{(1+x)^2}$ is

(a) $\sum_{n=1}^{\infty} (-1)^{n+1}\, n\, x^{n+3},\ |x| < 1$ ——— (correct)

(b) $\sum_{n=1}^{\infty} (-1)^{n}\, (n+1)\, x^{n+2},\ |x| < 1$

(c) $\sum_{n=1}^{\infty} (-1)^{n+1}\, (n+2)\, x^{n},\ |x| < 1$

(d) $\sum_{n=1}^{\infty} (-1)^{n}\, n\, x^{n+4},\ |x| < 1$

(e) $\sum_{n=2}^{\infty} (-1)^{n}\, n\, x^{n+2},\ |x| < 1$

21. The area of the region in the first quadrant enclosed by the curves $y = x$, $y = \dfrac{1}{x}$ and $y = \dfrac{1}{4}x$ is equal to

(a) $\ln 2$ ________ (correct)
(b) $\ln 3$
(c) $\ln 4$
(d) $\ln 5$
(e) $\ln 6$

22. Using the method of cylindrical shell, the volume of the solid generated by revolving about the line $x = 2$ the region bounded by $y = x - x^2$ and $y = 0$ is equal to

(a) $\dfrac{\pi}{2}$ ________ (correct)
(b) $\dfrac{\pi}{4}$
(c) $\dfrac{\pi}{3}$
(d) $\dfrac{\pi}{6}$
(e) $\dfrac{\pi}{5}$

23. $\displaystyle\int_0^1 \frac{dx}{1+x^6} =$

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{6n+1}$ ________________ (correct)

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{6n+2}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{6n+2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{6n+2}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{6n+2}$

24. Estimating the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and right points is equal to

(a) 8 ________________ (correct)

(b) 10

(c) 12

(d) 14

(e) 16

25. $-\ln 2+\frac{(\ln 2)^2}{2!}-\frac{(\ln 2)^3}{3!}+\ldots=$

(a) $-\dfrac{1}{2}$ ———— (correct)

(b) $\dfrac{1}{2}$

(c) 2

(d) -2

(e) $\dfrac{1}{4}$

26. The Taylor series for $f(x)=e^{2x}$ centered at $a=3$ is given by

(a) $\displaystyle\sum_{n=0}^{\infty}\frac{2^n e^6}{n!}(x-3)^n,\ R=\infty$ ———— (correct)

(b) $\displaystyle\sum_{n=0}^{\infty}\frac{2^n}{n!}(x-3)^n,\ R=\infty$

(c) $\displaystyle\sum_{n=0}^{\infty}\frac{e^6}{n!}(x-3)^n,\ R=\infty$

(d) $\displaystyle\sum_{n=0}^{\infty}2^n e^6(x-3)^n,\ R=\infty$

(e) $\displaystyle\sum_{n=0}^{\infty}2^n(x-3)^n,\ R=\infty$

27. The first three nonzero terms of the Maclaurin series for the function $f(x) = (1-x)^{1/4}$ are

(a) $1 - \frac{1}{4}x - \frac{3}{32}x^2$ ________________ (correct)

(b) $1 + \frac{1}{4}x - \frac{3}{32}x^2$

(c) $1 - \frac{1}{4}x + \frac{3}{32}x^2$

(d) $1 - \frac{1}{4}x - \frac{3}{16}x^2$

(e) $1 + \frac{1}{4}x - \frac{3}{16}x^2$

28. The Maclaurin series for $f(x) = x\,\cos\left(\frac{x^2}{2}\right)$ is

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(2n)!}\, x^{4n+1},\ R = \infty$ ________________ (correct)

(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\, x^{4n+1},\ R = \infty$

(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}}\, x^{4n+1},\ R = \infty$

(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n}(2n)!}\, x^{4n+1},\ R = \infty$

(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n}(2n)!}\, x^{4n+2},\ R = \infty$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE01 **CODE01**

Math 102
Final Exam
213
August 09, 2022
Net Time Allowed: 180 Minutes

Name	

ID		**Sec**	

Check that this exam has 28 questions.

Important Instructions:

1. All types of calculators, smart watches or mobile phones are NOT allowed during the examination.
2. Use HB 2.5 pencils only.
3. Use a good eraser. DO NOT use the erasers attached to the pencil.
4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The improper integral $\displaystyle\int_{-2}^{3} \frac{1}{x^4}\,dx$ is

(a) divergent

(b) convergent to $\frac{1}{4}$

(c) convergent to $-\frac{1}{2}$

(d) convergent to $\frac{1}{2}$

(e) convergent to $-\frac{1}{4}$

2. The average value of $f(x) = \sec^2\left(\frac{x}{2}\right)$ over $\left[0, \frac{\pi}{2}\right]$ is equal to

(a) $\frac{10}{\pi}$

(b) $\frac{12}{\pi}$

(c) $\frac{6}{\pi}$

(d) $\frac{4}{\pi}$

(e) $\frac{8}{\pi}$

3. $\displaystyle\int \frac{\cos^5 x}{\sqrt{\sin x}}\,dx =$

(a) $4\sqrt{\sin x} - \frac{1}{5}\sin^{4/5} x - \frac{2}{9}\sin^{9/2} x + c$

(b) $2\sqrt{\sin x} - \frac{4}{5}\sin^{5/2} x + \frac{2}{9}\sin^{9/2} x + c$

(c) $\sqrt{\sin x} - \frac{1}{5}\sin^{4/5} x + \frac{2}{9}\sin^{9/2} x + c$

(d) $\sqrt{\sin x} + \frac{4}{5}\sin^{5/2} x + \sin^{9/2} x + c$

(e) $2\sqrt{\sin x} + \sin^{5/2} x + \frac{1}{9}\sin^{9/2} x + c$

4. $\displaystyle\int \frac{x^3+4}{x^2+4}\,dx =$

(a) $4x^2 - 2\ln(x^2+4) - 2\tan^{-1}\left(\frac{x}{2}\right) + c$

(b) $\frac{1}{2}x^2 - 2\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + c$

(c) $\frac{1}{2}x^2 - 2\ln(x^2+4) - 4\tan^{-1}\left(\frac{x}{2}\right) + c$

(d) $x^2 - 2\ln(x^2+4) - \tan^{-1}\left(\frac{x}{2}\right) + c$

(e) $3x^2 + 2\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + c$

5. $\int x\sqrt{1-x^4}\,dx =$

(a) $\frac{1}{4}\sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1-x^4} + c$

(b) $\frac{1}{8}\sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1-x^4} + c$

(c) $\frac{1}{4}\sin^{-1}(x^2) - \frac{1}{8}x^2\sqrt{1-x^4} + c$

(d) $\frac{1}{8}\sin^{-1}(x^2) + \frac{1}{8}x^2\sqrt{1-x^4} + c$

(e) $\frac{1}{4}\sin^{-1}(x^2) + \frac{1}{8}x^2\sqrt{1-x^4} + c$

6. If $y = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du$, then $\frac{dy}{dx}$ at $x = 0$ is equal to

(a) -2

(b) -3

(c) -1

(d) 2

(e) 4

7. $\int \tan^{-1}(4x) =$

(a) $2\tan^{-1}(4x) - \dfrac{1}{8}\ln\left(1+16x^2\right) + c$

(b) $3x\tan^{-1}(4x) + \dfrac{7}{8}\ln\left(1+16x^2\right) + c$

(c) $3\tan^{-1}(4x) + \dfrac{1}{8}\ln\left(1+16x^2\right) + c$

(d) $2x\tan^{-1}(4x) - \dfrac{7}{8}\ln\left(1+16x^2\right) + c$

(e) $x\tan^{-1}(4x) - \dfrac{1}{8}\ln\left(1+16x^2\right) + c$

8. The series

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3+4n+3}}$$

(a) converges by comparing with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/6}}$

(b) diverges by comparing with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$

(c) diverges by comparing with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/6}}$

(d) diverges by limit comparison test

(e) converges by comparing with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$

9. The series

$$\sum_{n=2}^{\infty} \frac{2}{n^2 - 1}$$

is

(a) convergent and its sum is $\frac{5}{2}$

(b) convergent and its sum is $\frac{9}{2}$

(c) convergent and its sum is $\frac{3}{2}$

(d) convergent and its sum is $\frac{7}{2}$

(e) divergent

10. Let $a_n = \dfrac{(-1)^n n^3}{n^3 + 2n^2 + 1}$, then $\lim\limits_{n\to\infty} a_n$

(a) converges to 1

(b) converges to 0

(c) converges to -1

(d) converges to 2

(e) does not exist

11. The series $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ is

(a) convergent to $\frac{1}{3e}$

(b) convergent by integral test

(c) a series where the integral test is not applicable

(d) divergent by integral test

(e) divergent by divergence test

12. The area of the surface obtained by rotating the curve $y = \sqrt{1+e^x}$, $0 \le x \le 1$ about x-axis is equal to

(a) $\pi(e+1)$

(b) $3\pi(e+1)$

(c) $2\pi(e-1)$

(d) $\pi(e-1)$

(e) $2\pi(e+4)$

13. The series $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$ is

(a) convergent with sum $= \frac{3e}{3+e}$

(b) divergent

(c) convergent with sum $= \frac{e}{3-e}$

(d) convergent with sum $= \frac{3e}{3-e}$

(e) convergent with sum $= \frac{e}{3-2e}$

14. The length of the curve $f(x) = 3 + \frac{1}{2}\cosh 2x$ over $[0, 1]$ is equal to

(a) $\sinh 3$

(b) $\sinh 2$

(c) $\frac{1}{3}\sinh 3$

(d) $\frac{1}{2}\sinh 4$

(e) $\frac{1}{2}\sinh 2$

15. The power series representation for the function $f(x) = \dfrac{x^4}{(1+x)^2}$ is

(a) $\sum_{n=2}^{\infty}(-1)^n \, n \, x^{n+2}, \; |x| < 1$

(b) $\sum_{n=1}^{\infty}(-1)^{n+1} \, n \, x^{n+3}, \; |x| < 1$

(c) $\sum_{n=1}^{\infty}(-1)^n \, n \, x^{n+4}, \; |x| < 1$

(d) $\sum_{n=1}^{\infty}(-1)^n \, (n+1) \, x^{n+2}, \; |x| < 1$

(e) $\sum_{n=1}^{\infty}(-1)^{n+1} \, (n+2) \, x^{n}, \; |x| < 1$

16. Using the Integral Test Remainder Estimate for the series $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$, we find that the smallest number of terms need to be added such that $|\text{error}| < 10^{-2}$ is

(a) 97

(b) 99

(c) 103

(d) 101

(e) 105

17. The series

$$\sum_{n=1}^{\infty}(\tan^{-1} n)^n$$

(a) converges by the ratio test

(b) diverges by the root test

(c) a series where the root test is inconclusive

(d) converges by the root test

(e) a series where the ratio test is inconclusive

18. The series $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

(a) converges conditionally

(b) converges by integral test

(c) converges by the root test

(d) converges by the ratio test

(e) diverges by the ratio test

19. The area of the region in the first quadrant enclosed by the curves $y = x$, $y = \dfrac{1}{x}$ and $y = \dfrac{1}{4}x$ is equal to

(a) $\ln 5$
(b) $\ln 2$
(c) $\ln 6$
(d) $\ln 4$
(e) $\ln 3$

20. The interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$$

is

(a) $(-\infty, \infty)$
(b) $(1, 3)$
(c) $[1, 3]$
(d) $[1, 3)$
(e) $(1, 3]$

21. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is

(a) a convergent geometric series
(b) a divergent p-series
(c) a convergent p-series
(d) a convergent alternating series
(e) a divergent geometric series

22. Using the method of cylindrical shell, the volume of the solid generated by revolving about the line $x = 2$ the region bounded by $y = x - x^2$ and $y = 0$ is equal to

(a) $\frac{\pi}{6}$
(b) $\frac{\pi}{4}$
(c) $\frac{\pi}{3}$
(d) $\frac{\pi}{2}$
(e) $\frac{\pi}{5}$

23. $\displaystyle\int_0^1 \frac{dx}{1+x^6} =$

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{6n+2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{6n+2}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{6n+2}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{6n+2}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{6n+1}$

24. Estimating the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and right points is equal to

(a) 12

(b) 8

(c) 10

(d) 14

(e) 16

25. The Maclaurin series for $f(x) = x\cos\left(\dfrac{x^2}{2}\right)$ is

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(2n)!}\, x^{4n+1},\ R = \infty$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}}\, x^{4n+1},\ R = \infty$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n}(2n)!}\, x^{4n+2},\ R = \infty$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\, x^{4n+1},\ R = \infty$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n}(2n)!}\, x^{4n+1},\ R = \infty$

26. The first three nonzero terms of the Maclaurin series for the function $f(x) = (1-x)^{1/4}$ are

(a) $1 - \frac{1}{4}x - \frac{3}{32}x^2$

(b) $1 + \frac{1}{4}x - \frac{3}{32}x^2$

(c) $1 + \frac{1}{4}x - \frac{3}{16}x^2$

(d) $1 - \frac{1}{4}x - \frac{3}{16}x^2$

(e) $1 - \frac{1}{4}x + \frac{3}{32}x^2$

27. $-\ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \ldots =$

(a) -2

(b) $\dfrac{1}{2}$

(c) 2

(d) $\dfrac{1}{4}$

(e) $-\dfrac{1}{2}$

28. The Taylor series for $f(x) = e^{2x}$ centered at $a = 3$ is given by

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n e^6}{n!}(x-3)^n,\ R = \infty$

(b) $\displaystyle\sum_{n=0}^{\infty} 2^n e^6 (x-3)^n,\ R = \infty$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{e^6}{n!}(x-3)^n,\ R = \infty$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}(x-3)^n,\ R = \infty$

(e) $\displaystyle\sum_{n=0}^{\infty} 2^n (x-3)^n,\ R = \infty$

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Department of Mathematics

CODE02 **CODE02**

Math 102
Final Exam
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Name	
ID	**Sec**

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4. Write your name, ID number and Section number on the examination paper and in the upper left corner of the answer sheet.
5. When bubbling your ID number and Section number, be sure that the bubbles match with the numbers that you write.
6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. $\int \tan^{-1}(4x) =$

(a) $2x\tan^{-1}(4x) - \frac{7}{8}\ln\left(1+16x^2\right) + c$

(b) $x\tan^{-1}(4x) - \frac{1}{8}\ln\left(1+16x^2\right) + c$

(c) $2\tan^{-1}(4x) - \frac{1}{8}\ln\left(1+16x^2\right) + c$

(d) $3x\tan^{-1}(4x) + \frac{7}{8}\ln\left(1+16x^2\right) + c$

(e) $3\tan^{-1}(4x) + \frac{1}{8}\ln\left(1+16x^2\right) + c$

2. $\int \frac{\cos^5 x}{\sqrt{\sin x}}\, dx =$

(a) $\sqrt{\sin x} - \frac{1}{5}\sin^{4/5} x + \frac{2}{9}\sin^{9/2} x + c$

(b) $2\sqrt{\sin x} + \sin^{5/2} x + \frac{1}{9}\sin^{9/2} x + c$

(c) $2\sqrt{\sin x} - \frac{4}{5}\sin^{5/2} x + \frac{2}{9}\sin^{9/2} x + c$

(d) $4\sqrt{\sin x} - \frac{1}{5}\sin^{4/5} x - \frac{2}{9}\sin^{9/2} x + c$

(e) $\sqrt{\sin x} + \frac{4}{5}\sin^{5/2} x + \sin^{9/2} x + c$

3. $\displaystyle\int \frac{x^3+4}{x^2+4}\,dx =$

(a) $\frac{1}{2}x^2 - 2\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + c$

(b) $x^2 - 2\ln(x^2+4) - \tan^{-1}\left(\frac{x}{2}\right) + c$

(c) $3x^2 + 2\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + c$

(d) $\frac{1}{2}x^2 - 2\ln(x^2+4) - 4\tan^{-1}\left(\frac{x}{2}\right) + c$

(e) $4x^2 - 2\ln(x^2+4) - 2\tan^{-1}\left(\frac{x}{2}\right) + c$

4. The average value of $f(x) = \sec^2\left(\frac{x}{2}\right)$ over $\left[0, \frac{\pi}{2}\right]$ is equal to

(a) $\frac{8}{\pi}$

(b) $\frac{6}{\pi}$

(c) $\frac{12}{\pi}$

(d) $\frac{4}{\pi}$

(e) $\frac{10}{\pi}$

5. The improper integral $\displaystyle\int_{-2}^{3} \frac{1}{x^4}\,dx$ is

(a) convergent to $\frac{1}{2}$
(b) divergent
(c) convergent to $\frac{1}{4}$
(d) convergent to $-\frac{1}{2}$
(e) convergent to $-\frac{1}{4}$

6. If $y = \displaystyle\int_{2x}^{3x} \frac{u^2-1}{u^2+1}\,du$, then $\frac{dy}{dx}$ at $x = 0$ is equal to

(a) -2
(b) -3
(c) 2
(d) -1
(e) 4

7. $\displaystyle\int x\sqrt{1-x^4}\,dx =$

(a) $\dfrac{1}{4}\sin^{-1}(x^2) + \dfrac{1}{4}x^2\sqrt{1-x^4} + c$
(b) $\dfrac{1}{8}\sin^{-1}(x^2) + \dfrac{1}{4}x^2\sqrt{1-x^4} + c$
(c) $\dfrac{1}{8}\sin^{-1}(x^2) + \dfrac{1}{8}x^2\sqrt{1-x^4} + c$
(d) $\dfrac{1}{4}\sin^{-1}(x^2) - \dfrac{1}{8}x^2\sqrt{1-x^4} + c$
(e) $\dfrac{1}{4}\sin^{-1}(x^2) + \dfrac{1}{8}x^2\sqrt{1-x^4} + c$

8. The area of the surface obtained by rotating the curve $y = \sqrt{1+e^x}$, $0 \le x \le 1$ about x-axis is equal to

(a) $2\pi(e+4)$
(b) $2\pi(e-1)$
(c) $\pi(e-1)$
(d) $\pi(e+1)$
(e) $3\pi(e+1)$

9. The series

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3+4n+3}}$$

(a) diverges by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^{1/6}}$

(b) converges by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^{1/6}}$

(c) diverges by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$

(d) converges by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$

(e) diverges by limit comparison test

10. The series

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1}$$

is

(a) convergent and its sum is $\frac{5}{2}$

(b) convergent and its sum is $\frac{9}{2}$

(c) convergent and its sum is $\frac{3}{2}$

(d) divergent

(e) convergent and its sum is $\frac{7}{2}$

11. Let $a_n = \dfrac{(-1)^n n^3}{n^3 + 2n^2 + 1}$, then $\lim\limits_{n\to\infty} a_n$

(a) converges to 2

(b) does not exist

(c) converges to -1

(d) converges to 0

(e) converges to 1

12. The series $\sum\limits_{n=1}^{\infty} n^2 e^{-n^3}$ is

(a) divergent by integral test

(b) a series where the integral test is not applicable

(c) convergent by integral test

(d) convergent to $\dfrac{1}{3e}$

(e) divergent by divergence test

13. The series $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$ is

(a) convergent with sum $= \frac{3e}{3-e}$

(b) convergent with sum $= \frac{e}{3-2e}$

(c) divergent

(d) convergent with sum $= \frac{e}{3-e}$

(e) convergent with sum $= \frac{3e}{3+e}$

14. The length of the curve $f(x) = 3 + \frac{1}{2}\cosh 2x$ over $[0, 1]$ is equal to

(a) $\frac{1}{3}\sinh 3$

(b) $\sinh 3$

(c) $\frac{1}{2}\sinh 2$

(d) $\sinh 2$

(e) $\frac{1}{2}\sinh 4$

15. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is

(a) a convergent p-series

(b) a convergent geometric series

(c) a divergent geometric series

(d) a convergent alternating series

(e) a divergent p-series

16. The series $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

(a) converges conditionally

(b) diverges by the ratio test

(c) converges by the ratio test

(d) converges by the root test

(e) converges by integral test

17. Using the Integral Test Remainder Estimate for the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, we find that the smallest number of terms need to be added such that $|\text{error}| < 10^{-2}$ is

(a) 105
(b) 97
(c) 99
(d) 103
(e) 101

18. The series

$$\sum_{n=1}^{\infty} (\tan^{-1} n)^n$$

(a) a series where the ratio test is inconclusive
(b) a series where the root test is inconclusive
(c) converges by the root test
(d) diverges by the root test
(e) converges by the ratio test

19. The power series representation for the function $f(x) = \dfrac{x^4}{(1+x)^2}$ is

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} (n+2)\, x^n, \; |x| < 1$

(b) $\displaystyle\sum_{n=2}^{\infty} (-1)^{n}\, n\, x^{n+2}, \; |x| < 1$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n} (n+1)\, x^{n+2}, \; |x| < 1$

(d) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1}\, n\, x^{n+3}, \; |x| < 1$

(e) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n}\, n\, x^{n+4}, \; |x| < 1$

20. The area of the region in the first quadrant enclosed by the curves $y = x$, $y = \dfrac{1}{x}$ and $y = \dfrac{1}{4}x$ is equal to

(a) ln 6

(b) ln 4

(c) ln 2

(d) ln 3

(e) ln 5

21. The interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$$

is

(a) $(1, 3)$
(b) $[1, 3)$
(c) $[1, 3]$
(d) $(1, 3]$
(e) $(-\infty, \infty)$

22. The Taylor series for $f(x) = e^{2x}$ centered at $a = 3$ is given by

(a) $\sum_{n=0}^{\infty} \frac{2^n}{n!}(x-3)^n,\ R = \infty$
(b) $\sum_{n=0}^{\infty} 2^n e^6 (x-3)^n,\ R = \infty$
(c) $\sum_{n=0}^{\infty} \frac{e^6}{n!}(x-3)^n,\ R = \infty$
(d) $\sum_{n=0}^{\infty} 2^n (x-3)^n,\ R = \infty$
(e) $\sum_{n=0}^{\infty} \frac{2^n e^6}{n!}(x-3)^n,\ R = \infty$

23. The Maclaurin series for $f(x) = x \cos\left(\dfrac{x^2}{2}\right)$ is

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(2n)!}\, x^{4n+1},\ R = \infty$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!}\, x^{4n+1},\ R = \infty$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}}\, x^{4n+1},\ R = \infty$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n}(2n)!}\, x^{4n+2},\ R = \infty$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n}(2n)!}\, x^{4n+1},\ R = \infty$

24. Estimating the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and right points is equal to

(a) 8

(b) 16

(c) 14

(d) 10

(e) 12

25. The first three nonzero terms of the Maclaurin series for the function $f(x) = (1-x)^{1/4}$ are

(a) $1 - \frac{1}{4}x - \frac{3}{32}x^2$

(b) $1 - \frac{1}{4}x - \frac{3}{16}x^2$

(c) $1 - \frac{1}{4}x + \frac{3}{32}x^2$

(d) $1 + \frac{1}{4}x - \frac{3}{32}x^2$

(e) $1 + \frac{1}{4}x - \frac{3}{16}x^2$

26. $-\ln\, 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \ldots =$

(a) 2

(b) -2

(c) $-\frac{1}{2}$

(d) $\frac{1}{4}$

(e) $\frac{1}{2}$

27. $\displaystyle\int_0^1 \frac{dx}{1+x^6} =$

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{6n+2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{6n+2}$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{6n+2}$

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{6n+2}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{6n+1}$

28. Using the method of cylindrical shell, the volume of the solid generated by revolving about the line $x = 2$ the region bounded by $y = x - x^2$ and $y = 0$ is equal to

(a) $\frac{\pi}{6}$

(b) $\frac{\pi}{3}$

(c) $\frac{\pi}{5}$

(d) $\frac{\pi}{2}$

(e) $\frac{\pi}{4}$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE03 **CODE03**

Math 102
Final Exam
213
August 09, 2022
Net Time Allowed: 180 Minutes

Name	

ID		**Sec**	

Check that this exam has 28 questions.

Important Instructions:

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1. The average value of $f(x) = \sec^2\left(\frac{x}{2}\right)$ over $\left[0, \frac{\pi}{2}\right]$ is equal to

(a) $\frac{12}{\pi}$
(b) $\frac{6}{\pi}$
(c) $\frac{8}{\pi}$
(d) $\frac{4}{\pi}$
(e) $\frac{10}{\pi}$

2. If $y = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\, du$, then $\frac{dy}{dx}$ at $x = 0$ is equal to

(a) 4
(b) -3
(c) -2
(d) 2
(e) -1

3. The improper integral $\displaystyle\int_{-2}^{3} \frac{1}{x^4}\, dx$ is

(a) divergent

(b) convergent to $\dfrac{1}{2}$

(c) convergent to $\dfrac{1}{4}$

(d) convergent to $-\dfrac{1}{4}$

(e) convergent to $-\dfrac{1}{2}$

4. $\displaystyle\int x\sqrt{1-x^4}\, dx =$

(a) $\dfrac{1}{8}\sin^{-1}(x^2) + \dfrac{1}{4}x^2\sqrt{1-x^4} + c$

(b) $\dfrac{1}{4}\sin^{-1}(x^2) + \dfrac{1}{4}x^2\sqrt{1-x^4} + c$

(c) $\dfrac{1}{8}\sin^{-1}(x^2) + \dfrac{1}{8}x^2\sqrt{1-x^4} + c$

(d) $\dfrac{1}{4}\sin^{-1}(x^2) - \dfrac{1}{8}x^2\sqrt{1-x^4} + c$

(e) $\dfrac{1}{4}\sin^{-1}(x^2) + \dfrac{1}{8}x^2\sqrt{1-x^4} + c$

5. $\displaystyle\int \frac{x^3+4}{x^2+4}\,dx =$

(a) $\frac{1}{2}x^2 - 2\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + c$

(b) $x^2 - 2\ln(x^2+4) - \tan^{-1}\left(\frac{x}{2}\right) + c$

(c) $4x^2 - 2\ln(x^2+4) - 2\tan^{-1}\left(\frac{x}{2}\right) + c$

(d) $\frac{1}{2}x^2 - 2\ln(x^2+4) - 4\tan^{-1}\left(\frac{x}{2}\right) + c$

(e) $3x^2 + 2\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + c$

6. $\displaystyle\int \frac{\cos^5 x}{\sqrt{\sin x}}\,dx =$

(a) $2\sqrt{\sin x} + \sin^{5/2} x + \frac{1}{9}\sin^{9/2} x + c$

(b) $\sqrt{\sin x} + \frac{4}{5}\sin^{5/2} x + \sin^{9/2} x + c$

(c) $\sqrt{\sin x} - \frac{1}{5}\sin^{4/5} x + \frac{2}{9}\sin^{9/2} x + c$

(d) $4\sqrt{\sin x} - \frac{1}{5}\sin^{4/5} x - \frac{2}{9}\sin^{9/2} x + c$

(e) $2\sqrt{\sin x} - \frac{4}{5}\sin^{5/2} x + \frac{2}{9}\sin^{9/2} x + c$

7. $\int \tan^{-1}(4x) =$

(a) $2\tan^{-1}(4x) - \frac{1}{8}\ln(1+16x^2) + c$

(b) $x\tan^{-1}(4x) - \frac{1}{8}\ln(1+16x^2) + c$

(c) $2x\tan^{-1}(4x) - \frac{7}{8}\ln(1+16x^2) + c$

(d) $3x\tan^{-1}(4x) + \frac{7}{8}\ln(1+16x^2) + c$

(e) $3\tan^{-1}(4x) + \frac{1}{8}\ln(1+16x^2) + c$

8. The area of the surface obtained by rotating the curve $y = \sqrt{1+e^x}$, $0 \le x \le 1$ about x-axis is equal to

(a) $2\pi(e-1)$

(b) $2\pi(e+4)$

(c) $\pi(e+1)$

(d) $\pi(e-1)$

(e) $3\pi(e+1)$

9. The series

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3+4n+3}}$$

(a) converges by comparing with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$

(b) diverges by comparing with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$

(c) diverges by comparing with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/6}}$

(d) diverges by limit comparison test

(e) converges by comparing with $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{1/6}}$

10. The series $\displaystyle\sum_{n=1}^{\infty} n^2 e^{-n^3}$ is

(a) convergent to $\dfrac{1}{3e}$

(b) a series where the integral test is not applicable

(c) convergent by integral test

(d) divergent by integral test

(e) divergent by divergence test

11. The series $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$ is

(a) convergent with sum $= \frac{3e}{3-e}$

(b) convergent with sum $= \frac{3e}{3+e}$

(c) divergent

(d) convergent with sum $= \frac{e}{3-2e}$

(e) convergent with sum $= \frac{e}{3-e}$

12. The length of the curve $f(x) = 3 + \frac{1}{2}\cosh 2x$ over $[0, 1]$ is equal to

(a) $\frac{1}{2}\sinh 4$

(b) $\frac{1}{2}\sinh 2$

(c) $\sinh 2$

(d) $\frac{1}{3}\sinh 3$

(e) $\sinh 3$

13. The series

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1}$$

is

(a) convergent and its sum is $\frac{3}{2}$

(b) divergent

(c) convergent and its sum is $\frac{7}{2}$

(d) convergent and its sum is $\frac{9}{2}$

(e) convergent and its sum is $\frac{5}{2}$

14. Let $a_n = \dfrac{(-1)^n n^3}{n^3+2n^2+1}$, then $\lim\limits_{n\to\infty} a_n$

(a) converges to -1

(b) converges to 2

(c) converges to 0

(d) converges to 1

(e) does not exist

15. The area of the region in the first quadrant enclosed by the curves $y = x$, $y = \frac{1}{x}$ and $y = \frac{1}{4}x$ is equal to

(a) $\ln 2$
(b) $\ln 3$
(c) $\ln 6$
(d) $\ln 5$
(e) $\ln 4$

16. The series $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

(a) diverges by the ratio test
(b) converges conditionally
(c) converges by the root test
(d) converges by integral test
(e) converges by the ratio test

17. The series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$ is

   (a) a convergent alternating series
   (b) a divergent geometric series
   (c) a divergent p-series
   (d) a convergent geometric series
   (e) a convergent p-series

18. The interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$$

   is

   (a) $(-\infty, \infty)$
   (b) $[1, 3)$
   (c) $(1, 3]$
   (d) $(1, 3)$
   (e) $[1, 3]$

19. The series

$$\sum_{n=1}^{\infty}(\tan^{-1} n)^n$$

(a) converges by the root test
(b) diverges by the root test
(c) a series where the ratio test is inconclusive
(d) converges by the ratio test
(e) a series where the root test is inconclusive

20. Using the Integral Test Remainder Estimate for the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, we find that the smallest number of terms need to be added such that $|\text{error}| < 10^{-2}$ is

(a) 99
(b) 97
(c) 105
(d) 101
(e) 103

21. The power series representation for the function $f(x) = \dfrac{x^4}{(1+x)^2}$ is

(a) $\sum_{n=2}^{\infty} (-1)^n \, n \, x^{n+2}, \; |x| < 1$

(b) $\sum_{n=1}^{\infty} (-1)^n \, n \, x^{n+4}, \; |x| < 1$

(c) $\sum_{n=1}^{\infty} (-1)^{n+1} \, n \, x^{n+3}, \; |x| < 1$

(d) $\sum_{n=1}^{\infty} (-1)^n \, (n+1) \, x^{n+2}, \; |x| < 1$

(e) $\sum_{n=1}^{\infty} (-1)^{n+1} \, (n+2) \, x^{n}, \; |x| < 1$

22. $\displaystyle\int_0^1 \frac{dx}{1+x^6} =$

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{6n+2}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{6n+2}$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{6n+2}$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{6n+1}$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{1}{6n+2}$

23. Estimating the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and right points is equal to

(a) 10
(b) 12
(c) 8
(d) 14
(e) 16

24. $-\ln 2 + \frac{(\ln 2)^2}{2!} - \frac{(\ln 2)^3}{3!} + \ldots =$

(a) $\dfrac{1}{4}$
(b) 2
(c) $\dfrac{1}{2}$
(d) -2
(e) $-\dfrac{1}{2}$

25. The Maclaurin series for $f(x) = x \cos\left(\frac{x^2}{2}\right)$ is

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{4n+1}$, $R = \infty$

(b) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}} x^{4n+1}$, $R = \infty$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(2n)!} x^{4n+1}$, $R = \infty$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n}(2n)!} x^{4n+1}$, $R = \infty$

(e) $\displaystyle\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n}(2n)!} x^{4n+2}$, $R = \infty$

26. The first three nonzero terms of the Maclaurin series for the function $f(x) = (1 - x)^{1/4}$ are

(a) $1 - \frac{1}{4}x + \frac{3}{32}x^2$

(b) $1 - \frac{1}{4}x - \frac{3}{32}x^2$

(c) $1 + \frac{1}{4}x - \frac{3}{32}x^2$

(d) $1 + \frac{1}{4}x - \frac{3}{16}x^2$

(e) $1 - \frac{1}{4}x - \frac{3}{16}x^2$

27. Using the method of cylindrical shell, the volume of the solid generated by revolving about the line $x = 2$ the region bounded by $y = x - x^2$ and $y = 0$ is equal to

(a) $\frac{\pi}{5}$

(b) $\frac{\pi}{4}$

(c) $\frac{\pi}{2}$

(d) $\frac{\pi}{3}$

(e) $\frac{\pi}{6}$

28. The Taylor series for $f(x) = e^{2x}$ centered at $a = 3$ is given by

(a) $\sum_{n=0}^{\infty} 2^n e^6 (x-3)^n$, $R = \infty$

(b) $\sum_{n=0}^{\infty} \frac{e^6}{n!}(x-3)^n$, $R = \infty$

(c) $\sum_{n=0}^{\infty} \frac{2^n e^6}{n!}(x-3)^n$, $R = \infty$

(d) $\sum_{n=0}^{\infty} 2^n (x-3)^n$, $R = \infty$

(e) $\sum_{n=0}^{\infty} \frac{2^n}{n!}(x-3)^n$, $R = \infty$

King Fahd University of Petroleum and Minerals
Department of Mathematics

CODE04 **CODE04**

Math 102
Final Exam
213
August 09, 2022
Net Time Allowed: 180 Minutes

Name	

ID		**Sec**	

Check that this exam has 28 questions.

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6. The Test Code Number is already bubbled in your answer sheet. Make sure that it is the same as that printed on your question paper.
7. When bubbling, make sure that the bubbled space is fully covered.
8. When erasing a bubble, make sure that you do not leave any trace of penciling.

1. The average value of $f(x) = \sec^2\left(\frac{x}{2}\right)$ over $\left[0, \frac{\pi}{2}\right]$ is equal to

(a) $\frac{10}{\pi}$
(b) $\frac{8}{\pi}$
(c) $\frac{6}{\pi}$
(d) $\frac{4}{\pi}$
(e) $\frac{12}{\pi}$

2. The improper integral $\int_{-2}^{3} \frac{1}{x^4}\, dx$ is

(a) divergent
(b) convergent to $\frac{1}{4}$
(c) convergent to $-\frac{1}{4}$
(d) convergent to $-\frac{1}{2}$
(e) convergent to $\frac{1}{2}$

3. If $y = \int_{2x}^{3x} \frac{u^2 - 1}{u^2 + 1}\, du$, then $\frac{dy}{dx}$ at $x = 0$ is equal to

(a) -3
(b) -1
(c) -2
(d) 2
(e) 4

4. $\int x\sqrt{1 - x^4}\, dx =$

(a) $\frac{1}{8}\sin^{-1}(x^2) + \frac{1}{8}x^2\sqrt{1 - x^4} + c$
(b) $\frac{1}{8}\sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1 - x^4} + c$
(c) $\frac{1}{4}\sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1 - x^4} + c$
(d) $\frac{1}{4}\sin^{-1}(x^2) - \frac{1}{8}x^2\sqrt{1 - x^4} + c$
(e) $\frac{1}{4}\sin^{-1}(x^2) + \frac{1}{8}x^2\sqrt{1 - x^4} + c$

5. $\int \frac{\cos^5 x}{\sqrt{\sin x}}\,dx =$

(a) $\sqrt{\sin x} + \frac{4}{5}\sin^{5/2} x + \sin^{9/2} x + c$

(b) $2\sqrt{\sin x} + \sin^{5/2} x + \frac{1}{9}\sin^{9/2} x + c$

(c) $2\sqrt{\sin x} - \frac{4}{5}\sin^{5/2} x + \frac{2}{9}\sin^{9/2} x + c$

(d) $4\sqrt{\sin x} - \frac{1}{5}\sin^{4/5} x - \frac{2}{9}\sin^{9/2} x + c$

(e) $\sqrt{\sin x} - \frac{1}{5}\sin^{4/5} x + \frac{2}{9}\sin^{9/2} x + c$

6. $\int \tan^{-1}(4x) =$

(a) $2\tan^{-1}(4x) - \frac{1}{8}\ln(1+16x^2) + c$

(b) $x\tan^{-1}(4x) - \frac{1}{8}\ln(1+16x^2) + c$

(c) $2x\tan^{-1}(4x) - \frac{7}{8}\ln(1+16x^2) + c$

(d) $3\tan^{-1}(4x) + \frac{1}{8}\ln(1+16x^2) + c$

(e) $3x\tan^{-1}(4x) + \frac{7}{8}\ln(1+16x^2) + c$

7. $\displaystyle\int \frac{x^3+4}{x^2+4}\,dx =$

(a) $\frac{1}{2}x^2 - 2\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + c$

(b) $\frac{1}{2}x^2 - 2\ln(x^2+4) - 4\tan^{-1}\left(\frac{x}{2}\right) + c$

(c) $4x^2 - 2\ln(x^2+4) - 2\tan^{-1}\left(\frac{x}{2}\right) + c$

(d) $x^2 - 2\ln(x^2+4) - \tan^{-1}\left(\frac{x}{2}\right) + c$

(e) $3x^2 + 2\ln(x^2+4) + 2\tan^{-1}\left(\frac{x}{2}\right) + c$

8. The series

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1}$$

is

(a) convergent and its sum is $\frac{7}{2}$

(b) convergent and its sum is $\frac{3}{2}$

(c) divergent

(d) convergent and its sum is $\frac{5}{2}$

(e) convergent and its sum is $\frac{9}{2}$

9. The series $\sum_{n=1}^{\infty} \frac{e^n}{3^{n-1}}$ is

(a) convergent with sum $= \frac{e}{3-e}$

(b) divergent

(c) convergent with sum $= \frac{3e}{3-e}$

(d) convergent with sum $= \frac{3e}{3+e}$

(e) convergent with sum $= \frac{e}{3-2e}$

10. Let $a_n = \frac{(-1)^n n^3}{n^3 + 2n^2 + 1}$, then $\lim_{n\to\infty} a_n$

(a) converges to 1

(b) converges to -1

(c) does not exist

(d) converges to 2

(e) converges to 0

11. The series $\sum_{n=1}^{\infty} n^2 e^{-n^3}$ is

(a) divergent by integral test
(b) convergent to $\frac{1}{3e}$
(c) convergent by integral test
(d) divergent by divergence test
(e) a series where the integral test is not applicable

12. The length of the curve $f(x) = 3 + \frac{1}{2}\cosh 2x$ over $[0, 1]$ is equal to

(a) $\sinh 3$
(b) $\frac{1}{3}\sinh 3$
(c) $\frac{1}{2}\sinh 4$
(d) $\sinh 2$
(e) $\frac{1}{2}\sinh 2$

13. The series

$$\sum_{n=1}^{\infty} \frac{\sqrt[3]{n}}{\sqrt{n^3+4n+3}}$$

(a) diverges by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$

(b) diverges by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^{1/6}}$

(c) converges by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^{1/6}}$

(d) converges by comparing with $\sum_{n=1}^{\infty} \frac{1}{n^{7/6}}$

(e) diverges by limit comparison test

14. The area of the surface obtained by rotating the curve $y = \sqrt{1+e^x}$, $0 \le x \le 1$ about x-axis is equal to

(a) $2\pi(e+4)$

(b) $\pi(e+1)$

(c) $2\pi(e-1)$

(d) $3\pi(e+1)$

(e) $\pi(e-1)$

15. Using the Integral Test Remainder Estimate for the series $\sum_{n=1}^{\infty} \frac{1}{n^2}$, we find that the smallest number of terms need to be added such that $|\text{error}| < 10^{-2}$ is

(a) 105
(b) 101
(c) 99
(d) 103
(e) 97

16. The series $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2}$

(a) converges by the ratio test
(b) converges by the root test
(c) diverges by the ratio test
(d) converges by integral test
(e) converges conditionally

17. The power series representation for the function $f(x) = \dfrac{x^4}{(1+x)^2}$ is

(a) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\, n\, x^{n+3},\ |x| < 1$

(b) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\,(n+2)\, x^{n},\ |x| < 1$

(c) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\, n\, x^{n+4},\ |x| < 1$

(d) $\displaystyle\sum_{n=2}^{\infty}(-1)^{n}\, n\, x^{n+2},\ |x| < 1$

(e) $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\,(n+1)\, x^{n+2},\ |x| < 1$

18. The series $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ is

(a) a divergent p-series

(b) a convergent alternating series

(c) a convergent p-series

(d) a convergent geometric series

(e) a divergent geometric series

19. The series

$$\sum_{n=1}^{\infty} (\tan^{-1} n)^n$$

(a) converges by the ratio test
(b) a series where the ratio test is inconclusive
(c) a series where the root test is inconclusive
(d) converges by the root test
(e) diverges by the root test

20. The area of the region in the first quadrant enclosed by the curves $y = x$, $y = \dfrac{1}{x}$ and $y = \dfrac{1}{4}x$ is equal to

(a) ln 6
(b) ln 3
(c) ln 5
(d) ln 4
(e) ln 2

21. The interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$$

is

(a) $(-\infty, \infty)$
(b) $[1, 3]$
(c) $(1, 3]$
(d) $(1, 3)$
(e) $[1, 3)$

22. The Maclaurin series for $f(x) = x \cos\left(\frac{x^2}{2}\right)$ is

(a) $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n}(2n)!} x^{4n+1}$, $R = \infty$
(b) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}} x^{4n+1}$, $R = \infty$
(c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{4^{2n}(2n)!} x^{4n+2}$, $R = \infty$
(d) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{4n+1}$, $R = \infty$
(e) $\sum_{n=0}^{\infty} \frac{(-1)^n}{2^{2n}(2n)!} x^{4n+1}$, $R = \infty$

23. $-\ln\,2+\frac{(\ln 2)^2}{2!}-\frac{(\ln 2)^3}{3!}+\ldots=$

(a) 2

(b) -2

(c) $\dfrac{1}{2}$

(d) $\dfrac{1}{4}$

(e) $-\dfrac{1}{2}$

24. $\displaystyle\int_0^1 \frac{dx}{1+x^6}=$

(a) $\displaystyle\sum_{n=1}^{\infty}\frac{1}{6n+2}$

(b) $\displaystyle\sum_{n=0}^{\infty}\frac{1}{6n+2}$

(c) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{6n+2}$

(d) $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{6n+2}$

(e) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n}{6n+1}$

25. Estimating the area under the graph of $f(x) = 1 + x^2$ from $x = -1$ to $x = 2$ using three rectangles and right points is equal to

    (a) 8
    (b) 12
    (c) 10
    (d) 16
    (e) 14

26. Using the method of cylindrical shell, the volume of the solid generated by revolving about the line $x = 2$ the region bounded by $y = x - x^2$ and $y = 0$ is equal to

    (a) $\frac{\pi}{2}$
    (b) $\frac{\pi}{5}$
    (c) $\frac{\pi}{4}$
    (d) $\frac{\pi}{6}$
    (e) $\frac{\pi}{3}$

27. The Taylor series for $f(x) = e^{2x}$ centered at $a = 3$ is given by

(a) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n}{n!}(x-3)^n,\ R = \infty$

(b) $\displaystyle\sum_{n=0}^{\infty} 2^n e^6 (x-3)^n,\ R = \infty$

(c) $\displaystyle\sum_{n=0}^{\infty} \frac{e^6}{n!}(x-3)^n,\ R = \infty$

(d) $\displaystyle\sum_{n=0}^{\infty} \frac{2^n e^6}{n!}(x-3)^n,\ R = \infty$

(e) $\displaystyle\sum_{n=0}^{\infty} 2^n (x-3)^n,\ R = \infty$

28. The first three nonzero terms of the Maclaurin series for the function $f(x) = (1-x)^{1/4}$ are

(a) $1 + \frac{1}{4}x - \frac{3}{16}x^2$

(b) $1 - \frac{1}{4}x + \frac{3}{32}x^2$

(c) $1 - \frac{1}{4}x - \frac{3}{32}x^2$

(d) $1 + \frac{1}{4}x - \frac{3}{32}x^2$

(e) $1 - \frac{1}{4}x - \frac{3}{16}x^2$

Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	A 7	B 2	D 1	D 1
2	A	D 1	C 6	E 3	A 7
3	A	B 6	A 4	A 7	B 3
4	A	B 4	D 1	B 5	C 5
5	A	A 5	B 7	A 4	C 6
6	A	C 3	D 3	E 6	B 2
7	A	E 2	A 5	B 2	A 4
8	A	E 14	D 9	C 9	B 11
9	A	C 11	D 14	A 14	C 12
10	A	E 10	C 11	C 13	C 10
11	A	B 13	B 10	A 12	C 13
12	A	A 9	C 13	B 8	E 8
13	A	D 12	A 12	A 11	D 14
14	A	E 8	C 8	E 10	B 9
15	A	B 20	D 16	A 21	B 15
16	A	D 15	B 17	A 17	C 17
17	A	B 18	E 15	A 16	A 20
18	A	E 17	D 18	E 19	B 16
19	A	B 21	D 20	B 18	E 18
20	A	C 19	C 21	D 15	E 21
21	A	D 16	C 19	C 20	B 19
22	A	D 22	E 26	D 23	E 28
23	A	E 23	A 28	C 24	E 25
24	A	B 24	A 24	E 25	E 23
25	A	A 28	A 27	C 28	A 24
26	A	A 27	C 25	B 27	A 22
27	A	E 25	E 23	C 22	D 26
28	A	A 26	D 22	C 26	C 27

Answer Counts

V	A	B	C	D	E
1	6	7	3	5	7
2	6	4	7	8	3
3	8	5	7	3	5
4	5	7	7	3	6