1. $\int_0^{\frac{3\pi}{2}} |\sin x|\, dx =$

#46/404

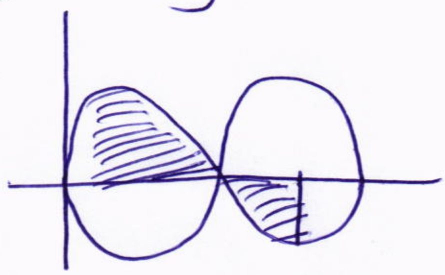
(a) 3 ______________________________ (correct)
(b) 1
(c) 2
(d) −1
(e) −2

$$\int_0^{\pi} (\sin x)\,dx + \int_{\pi}^{3\pi/2} (-\sin x)\,dx$$

$$= \left[-\cos x\right]_0^{\pi} + \left[\cos x\right]_{\pi}^{3\pi/2}$$

$$= \left[1-(-1)\right] + \left[0-(-1)\right]$$

$$= \boxed{3}$$



2. $\int_1^2 \frac{(x-1)^3}{x^2}\, dx =$

(a) $-2+3\ln 2$ ______________________________ (correct)
(b) $\ln 9$
(c) $\frac{3}{2}+\ln 2$
(d) $2\ln 3-\sqrt{2}$
(e) $1-2\ln 2$

#42/404

$$\int_1^2 \frac{x^3-3x^2+3x-1}{x^2}\,dx$$

$$= \int_1^2 \left(x-3+\frac{3}{x}-\frac{1}{x^2}\right)dx$$

$$= \left[\frac{x^2}{2}-3x+3\ln|x|+\frac{1}{x}\right]_1^2$$

$$= 2-6+3\ln 2+\frac{1}{2}-\frac{1}{2}+3-1$$

$$\boxed{-2+3\ln 2}$$

3. $\int \frac{\cos\left(\frac{\pi}{x}\right)}{x^2}\,dx =$

#34/4.4

(a) $-\frac{1}{\pi}\sin\left(\frac{\pi}{x}\right)+c$ ________ (correct)

(b) $\sin\left(\frac{\pi}{x}\right)+c$

(c) $-\frac{1}{x}\sin(\pi x)+c$

(d) $-\frac{1}{\pi}\sin(\pi x)+c$

(e) $-\frac{x}{\pi}\sin\left(\frac{\pi}{x}\right)+c$

let $u=\frac{\pi}{x} \Rightarrow du=-\frac{\pi}{x^2}dx$

$$\int \frac{\cos(\pi/x)}{x^2}dx = \int \cos u\left(-\frac{du}{\pi}\right)$$

$$= -\frac{1}{\pi}\sin(u)+C$$

$$= -\frac{1}{\pi}\sin\left(\frac{\pi}{x}\right)+C$$

4. If $\int_0^4 e^{(x-2)^4}\,dx = k$, then the value of $\int_0^4 xe^{(x-2)^4}\,dx =$

end of ch#5 Problems

(a) $2k$ ________ (correct)

(b) k

(c) k^4

(d) k^2

(e) $4k^2$

starts with $u=x-2 \Rightarrow du=dx$

$\boxed{u+2=x}$ $\quad x=0 \Rightarrow u=-2$

$x=4 \Rightarrow u=2$

$$\int_0^4 xe^{(x-2)^4}dx = \int_{-2}^{2}(u+2)e^{u^4}du$$

$$= \underbrace{\int_{-2}^{2} ue^{u^4}du}_{\substack{\text{zero} \\ \text{odd} \\ \text{fun}}} + \underbrace{\int_{-2}^{2} 2e^{u^4}du}_{\text{even}} = \int_0^4 e^{(x-2)^4}du = 2k$$

5. If $x\sin(\pi x) = \int_0^{x^2} f(t)\,dt$, where f is a continuous function, then $f(4) =$

end of ch#5, Problems

(a) $\frac{\pi}{2}$ (correct)

(b) 2

(c) $\frac{2}{3}\pi$

(d) 2π

(e) $\frac{3}{2}\pi$

Diff Bothsides:

$\sin(\pi x) + x\pi\cos(\pi x) = 2x f(x^2)$

let $x = 2 \Rightarrow$

$\sin(2\pi) + 2\pi\cos(2\pi) = 4f(4)$

$0 + 2\pi = 4f(4)$

$f(4) = \pi/2$

6. $\int_1^5 \frac{x\,dx}{\sqrt{2x-1}} =$

Sec# 5,3

(a) $\frac{16}{3}$ (correct)

(b) $\frac{8}{3}$

(c) $\frac{13}{3}$

(d) $\frac{14}{3}$

(e) $\frac{11}{3}$

let $u = \sqrt{2x-1} \Rightarrow u^2 = 2x-1$

$2u\,du = 2dx$

$x=1 \Rightarrow u=1, \quad x=5 \Rightarrow u=3$

$$\int_1^5 \frac{x\,dx}{\sqrt{2x-1}} = \int_1^3 \frac{1}{u}\left(\frac{u^2+1}{2}\right)u\,du =$$

$$\frac{1}{2}\int_1^3 (u^2+1)\,du = \frac{1}{2}\left[\frac{u^3}{3}+u\right]_1^3 = \frac{1}{2}\left((9+3)-\left(\frac{1}{3}+1\right)\right) = \boxed{16/3}$$

7. The curve $y = \int_1^x \frac{t^2+1}{\sqrt{t}}\,dt$ concave up on the interval:

5.3

(a) $(1, \infty)$ ________________ (correct)

(b) $\left(\frac{1}{2}, \infty\right)$

(c) $(0, \sqrt{3})$

(d) $(-\infty, \sqrt{3})$

(e) $(0, 1)$

concave up $y'' > 0$

$$\frac{dy}{dx} = \frac{x^2+1}{\sqrt{x}} = x^{3/2} + x^{-1/2}$$

$$\frac{d^2y}{dx^2} = \frac{3}{2}x^{1/2} - \frac{1}{2}x^{-3/2} = \frac{3}{2}x^{1/2} - \frac{1}{2x^{3/2}}$$

$$\frac{3x^2-1}{2x^{3/2}} = 0 \Rightarrow x = \frac{1}{\sqrt{3}}$$

$$\left(\frac{1}{\sqrt{3}}, \infty\right)$$

$-- \mid ++$ at $\frac{1}{\sqrt{3}}$

8. $\frac{d}{dx}\left[\int_{1-3x}^{1} \frac{u^3}{1+u^2}\,du\right] =$

(a) $\frac{3(1-3x)^3}{1+(1-3x)^2}$ ________________ (correct)

(b) $\frac{3x^3}{1+x^2}$

(c) $\frac{-3(1-3x)^3}{1+(1-3x)^2}$

(d) $\frac{(1-3x)^3}{1+(1-3x)^2}$

(e) $\frac{1}{2}$

$$0 - \left(\frac{(1-3x)^3}{1+(1-3x)^2}\right)(-3)$$

$$= \frac{3(1-3x)^3}{1+(1-3x)^2}$$

9. $\lim_{n\to\infty} \frac{3}{n} \sum_{i=1}^{n} \left[\left(\frac{3i}{n}\right)^3 - 6\left(\frac{3i}{n}\right) \right] =$

(a) $-\frac{27}{4}$ ______________________________ (correct)

(b) $\frac{27}{4}$

(c) $\frac{29}{4}$

(d) $-\frac{29}{4}$

(e) $-\frac{26}{4}$

See Book, Sec 5.2 example 2, about $f(x) = x^3 - 6x$ over $[0, 3]$, $n=6$

10. $\int_0^5 (x - \sqrt{25 - x^2})\, dx =$

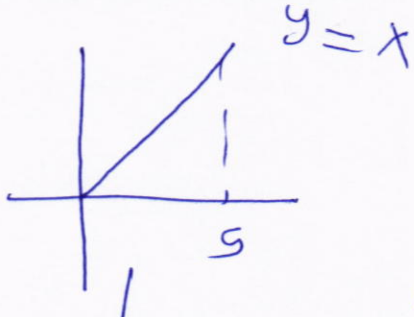
(a) $\frac{50 - 25\pi}{4}$ ______________________________ (correct)

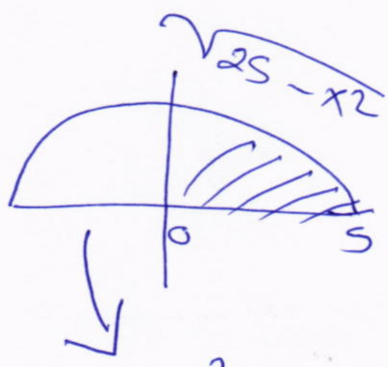
(b) $\frac{40 - 25\pi}{4}$

(c) $\frac{25 - 25\pi}{2}$

(d) $\frac{25 - 50\pi}{2}$

(e) $\frac{50 + \pi}{4}$





$\frac{1}{2}(5)(5)$

$\frac{1}{4}\pi(5)^2 = \frac{25\pi}{4}$

So $\frac{25}{2} - \frac{25\pi}{4} = \frac{50 - 25\pi}{4}$

11. $\int_1^3 \sqrt{4+x^2}\,dx =$

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\sqrt{4+\left(1+\frac{2i}{n}\right)^2}\right) \frac{2}{n}$ ———————— (correct)

(b) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\sqrt{4+\frac{4i^2}{n^2}}\right) \frac{2}{n}$

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\sqrt{4+\left(1+\frac{i}{n}\right)^2}\right) \frac{1}{n}$

(d) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\sqrt{4+\left(1-\frac{2i}{n}\right)^2}\right) \frac{2}{n}$

(e) $\lim_{n\to\infty} \sum_{i=1}^{n} \left(\sqrt{4+\left(1+\frac{2i}{n}\right)}\right) \frac{2}{n}$

Riemann Sum

12. If U and L are upper and lower sums for $f(x) = x^2$, $0 \le x \le 2$, with $n = 4$ then $\frac{U+L}{2} =$

$L: x_1^* = 0,\ x_2^* = \frac{1}{2},\ x_3^* = 1,\ x_4^* = 1.5$

(a) $\frac{11}{4}$ ———————— (correct)

(b) $\frac{15}{4}$

(c) $\frac{7}{4}$

(d) $\frac{13}{4}$

(e) 3

$L_4 = (0)\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{1}{4}\right) + \frac{1}{2}(1) + \frac{1}{2}\left(\frac{3}{2}\right)^2$

$= 7/4$

$U: \frac{1}{2}, 1, 1\frac{1}{2}, 2$

$U_4 = \frac{1}{2}\left(\frac{1}{2}\right)^2 + \frac{1}{2}(1)^2 + \frac{1}{2}\left(\frac{3}{2}\right)^2 + \frac{1}{2}(2)^2$

$1/8 + \frac{1}{2} + \frac{9}{8} + 2 = \frac{15}{4} \Rightarrow \frac{\frac{7}{4}+\frac{15}{4}}{2} = \boxed{\frac{11}{4}}$

13. Find the number b such that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two equal halves.

#58/6.1

(a) $2\sqrt[3]{2}$ ________________________________ (correct)

(b) 2

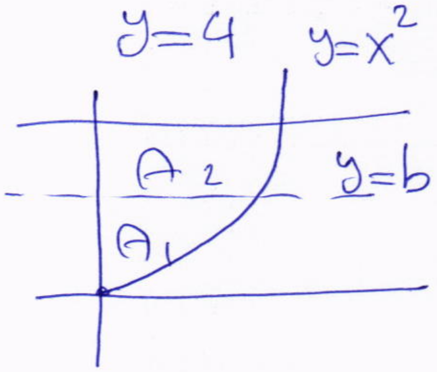
(c) $2\sqrt[3]{4}$

(d) 4

(e) 8

$A_1 = A_2$

$$A_1 = \int_0^b (R-L)\,dy = \int_0^b \sqrt{y}\,dy = \frac{2}{3} b^{3/2}$$



$$A_2 = \int_b^4 (R-L)\,dy = \int_b^4 \sqrt{y}\,dy = \frac{3}{2}\left(4^{3/2} - b^{3/2}\right) = \frac{2}{3} b^{3/2}$$

$$2b^{3/2} = 8 \Longrightarrow b = 2\sqrt[3]{2}$$

14. Find the area of the region enclosed by the curves

$y = x^2$ and $y = 2 - |x|$

#25/6.1

(a) 7/3 ________________________________ (correct)

(b) 13/3

(c) 5/3

(d) 7/6

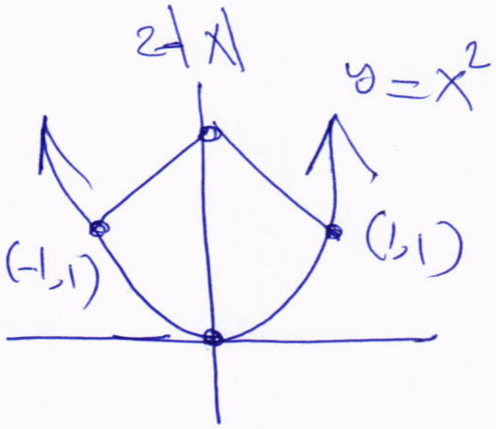
(e) 5/6

$$\int_{-1}^{1} \left((2-|x|) - (x^2)\right) dx$$

even: $2\int_0^1 (2 - |x| - x^2)\,dx$

$$= 2\int_0^1 (2 - x - x^2)\,dx$$

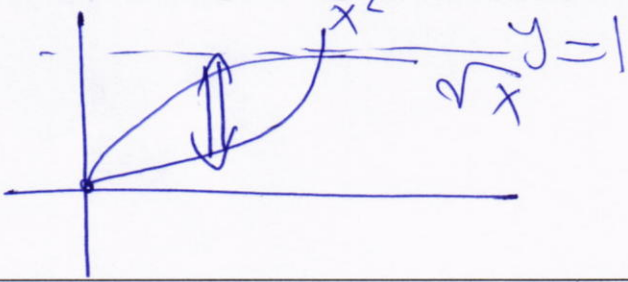
$$= 2\left[2x - \frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left(2 - \frac{1}{2} - \frac{1}{3}\right) = \frac{7}{3}$$



15. Find the volume of the solid obtained by rotating the region bounded by the curves

$y = x^2$ and $x = y^2$

about the line $y = 1$.



$\boxed{11/6.2}$

(a) $\dfrac{11}{30}\pi$ ______ (correct)

(b) $\dfrac{\pi}{3}$

(c) $\dfrac{\pi}{6}$

(d) $\dfrac{13}{30}\pi$

(e) $\dfrac{\pi}{2}$

inner: $1-\sqrt{x}$, out: $1-x^2$

$(0,0), (1,1)$: $0 \le x \le 1$

$$V = \pi\int_0^1 \left[(1-x^2)^2 - (1-\sqrt{x})^2\right] dx$$

$$= \pi\int_0^1 (x^4 - 2x^2 - x + 2\sqrt{x})\, dx = \pi\left(\frac{1}{5} - \frac{2}{3} - \frac{1}{2} + \frac{4}{3}\right) = \frac{11\pi}{30}$$

16. The volume of the solid obtained by rotating the region bounded by the curves

$y = \ln x,\ y = -1$ and $y = 1$,

about $x = -1$ is given by

$\approx \#4/6.2$

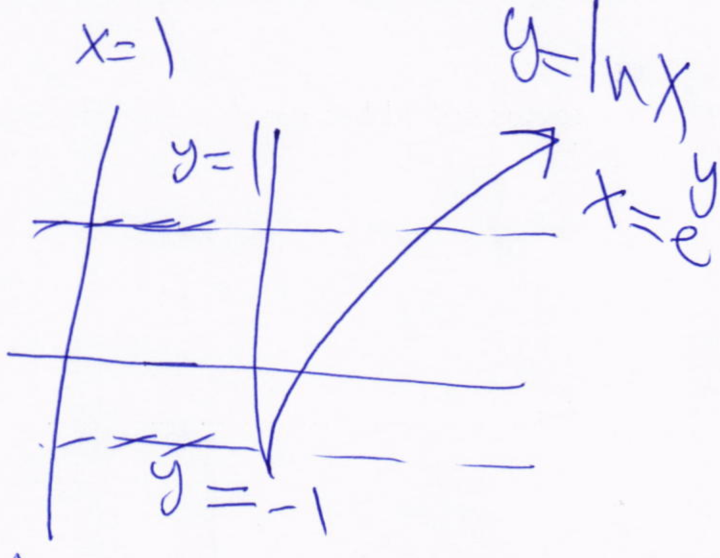
(a) $\pi \displaystyle\int_{-1}^{1} (e^{2y} + 2e^{y})\, dy$ ______ (correct)

(b) $\pi \displaystyle\int_{-1}^{1} [(1+e^{y})^2 - e^{2y}]\, dy$

(c) $2\pi \displaystyle\int_{0}^{1} (e^{2y} + 2e^{y})\, dy$

(d) $2\pi \displaystyle\int_{0}^{1} [(1+\ln x)^2 - 1]\, dx$

(e) $\pi \displaystyle\int_{-1}^{1} [(1+\ln x)^2 - e^{x}]\, dx$



inn: 1

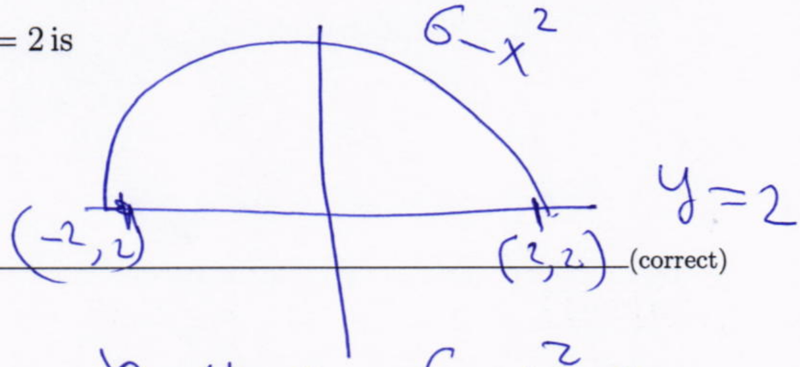
out: $1+x = 1+e^y$, $|y| \le 1$

$$V = \pi\int_{-1}^{1} \left((1+e^y)^2 - 1^2\right) dy = \pi\int_{-1}^{1} (e^{2y} + 2e^{y})\, dy$$

17. The volume of the solid obtained by rotating the region bounded by the curves

$y = 6 - x^2$ and $y = 2$ about $y = 2$ is

8/6.2



(a) $2\pi \int_0^2 (4 - x^2)^2 \, dx$ ______ (correct)

(b) $\pi \int_{-2}^{2} [(6 - x^2)^2 - 4] \, dx$

(c) $2\pi \int_0^2 [(4 - x^2)^2 - 4] \, dx$

(d) $2\pi \int_0^2 (x^2 + 4)^2 \, dx$

(e) $\pi \int_{-2}^{2} (x^2 - 4) \, dx$

$r = y - 2 = 6 - x^2 - 2$

$= 4 - x^2$

$|x| \le 2$

$\pi \int_{-2}^{2} (4 - x^2)^2 \, dx$

$= 2\pi \int_0^2 (4 - x^2)^2 \, dx$

18. The volume of the solid obtained by rotating the region bounded by the curves $x = \sqrt{y}$, $x = 0$ and $y = 1$ about $y = 1$ is

5/6.2

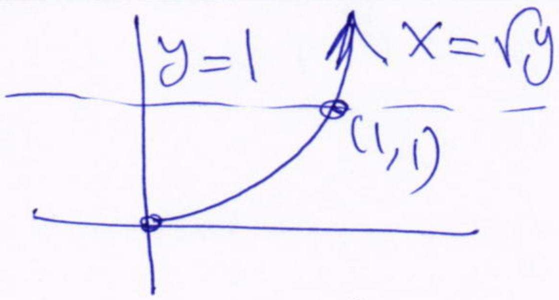
(a) $\frac{8\pi}{15}$ ______ (correct)

(b) $\frac{11\pi}{15}$

(c) $\frac{13\pi}{15}$

(d) $\frac{7}{13}\pi$

(e) $\frac{3}{5}\pi$



$0 \le x \le 1$

$1 - y = 1 - x^2$

$V = \pi \int_0^1 (1 - x^2)^2 \, dx = \pi \int_0^1 (1 - 2x^2 + x^4) \, dx$

$\pi \left[x - \frac{2x^3}{3} + \frac{1}{5}x^5 \right]_0^1 = \pi \left[1 - \frac{2}{3} + \frac{1}{5} \right] = \boxed{\frac{8\pi}{15}}$