1. The volume of the solid obtained by rotating about the x-axis, the region bounded by $x + y = 3$, $x = 4 - (y-1)^2$ is

(a) $\frac{27}{2}\pi$ ______ (correct)

(b) $\frac{81}{4}\pi$

(c) $\frac{16}{3}\pi$

(d) $\frac{13}{4}\pi$

(e) $\frac{31}{3}\pi$

$4-(y-1)^2=0 \Rightarrow y-1=\pm 2$

$y=3, -1$, Vertex $(4,1)$

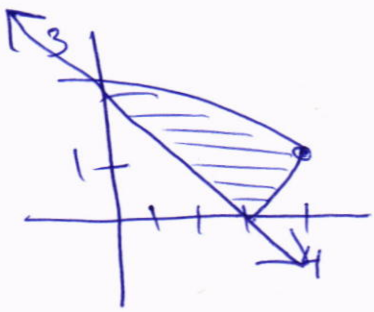
$x+y=3$: $(0,3), (3,0)$

$\boxed{\#14/6.3}$

$V=\int_0^3 2\pi y\left(4-(y-1)^2\right)-3+y)\,dy$

$2\pi\int_0^3 y(4-(y^2-2y+1)-3+y)\,dy$

$2\pi\int_0^3 y(-y^2+3y)\,dy = \ldots = \frac{27}{2}\pi$



2. The average value of $f(x) = \sec^2\left(\frac{\theta}{2}\right)$ over $\left[0, \frac{\pi}{2}\right]$ is equal to:

(a) $\frac{4}{\pi}$ ______ (correct)

(b) $\frac{2}{\pi}$

(c) 2

(d) 4

(e) $\frac{\pi}{2}$

$f_{avg} = \frac{1}{\frac{\pi}{2}-0}\int_0^{\pi/2} \sec^2\left(\frac{\theta}{2}\right)d\theta$

$=\frac{2}{\pi}\int_0^{\pi/2} \sec^2\left(\frac{\theta}{2}\right)d\theta = \frac{2}{\pi}\left[2\tan\left(\frac{\theta}{2}\right)\right]_0^{\pi/2}$

$=\frac{4}{\pi}[1-0]$

$\boxed{\#6/6.5}$

3. $\int_0^1 \frac{x^3}{\sqrt{4+x^2}}\,dx =$ $\int \frac{x\,x^2}{\sqrt{4-x^2}}dx$, let $u=x^2$ $\Rightarrow du = 2x\,dx$

(a) $\frac{16}{3} - \frac{7}{3}\sqrt{5}$ (correct)

(b) $\frac{8-\sqrt[3]{7}}{3}$

(c) $10\sqrt{5} - \frac{1}{3}$

(d) $5\sqrt{5} - \frac{8}{3}\sqrt{7}$

(e) 0

$dv = \frac{x\,dx}{\sqrt{x^2+4}} \Rightarrow v = \sqrt{4+x^2}$

$\int \frac{x^3 dx}{\sqrt{4+x^2}} = x^2\sqrt{4+x^2} - 2\int x\sqrt{4+x^2}\,dx$

$= \left[x^2\sqrt{4+x^2} - \frac{2}{3}(4+x^2)^{3/2}\right]_0^1$ #34/7.1

$\left(\sqrt{5} - \frac{2}{3}(4+1)^{3/2}\right) - \left(-\frac{2}{3}\sqrt{64}\right) = \cdots = \frac{16}{3} - \frac{7\sqrt{5}}{3}$

4. $\int_0^{\frac{1}{2}} x\cos(\pi x)\,dx =$

(a) $\frac{1}{2\pi} - \frac{1}{\pi^2}$ (correct)

(b) $\frac{1-\pi}{2}$

(c) $\frac{\pi-1}{3}$

(d) $\frac{2\pi-1}{3}$

(e) $\frac{1+\pi}{\pi}$

let $u = x \Rightarrow du = dx$

$dv = \cos(\pi x)dx \Rightarrow v = \frac{1}{\pi}\sin(\pi x)$

$\int x\cos(\pi x)dx = \frac{1}{\pi}x\sin(\pi x) - \int \frac{1}{\pi}\sin(\pi x)\,dx$

$= \frac{1}{\pi}x\sin(\pi x) - \frac{1}{\pi}\left[\frac{-\cos(\pi x)}{\pi}\right]$ #23/7.1

$= \left[\frac{x}{\pi}\sin(\pi x)\right]_0^{1/2} + \left[\frac{1}{\pi^2}\cos(\pi x)\right]_0^{1/2}$

$\frac{1}{\pi}\left(\frac{1}{2} - 0\right) + \frac{1}{\pi^2}(0-1)$

5. $\int x \ln(1+x)\,dx =$ $\boxed{4.1/7.1}$

$u = \ln(1+x) \Rightarrow du = \frac{dx}{1+x}$

$dv = x\,dx \Rightarrow v = \frac{1}{2}x^2$

(a) $\frac{1}{2}(x^2-1)\ln(1+x) - \frac{1}{4}x^2 + \frac{1}{2}x + c$ ______ (correct)

(b) $x\ln(1+x) + x^2 + \ln(1+x) + c$

(c) $x^2\ln(x+1) - x\ln(1+x) + c$

(d) $x\ln(1+x) - x^2 - x + c$

(e) $\frac{1}{2}x^2\ln(x+1) - \frac{1}{2}x^2 + x + c$

$\int x\ln(1+x)\,dx = \frac{1}{2}x^2\ln(1+x) - \frac{1}{2}\int \frac{x^2}{1+x}\,dx$

Now $1+x \overline{\big) x^2}$ with quotient $x-1$ $\Rightarrow \frac{x^2}{x+1} = x - 1 + \frac{1}{x-1}$

so $\int \left(x - 1 + \frac{1}{x-1}\right)dx = \frac{1}{2}x^2 - x + \ln|x-1|$

$\int x\ln(1+x)\,dx = \left(\frac{1}{2}x^2 - \frac{1}{2}\right)\ln|1+x| - \frac{1}{4}x^2 + \frac{1}{2}x + C$

6. $\int \tan^3 x\,dx =$

(a) $\frac{1}{2}\tan^2 x - \ln|\sec x| + c$ ______ (correct)

(b) $\frac{1}{4}(\tan x)^4 + c$

(c) $\tan x \sec^2 x + c$

(d) $\ln|\sec x + \tan x| + c$

(e) $\tan x - \ln|\tan x| + c$

$\boxed{\text{ex\#7/7.2}}$

$\int \tan^3 x\,dx = \int \tan^2 x \tan x\,dx$

$= \int \tan x(\sec^2 x - 1)\,dx = \int \tan x \sec^2 x - \int \tan x$

$= \frac{1}{2}(\tan x)^2 + \ln|\sec x| + C$

7. $\int_0^{\frac{\pi}{4}} \sqrt{1+\cos 4x}\, dx =$

$\cos^2\theta = \dfrac{1+\cos(2\theta)}{2}$

(a) $\dfrac{\sqrt{2}}{2}$ ________ (correct)

(b) $\sqrt{2}$

(c) $2\sqrt{2}$

(d) $2+\sqrt{2}$

(e) $4+\sqrt{2}$

$2\cos^2\theta = 1+\cos(2\theta)$

$2\cos^2(2\theta) = 1+\cos(4\theta)$

$\int_0^{\pi/4} \sqrt{2\cos^2(2\theta)}\, d\theta$

$\boxed{46/7.2}$

$= \sqrt{2}\int_0^{\pi/4} \cos(2\theta)\, d\theta \quad (>0)$

$= \sqrt{2}\left[\dfrac{\sin(2\theta)}{2}\right]_0^{\pi/4} = \sqrt{2}/2$

8. $\int \sin^6 x \cos^3 x\, dx =$

(a) $\dfrac{(\sin x)^7}{7} - \dfrac{(\sin x)^9}{9} + c$ ________ (correct)

$\boxed{2/7.2}$

(b) $-\dfrac{(\sin x)^7}{7} + \dfrac{(\sin x)^9}{9} + c$

(c) $\dfrac{(\sin x)^5}{5} + \dfrac{(\sin x)^7}{7} - \dfrac{(\sin x)^9}{9} + c$

(d) $\dfrac{(\sin x)^3}{3} + \dfrac{(\sin x)^6}{6} + c$

(e) $\dfrac{(\sin x)^7}{7} - \dfrac{(\sin x)^9}{9} + \dfrac{(\sin x)^3}{3} + c$

$\int \sin^6 x\, \cos^2 x \cos x\, dx = \int \sin^6 x\,(1-\sin^2 x)\cos x\, dx$

$u = \sin x \Rightarrow du = \cos x\, dx$

$\int u^6(1-u^2)\, dx = \int (u^6 - u^8)\, du = \dfrac{u^7}{7} - \dfrac{u^9}{9} + C$

$-\dfrac{(\sin x)^9}{9} + \dfrac{(\sin x)^7}{7} + C$

5/7.3

9. $\displaystyle\int_{\sqrt{2}}^{2} \frac{dx}{x^3\sqrt{x^2-1}} =$, let $x = \sec\theta \Rightarrow dx = \sec\theta\tan\theta\, d\theta$

$x=\sqrt{2} \Rightarrow \theta = \frac{\pi}{4}\ (45^\circ)$

(a) $\frac{\pi}{24} + \frac{\sqrt{3}}{8} - \frac{1}{4}$ (correct)

$x = 2 \Rightarrow \theta = \frac{\pi}{3}\ (60^\circ)$

(b) 0

(c) $\frac{\pi}{13} - \frac{\sqrt{3}}{8} + 1$

(d) $\frac{\pi}{2} - 2\sqrt{2} + \frac{1}{4}$

(e) $2\sqrt{2}\pi + \frac{1}{12}$

$$\int_{\sqrt{2}}^{2} \frac{dx}{x^3\sqrt{x^2-1}} = \int_{\pi/4}^{\pi/3} \frac{\sec\theta\tan\theta\, d\theta}{\sec^3\theta \cdot \tan\theta}$$

$$\int_{\pi/4}^{\pi/3} \frac{d\theta}{\sec^2\theta} = \int_{\pi/4}^{\pi/3} \cos^2\theta\, d\theta = \int_{\pi/4}^{\pi/3} \left(\frac{1+\cos 2\theta}{2}\right) d\theta$$

$$= \frac{1}{2}\left[\theta + \frac{\sin 2\theta}{2}\right]_{\pi/4}^{\pi/3} = \cdots = \frac{\pi}{24} + \frac{\sqrt{3}}{8} - \frac{1}{4}$$

10. $\displaystyle\int \frac{\sqrt{1+x^2}}{x}\, dx$

(a) $\ln\left|\frac{\sqrt{1+x^2}-1}{x}\right| + \sqrt{1+x^2} + c$ (correct)

let $x = \tan\theta$

$dx = \sec^2\theta\, d\theta$

(b) $\ln|x\sqrt{1+x^2}| - \frac{1}{x} + c$

(c) $\frac{1}{x} - \sqrt{1+x^2} + \frac{1}{2}\ln(1+x^2) + c$

(d) $\ln\left|\frac{1}{x} - \sqrt{1+x^2}\right| + x + c$

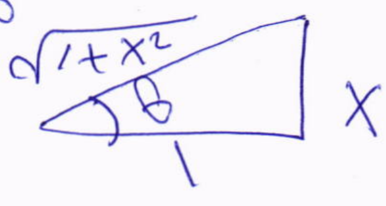
(e) $\frac{\sqrt{1+x^2}}{x} + \ln\left|\frac{1+\sqrt{x^2+1}}{x}\right| + c$

$$\int \frac{\sqrt{1+x^2}}{x}\, dx = \int \frac{\sqrt{1+\tan^2\theta}}{\tan\theta} \cdot \sec^2\theta$$

19/7.3

$$\int \frac{\sec\theta}{\tan\theta} \cdot \sec^2\theta\, d\theta = \int \frac{\sec\theta}{\tan\theta}(1+\tan^2\theta)\, d\theta$$

$$\int (\csc\theta + \tan\theta\sec\theta)\, d\theta$$



$$\ln|\csc\theta - \cot\theta| + \sec\theta + C$$

11. The volume of the solid obtained by rotating the region, in the first quadrant, bounded by $y = x$, $y = \dfrac{2x}{1+x^3}$ about $x = -1$, is given by:

(a) $2\pi \int_0^1 (x+1)\left(\frac{2x}{1+x^3} - x\right) dx$ ______ (correct)

(b) $2\pi \int_{-1}^1 (x+1)\left(x - \frac{2x}{1+x^3}\right) dx$

(c) $2\pi \int_0^1 (x-1)\left(\frac{2x}{1+x^3} + x\right) dx$

(d) $\pi \int_0^2 (x+1)\left(\frac{2x}{1+x^3} - x\right) dx$

(e) $\pi \int_{-1}^1 (-x-1)\left(\frac{2x}{1+x^3} - x\right) dx$

$X = \frac{2X}{1+X^3} \iff$

$X + X^4 = 2X \iff$

$X^4 - X = 0 \iff X = 0, 1$

6.3

$V = 2\pi \int_0^1 (X+1)\left(\frac{2X}{1+X^3} - X\right) dX$

12. If the fraction $\dfrac{10}{x^3 + 9x - x^2 - 9}$ can be decomposed into two subfractions:

$\dfrac{A}{x+K}$ and $\dfrac{Bx+C}{x^2+9}$, then $K + A + B + C =$

$\frac{10}{(X-1)(X^2+9)} = \frac{A}{X-1} + \frac{BX+C}{X^2+9}$

(a) -2 ______ (correct)

(b) 0

(c) 1

(d) -1

(e) 2

$\boxed{K = -1}$

$10 = A(X^2+9) + (BX+C)(X-1)$

$X = 1 \Rightarrow \boxed{A = 1}$

$X = 0 \Rightarrow \boxed{C = -1}$

7.4

also, $\boxed{A + B = 0}$, $B = -1$

$(-1) + (1) + (-1) + (-1) = \boxed{-2}$

13. $\int \frac{x^2+1}{(x-3)(x-2)^2}\,dx =$ $\frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2}$

(a) $10\ln|x-3| - 9\ln|x-2| + \frac{5}{x-2} + c$ ____________ (correct)

(b) $9\ln|x-3| + 10\ln|x-2| - \frac{5}{x-2} + c$

(c) $\frac{9}{10}\ln\left|\frac{x-3}{x-2}\right| + \frac{5}{(x-2)^2} + c$

(d) $\frac{10}{9}\ln|x-2| + 5\ln|x-3| + \frac{1}{(x-2)^2} + c$

(e) $\frac{9}{10}\ln|x-3| + \frac{10}{9}\ln|x-2| + \frac{5}{(x-2)^2} + c$

19/7.4

$x^2+1 = A(x-2)^2 + B(x-3)(x-2) + C(x-3)$

$x=2 \Rightarrow C=-5$

$x=3 \Rightarrow A=10,\ A+B=1 \Rightarrow B=-9$

$10\ln|x-3| - 9\ln|x-2| + \frac{5}{x-2} + C$

14. $\int \frac{x^2+x+1}{(x^2+1)^2}\,dx =$

(a) $\tan^{-1}(x) - \frac{1}{2(x^2+1)} + c$ ____________ (correct)

(b) $\tan^{-1}(x^2+1) - \ln|x| + c$

(c) $2\tan^{-1}(x) - \frac{1}{2}\ln(x^2+1) + c$

(d) $\tan^{-1}(x^2+x+1) - \frac{1}{2}(x^2+1) + c$

(e) $\frac{1}{2}\tan^{-1}(x^2+1) - \ln|x^2+x+1| + c$

26/7.4

$\frac{x^2+x+1}{(x^2+1)^2} = \frac{x^2+1}{(x^2+1)^2} + \frac{x}{(x^2+1)^2} = \frac{1}{x^2+1} + \frac{x}{(x^2+1)^2}$

$\tan^{-1}x - \frac{1}{2(x^2+1)} + C$

15. $\int_1^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$ converges to:

$= \lim_{t\to\infty} \int_1^t \frac{e^{-\sqrt{x}}}{\sqrt{x}}\,dx$

(a) $\frac{2}{e}$ ______ (correct)

(b) $\sqrt{e}$

(c) $\frac{1}{2}e$

(d) $\sqrt{2e}$

(e) $2\sqrt{e}$

$u=\sqrt{x} \Rightarrow du = \frac{dx}{2\sqrt{x}}$

7.8

$\lim_{t\to\infty} \int_1^{\sqrt{t}} 2e^{-u}\,du$

$2\lim_{t\to\infty}\left[-e^{-u}\right]_1^{\sqrt{t}} = -2\lim_{t\to\infty}\left[e^{-\sqrt{t}} - 1/e\right]$

$= -2(e^{-\infty} - 1/e) = 2/e$

16. The Improper Integral $\int_0^\infty \sin^2 x\,dx$ $= \lim_{t\to\infty} \int_0^t \sin^2 x\,dx$

(a) Diverges to $+\infty$ ______ (correct)

(b) Diverges to $-\infty$

(c) Converges to $\frac{1}{\sqrt{3}}$

(d) Converges to $\frac{1}{3}$

(e) Diverges to $\pm\frac{1}{3}$

$= \int_0^t \left(\frac{1-\cos 2x}{2}\right)dx$

15/7.8

$= \int_0^t \left(\frac{1}{2} - \frac{\cos 2x}{2}\right)dx$

$= \left[\frac{1}{2}x - \frac{\sin(2x)}{4}\right]_0^t = \left(\frac{1}{2}t - \frac{\sin(2t)}{4}\right) - (0)$

Now $\lim_{t\to\infty}\left(\frac{1}{2}t - \frac{1}{4}\sin(2t)\right) = \boxed{\infty}$

17. $\int_0^\infty \frac{e^x}{e^{2x}+3}\,dx =$ $\lim_{t\to\infty} \int_0^t \frac{e^x}{(e^x)^2+3}\,dx$

(a) $\frac{\pi}{3\sqrt{3}}$ ______ (correct)
(b) $\frac{2\pi}{3\sqrt{3}}$
(c) ∞
(d) $3\pi\sqrt{e}$
(e) $\frac{3}{2}\pi\sqrt{e}$

$\lim_{t\to\infty}\left[\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{e^x}{\sqrt{3}}\right)\right]_0^t$

$\frac{1}{\sqrt{3}}\left[\lim_{t\to\infty}\tan^{-1}\left(\frac{e^t}{\sqrt{3}}\right)-\tan^{-1}\left(\frac{1}{\sqrt{3}}\right)\right]$

24/7.8 $\frac{1}{\sqrt{3}}\left[\pi/2-\pi/6\right]$

18. $\int \tan^{-1}(\sqrt{x})\,dx =$

21/7.5

(a) $x\tan^{-1}(\sqrt{x})-\sqrt{x}+\tan^{-1}(\sqrt{x})+c$ ______ (correct)
(b) $x\tan^{-1}x-\sqrt{x}+x+c$
(c) $(1+\tan^{-1}(\sqrt{x}))\,x+\frac{1}{x}+c$
(d) $\tan^{-1}(x)-x\sqrt{x}+c$
(e) $\sqrt{x}\tan^{-1}(\sqrt{x})-\sqrt{x}+c$

let $t=\sqrt{x} \Rightarrow t^2=x$
$2t\,dt=dx$
$2\int \tan^{-1}(t)(t\,dt)$ Now

let $u=\tan^{-1}(t) \Longrightarrow du=\frac{dt}{1+t^2}$, $dv=t\,dt$, $v=\frac{1}{2}t^2$

$2\int \tan^{-1}(t)\,dt = \frac{1}{2}t^2\tan^{-1}(t)-\int\frac{\frac{1}{2}t^2}{1+t^2}\,dt$

$= t^2\tan^{-1}(t)-\left[\int\left(1-\frac{1}{1+t^2}\right)dt\right]$

$= t^2\tan^{-1}(t)-t+\tan^{-1}t+c$