1. $\sin^2 x =$

(a) $\frac{2}{2!}x^2 - \frac{2^3}{4!}x^4 + \frac{2^5}{6!}x^6 - \frac{2^7}{8!}x^8 + \dots$ ________________ (correct)

(b) $x^2 + \frac{x^6}{(3!)^2} + \frac{x^{10}}{(5!)^2} + \frac{x^4}{(7!)^2} + \dots$

(c) $\frac{1}{2} - \frac{2}{2!}x^2 + \frac{2^3}{4!}x^4 - \frac{2^5}{6!}x^6 + \dots$

(d) $1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \dots$

(e) $1 - \left(\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}\right)^2$

2. $\lim_{h \to 0} \frac{1}{h} \int_2^{2+h} \sqrt{1+t^3}\, dt =$

(a) 3 ________________ (correct)

(b) 0

(c) $\sqrt{2}$

(d) $\sqrt{3}$

(e) 4

3. $\int_0^3 |x^2 - 4|\, dx =$

(a) $\frac{23}{3}$ ——————————————————————(correct)

(b) $\frac{29}{3}$

(c) 21

(d) $\frac{17}{2}$

(e) $\frac{16}{3}$

4. $\int_0^1 \frac{dx}{(1+\sqrt{x})^4} =$

(a) $\frac{1}{6}$ ——————————————————————(correct)

(b) $\frac{2}{6}$

(c) $\frac{1}{16}$

(d) $\frac{3}{8}$

(e) $\frac{3}{16}$

5. The area of the region bounded by $y = x^2 \ln x$ and $y = 4 \ln x$ is equal to:

(a) $\dfrac{16}{3} \ln 2 - \dfrac{29}{9}$ ________________ (correct)

(b) $\dfrac{16 \ln 2 - 3}{9}$

(c) $2 \ln 2 - 1$

(d) $\dfrac{8}{3}e - \dfrac{1}{2}e$

(e) $\dfrac{e^2}{2} - e \ln 2$

6. $\displaystyle\int_1^9 \frac{2x^2 + x^2\sqrt{x} - 1}{x^2}\, dx$

(a) $32\,\dfrac{4}{9}$ ________________ (correct)

(b) $29\,\dfrac{1}{4}$

(c) $34\,\dfrac{1}{2}$

(d) $36\,\dfrac{1}{4}$

(e) $33\,\dfrac{3}{4}$

7. Given that the base of a solid is the region bounded by the parabolas $y = x^2$ and $y = 2 - x^2$. The volume of the solid if the cross-sections perpendicular to the x-axis are squares with one side lying along the base is :

(a) $\dfrac{64}{15}$ ———————————————————— (correct)

(b) $\dfrac{8}{15}$

(c) $\dfrac{32}{15}$

(d) $\dfrac{28}{15}$

(e) $\dfrac{16}{15}$

8. Using the method of cylindrical shell, the volume generated by rotating the region bounded by $y = x^3$, $y = 0$, $x = 1$ about $y = 1$ is equal to:

(a) $\dfrac{5\pi}{14}$ ———————————————————— (correct)

(b) $\dfrac{10\pi}{14}$

(c) $\dfrac{13}{14}\pi$

(d) $\dfrac{16}{14}\pi$

(e) $\dfrac{3}{14}\pi$

9. The average value of $f(x) = \sin^2 x \cos^3 x$ over $[-\pi, \pi]$ is equal to:

(a) 0 ______ (correct)

(b) $\dfrac{1}{2\pi}$

(c) $\dfrac{1}{\pi 15}$

(d) $\dfrac{1}{15}(\pi^3 - \pi^5)$

(e) $\dfrac{\pi}{15}(\pi^5 - \pi^3)$

10. $\displaystyle\int \frac{1}{\sqrt{x} - \sqrt[3]{x}}\, dx =$

(Hint: Substitute $u = \sqrt[6]{x}$)

(a) $2\sqrt{x} + 3\sqrt[3]{x} + 6\sqrt[6]{x} + 6\ln|\sqrt[6]{x} - 1| + c$ ______ (correct)

(b) $\sqrt{x} - \dfrac{3}{2}\sqrt[4]{x} + 3\ln|\sqrt[6]{x - 1}| + c$

(c) $\sqrt{x}\sqrt[6]{x - 1} + 3\sqrt[4]{x} + \dfrac{1}{6}\ln|\sqrt[3]{x - 1}| + c$

(d) $6\sqrt[3]{x} + 6\ln|1 - \sqrt[4]{x}| + c$

(e) $\sqrt[4]{x} - \dfrac{3}{2}\sqrt[3]{x} + 6\ln\left|\sqrt[3]{x} - \dfrac{1}{2}\right| + c$

11. The length of $y = \ln(1 - x^2)$ over $\left[0, \frac{1}{2}\right]$ is

(a) $\ln 3 - \frac{1}{2}$ ______________________________ (correct)

(b) $2 \ln 3$

(c) $\ln 3 - \ln 2$

(d) $\frac{1}{2} \ln(1 + e)$

(e) $\ln(e - 1)$

12. The surface area that obtained when the curve $y = \sin(\pi x)$, $0 \leq x \leq 1$ is revolved about the x-axis is equal to:

(a) $2\sqrt{1 + \pi^2} + \frac{2}{\pi} \ln(\pi + \sqrt{1 + \pi^2})$ ______________________________ (correct)

(b) $\frac{4}{\pi}\sqrt{1 + \pi^2} + \ln(\sqrt{1 + \pi^2})$

(c) $\frac{2}{\pi} \ln(\sqrt{\pi + \sqrt{1 + \pi^2}})$

(d) $\frac{2}{\pi} \ln(\pi^2 + \sqrt{1 + \pi})$

(e) $\frac{1}{\pi} \ln(\pi + \sqrt{\pi^2 + \pi})$

13. $\displaystyle\int \frac{dx}{\sqrt{e^x - 1}} =$

(a) $2\tan^{-1}(\sqrt{e^x - 1}) + c$ ______________________________ (correct)

(b) $\dfrac{1}{2}\tan^{-1}\left(\dfrac{1}{2}x\right) + c$

(c) $\dfrac{e}{2}\tan^{-1}(\sqrt{e^x} - 1) + c$

(d) $\dfrac{2}{e}\tan^{-1}\left(\dfrac{x}{2}\right) + c$

(e) $\dfrac{2}{e}\tan^{-1}(\sqrt{e^x - 1}) + c$

14. $\displaystyle\int_0^\infty x^2 e^{-x^2}\,dx =$

(a) $\displaystyle\frac{1}{2}\int_0^\infty e^{-x^2}\,dx$ ______________________________ (correct)

(b) $\displaystyle 2\int_0^\infty x e^{-x^2}\,dx$

(c) 0

(d) $\displaystyle\int_{-\infty}^\infty x e^{-x^2}\,dx$

(e) $\displaystyle\frac{1}{2}\int_0^\infty x e^{-x}\,dx$

15. A formula for the general term a_n of the sequence

$$\{\frac{3}{5}, \frac{-4}{25}, \frac{5}{125}, -\frac{6}{625}, \frac{7}{3125}, \ldots\}$$

is

(a) $(-1)^{n-1}\frac{n+2}{5^n}$, $n \geq 1$ ______________________________ (correct)

(b) $(-1)^{n-1}\frac{n+2}{5^n}$, $n \geq 0$

(c) $(-1)^{n}\frac{n+2}{5^n}$, $n > 1$

(d) $(-1)^{n-1}\frac{n+3}{5^{n+1}}$, $n \geq 0$

(e) $(-1)^{n}\frac{n+3}{5^n}$, $n \geq 0$

16. The sequence $a_n = \dfrac{\sin 2n}{1+\sqrt{n}}$

(a) converges to 0 ______________________________ (correct)

(b) converges to 1

(c) converges to 2

(d) diverges

(e) converges to -1

17. The series $\displaystyle\sum_{n=1}^{\infty}\left(\frac{2}{e^n}+\frac{1}{2n(n+1)}\right)$

(a) converges and the sum is $\dfrac{3+e}{2e-2}$ ________________________________ (correct)

(b) converges and the sum is $\dfrac{e}{e-1}$

(c) converges and the sum is $\dfrac{1}{e-1}$

(d) converges and the sum is $\dfrac{2}{e-1}$

(e) converges and the sum is $\dfrac{e+1}{e-1}$

18. The series $\displaystyle\sum_{n=1}^{\infty}\ln\left(1+\frac{1}{n}\right)$ is

(a) diverges ________________________________ (correct)

(b) converges to 0

(c) converges to $\dfrac{1}{2}$

(d) converges to 2

(e) converges to 3

19. The values of p for which the series $\sum_{n=1}^{\infty} n((1+(2n)^2)^{\frac{p}{2}}$ is convergent.

(a) $p < -2$ ______________________________ (correct)

(b) $p < 1$

(c) $p < 0$

(d) $p < -1$

(e) $p > -2$

20. The series $\sum_{i=1}^{\infty} \frac{\ln n}{n^2}$

(a) converges by integral test $\int_1^{\infty} \frac{\ln x}{x^2}\, dx = 1$ ______________________________ (correct)

(b) converges by integral test $\int_1^{\infty} \frac{\ln x}{x^2}\, dx = \frac{1}{2}$

(c) converges by integral test $\int_1^{\infty} \frac{\ln x}{x^2}\, dx = \frac{1}{8}$

(d) diverges by integral test $\int_1^{\infty} \frac{\ln x}{x^2}\, dx = \infty$

(e) diverges by integral test $\int_1^{\infty} \frac{\ln x}{x^2}\, dx = -\infty$

21. The series $\displaystyle\sum_{n=1}^{\infty} \frac{\operatorname{arc\,cot} n}{n^{2\cdot 1}}$

(a) converges by comparing with $\pi \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{2\cdot 1}}$ ——————————(correct)

(b) converges by comparing with $\pi \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{0\cdot 1}}$

(c) diverges by comparing with $\pi \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{2\cdot 1}}$

(d) diverges by comparing with $\pi \displaystyle\sum_{n=1}^{\infty} \frac{1}{n^{0\cdot 1}}$

(e) diverges by limit comparison test

22. The series $\displaystyle\sum_{n=1}^{\infty} a_n$ where $a_n = \dfrac{3n^3 + n}{\sqrt{4 + n^7}}$

(a) diverges by limit comparison test with $\left(b_n = \dfrac{3}{n^{\frac{1}{2}}}\right)$ ——————————(correct)

(b) diverges by limit comparison test with $\left(b_n = \dfrac{3}{n^{\frac{7}{2}}}\right)$

(c) converges by limit comparison test with $\left(b_n = \dfrac{3}{n^{\frac{1}{2}}}\right)$

(d) converges by limit comparison test with $\left(b_n = \dfrac{3}{n}\right)$

(e) converges by limit comparison test with $\left(b_n = \dfrac{3}{n^{\frac{3}{2}}}\right)$

23. The series $\sum_{n=1}^{\infty} \frac{n \cos n\pi}{3^n}$

(a) a convergent alternating series ______________________________ (correct)

(b) a convergent geometric series

(c) a divergent p-series

(d) a divergent geometric series

(e) a divergent alternating series

24. The series $\sum_{n=1}^{\infty} \frac{10^n}{(n+1)3^{2n+1}}$

(a) diverges by the ratio test ______________________________ (correct)

(b) converges by the ratio test

(c) converge by the root test

(d) converge by integral test

(e) converge conditionally

25. The series $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$ converges for

(a) $x \in [1,3]$ ______________________________ (correct)
(b) $x \in [1,3)$ only
(c) $x \in (1,3]$ only
(d) all values of x
(e) $x \in [1,4]$

26. The series $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{n!(n+1)!2^{2n+1}}$ converges for

(a) all values of x ______________________________ (correct)
(b) $(0, \infty)$ only
(c) $(-\infty, 0)$ only
(d) $(1,3]$ only
(e) $[1,3]$ only

27. The power series representation for the function $\dfrac{x}{9+x^2}$ is

(a) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{9^{n+1}}$ ———————— (correct)

(b) $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n x^{2n}}{9^n}$

(c) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{9^n}$

(d) $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n+1}}{9^{n+1}}$

(e) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{3^{n+1}}$

28. The Maclaurin Series for $f(x) = \sinh x$ is

(a) $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}$ ———————— (correct)

(b) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!}$

(c) $\displaystyle\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!}$

(d) $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}$

(e) $\displaystyle\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n)!}$