Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	A 1	C 2	B 3	A 4
2	A	E 3	A 1	C 4	B 3
3	A	A 4	E 3	D 1	E 2
4	A	E 2	D 4	D 2	D 1
5	A	A 6	E 7	C 6	D 6
6	A	E 5	E 5	B 10	E 9
7	A	C 7	A 10	B 8	A 7
8	A	A 9	E 8	C 9	C 5
9	A	D 8	C 9	C 7	B 10
10	A	B 10	A 6	E 5	C 8
11	A	A 13	C 12	A 13	B 14
12	A	B 11	D 14	A 14	E 13
13	A	E 14	C 13	A 11	E 12
14	A	A 12	B 11	B 12	A 11
15	A	B 15	D 15	C 15	D 16
16	A	B 18	C 18	E 17	D 15
17	A	B 16	B 17	B 16	B 18
18	A	E 17	B 16	D 18	E 17

Detailed Solution →

1. If $\int_{-1}^{1} f(x)\,dx = -3$ and $\int_{0}^{1} f(x)\,dx = 5$, then $\int_{-1}^{0} f(x)\,dx =$

(a) -8

(b) $-\frac{3}{2}$

(c) 5

(d) 2

(e) 8

$$\int_{-1}^{1} f(x)\,dx = \int_{-1}^{0} f(x)\,dx + \int_{0}^{1} f(x)\,dx$$

$$-3 = \int_{-1}^{0} f(x)\,dx + 5$$

$$\Rightarrow \int_{-1}^{0} f(x)\,dx = -3-5 = -8$$

~ #48(a)
Sec 5.3

2. $\sum_{i=1}^{n} \frac{8i+3}{n^2} =$

(a) $4+\frac{7}{n}$

(b) $4+\frac{4}{n}$

(c) $4+\frac{2}{n}+\frac{3}{n^2}$

(d) $4+\frac{3}{n^2}$

(e) $4+\frac{3}{n}$

$$\frac{1}{n^2}\sum_{i=1}^{n}(8i+3) = \frac{1}{n^2}\left[8\sum_{i=1}^{n} i + 3\sum_{i=1}^{n} 1\right]$$

$$= \frac{1}{n^2}\left[8\cdot\frac{n(n+1)}{2} + 3n\right]$$

$$= \frac{1}{n^2}\left[4n(n+1)+3n\right]$$

$$= \frac{1}{n^2}\left[4n^2+7n\right]$$

$$= 4+\frac{7}{n}$$

~ #26
Sec 5.2

3. Using the **Midpoint Rule** with $n = 3$, the **area** of the region bounded by the graph of $f(x) = \sin^2 x$ and the x-axis over the interval $[0, \pi]$ is approximately equal to

$\Delta x = \frac{\pi - 0}{3} = \frac{\pi}{3}$

$\frac{\pi}{6}$ $\frac{3\pi}{6}$ $\frac{5\pi}{6}$

$\frac{\pi}{3}$ $\frac{2\pi}{3}$ π

(a) $\frac{\pi}{2}$
(b) $\frac{2\pi}{3}$
(c) $\frac{\pi}{3}$
(d) $\frac{\pi}{\sqrt{3}}$
(e) $\frac{\pi}{4}$

$$A \approx f(\tfrac{\pi}{6})\Delta x + f(\tfrac{3\pi}{6})\Delta x + f(\tfrac{5\pi}{6})\Delta x$$
$$= \frac{\pi}{3}\left[\sin^2(\tfrac{\pi}{6}) + \sin^2(\tfrac{\pi}{2}) + \sin^2(\tfrac{5\pi}{6})\right]$$
$$= \frac{\pi}{3}\left[(\tfrac{1}{2})^2 + (1)^2 + (\tfrac{1}{2})^2\right]$$
$$= \frac{\pi}{3} \cdot \frac{3}{2} = \frac{\pi}{2}$$

~ #66
Sec 5.2

4. If $F(x) = \int_3^{x^3} \frac{3}{t^3+3}\,dt$, then $F'(x) =$ $\frac{3}{(x^3)^3+3} \cdot 3x^2$

$= \frac{9x^2}{x^9+3}$

(a) $\frac{9x^2}{x^9+3}$
(b) $\frac{9x^2}{x^6+3}$
(c) $\frac{3}{x^9+3}$
(d) $\frac{3}{x^9+3} - \frac{1}{10}$
(e) $\frac{-9x^2}{x^9+3}$

~ # 84
Sec 5.4

114
Sec 5.4

5. If $f(x) = \int_{\sqrt{x}}^{x} e^{t^2}\, dt$, then $f'(4) =$

$f'(x) = e^{x^2} \cdot 1 - e^{(\sqrt{x})^2} \cdot \frac{1}{2\sqrt{x}}$

$= e^{x^2} - \frac{e^x}{2\sqrt{x}}$

$f'(4) = e^{16} - \frac{e^4}{4}$

(a) $e^{16} - \frac{e^4}{4}$

(b) $\frac{e^4}{4} - e^{16}$

(c) $e^{16} - e^4$

(d) $4e^{16} - e^4$

(e) $8e^{16} - 4e^4$

51
Sec 5.3

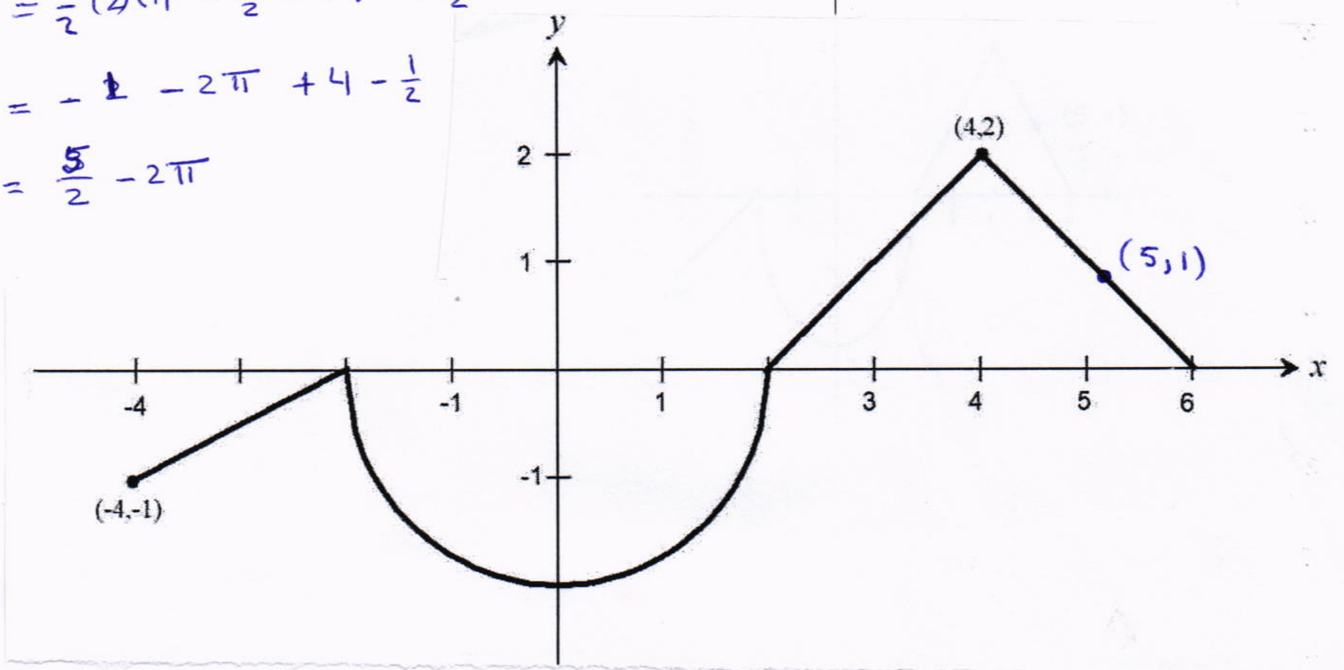
6. $\int_{-4}^{5} f(x)\, dx =$ $\int_{-4}^{-2} f(x)\,dx + \int_{-2}^{2} f(x)\,dx + \int_{2}^{6} f(x)\,dx - \int_{5}^{6} f(x)\,dx$

(The graph of $y = f(x)$ is given below)

$= \frac{-1}{2}(2)(1) - \frac{1}{2}\pi(2)^2 + \frac{1}{2}(4)(2) - \frac{1}{2}(1)(1)$

$= -1 - 2\pi + 4 - \frac{1}{2}$

$= \frac{5}{2} - 2\pi$



(a) $\frac{5}{2} - 2\pi$

(b) $6 + 2\pi$

(c) $3 - 2\pi$

(d) $5 + 2\pi$

(e) $\frac{3}{2} - 2\pi$

7. The **average value** of $f(x) = \sqrt{x}$ over the interval $[1, 4]$ is

~ #51
Sec 5.4

(a) $\frac{14}{9}$
(b) $\frac{7}{9}$
(c) $\frac{8}{3}$
(d) $\frac{2}{9}$
(e) $\frac{7}{3}$

$$f_{ave} = \frac{1}{4-1} \int_1^4 \sqrt{x}\, dx$$
$$= \frac{1}{3} \cdot \frac{2}{3} x^{3/2} \Big]_1^4$$
$$= \frac{2}{9} \left(\sqrt{4^3} - 1 \right)$$
$$= \frac{2}{9} (8-1) = \frac{14}{9}$$

8. If f is an **even** function and $\int_0^1 f(x)\, dx = 5$, then $\int_{-1}^1 (3x-4)\, f(x)\, dx =$

properties
§5.5

(a) -40
(b) 40
(c) 20
(d) -20
(e) 0

$$= \int_{-1}^1 (3x f(x) - 4 f(x))\, dx$$
$$= 3 \int_{-1}^1 \underbrace{x f(x)}_{\text{odd}}\, dx - 4 \int_{-1}^1 \underbrace{f(x)}_{\text{even}}\, dx$$
$$= 3(0) - 4 \cdot 2 \int_0^1 f(x)\, dx$$
$$= 0 - 4 \cdot 2 \cdot 5$$
$$= -40$$

9. $\int \frac{2x-1}{\sqrt{x+3}}\,dx =$

~ #70
Sec 5.5

(a) $\frac{2}{3}(2x-15)\sqrt{x+3}+C$
(b) $\frac{2}{3}(2x-1)\sqrt{x+3}+C$
(c) $(2x-15)\sqrt{(x+3)^3}+C$
(d) $2(2x-15)\sqrt{x+3}+C$
(e) $\frac{1}{3}(x+3)(2x-1)+C$

Let $u = x+3$. Then $du = dx$ & $x = u-3$

$= \int \frac{2(u-3)-1}{\sqrt{u}}\,du = \int \frac{2u-7}{\sqrt{u}}\,du$

$= \int 2\sqrt{u} - 7u^{-1/2}\,du$

$= 2\cdot\frac{2}{3}u^{3/2} - 7\cdot 2u^{1/2} + C$

$= \frac{4}{3}(x+3)^{3/2} - 14(x+3)^{1/2} + C$

$= (x+3)^{1/2}\left[\frac{4}{3}(x+3) - 14\right] + C$

$= (x+3)^{1/2}\left(\frac{4}{3}x + 4 - 14\right] + C$

$= (x+3)^{1/2}\left(\frac{4}{3}x - 10\right) + C$

$= \frac{2}{3}(x+3)^{1/2}(2x-15) + C$

$= \frac{2}{3}(2x-15)\sqrt{x+3} + C$

#55, Sec 5.9

10. $\int_0^{\ln 2} \tanh x\,dx =$

(a) $\ln\left(\frac{5}{4}\right)$
(b) $\ln\left(\frac{3}{2}\right)$
(c) $\ln(2)$
(d) 1
(e) 0

$\int_0^{\ln 2} \frac{\sinh x}{\cosh x}\,dx$; $u = \cosh x \Rightarrow du = \sinh x\,dx$

. $x=0 \Rightarrow u = \cosh(0) = 1$

. $x = \ln 2 \Rightarrow u = \cosh(\ln 2)$

$= \frac{e^{\ln 2} + e^{-\ln 2}}{2}$

$= \frac{1}{2}\left(2+\frac{1}{2}\right) = \frac{5}{4}$

$= \int_1^{5/4} \frac{1}{u}\,du$

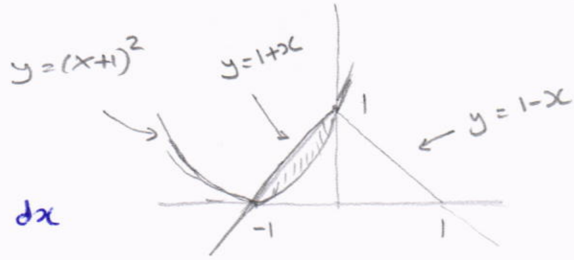
$= \ln|u|\Big]_1^{5/4}$

$= \ln\left(\frac{5}{4}\right).$

~ # 22

Sec 7.1

11. The **area** of the region bounded by the curves $y = (x+1)^2$ and $y = 1 - |x|$ is equal to



(a) $\frac{1}{6}$
(b) $\frac{5}{6}$
(c) $\frac{2}{3}$
(d) 1
(e) $\frac{1}{3}$

$$A = \int_{-1}^{0} [(1+x) - (x+1)^2]\, dx$$

$$= \int_{-1}^{0} (1+x - x^2 - 2x - 1)\, dx$$

$$= \int_{-1}^{0} (-x^2 - x)\, dx$$

$$= -\frac{x^3}{3} - \frac{x^2}{2}\Big]_{-1}^{0} = 0 - \left(\frac{1}{3} - \frac{1}{2}\right) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$

#24

Sec 5.7

12. $\displaystyle\int_e^{e^2} \frac{1}{x(\ln x^2)^3}\, dx =$

(a) $\frac{3}{64}$
(b) $\frac{3}{16}$
(c) $\frac{1}{64}$
(d) $\frac{5}{32}$
(e) $\frac{5}{16}$

Let $u = \ln x^2$. Then

$$du = \frac{1}{x^2} \cdot 2x\, dx = \frac{2}{x}\, dx$$

$$x = e \Rightarrow u = \ln e^2 = 2$$

$$x = e^2 \Rightarrow u = \ln e^4 = 4$$

$$= \frac{1}{2} \int_2^4 \frac{1}{u^3}\, du$$

$$= \frac{1}{2} \int_2^4 u^{-3}\, du$$

$$= \frac{1}{2} \cdot \frac{u^{-2}}{-2}\Big]_2^4 = -\frac{1}{4} \cdot \frac{1}{u^2}\Big]_2^4$$

$$= -\frac{1}{4}\left(\frac{1}{16} - \frac{1}{4}\right) = -\frac{1}{4} \cdot \frac{1-4}{16} = \frac{3}{64}$$

~ #42
sec 5.7

13. $\int t \sec t^2 \cdot (2\sec t^2 + \tan t^2)\, dt$

$u = t^2 \Rightarrow du = 2t\, dt$

$= \frac{1}{2} \int \sec u \cdot (2\sec u + \tan u)\, du$

$= \frac{1}{2} \int (2\sec^2 u + \sec u \tan u)\, du$

$= \frac{1}{2} [2\tan u + \sec u] + C$

$= \tan t^2 + \frac{1}{2}\sec t^2 + C$

(a) $\tan t^2 + \frac{1}{2}\sec t^2 + C$

(b) $\frac{1}{2}\tan t^2 + \frac{1}{2}\sec t^2 + C$

(c) $\tan^2 t^2 + \sec^2 t^2 + C$

(d) $\sec t^2 \cdot \tan t^2 + C$

(e) $\ln|\sec t^2 + \tan t^2| + C$

$$\begin{array}{r} x - 1 \\ x+1 \,\big)\, \overline{x^2 + 2} \\ \underline{x^2 + x} \\ -x + 2 \\ \underline{-x - 1} \\ 3 \end{array}$$

~ #55
sec 5.7

14. $\int_0^3 \frac{x^2+2}{x+1}\, dx = \int_0^3 \left(x - 1 + \frac{3}{x+1}\right) dx$

$= \left[\frac{1}{2}x^2 - x + 3\ln|x+1|\right]_0^3$

$= \frac{9}{2} - 3 + 3\ln 4 - (0 - 0 + 0)$

$= \frac{3}{2} + 3\ln 4$

(a) $\frac{3}{2} + 3\ln 4$

(b) $\frac{5}{2}$

(c) $2 - \ln 4$

(d) $3 + 3\ln 4$

(e) $\frac{1}{2} - 2\ln 4$

Example 6
§ 5.2

15. The **area** of the region bounded by the graph of $f(x) = 4 - x^2$, the x-axis and the vertical lines $x = 1$ and $x = 2$ is given by
(Choose c_i to be the right endpoint)

$\Delta x = \frac{2-1}{n} = \frac{1}{n}$; $c_i = 1 + i\Delta x = 1 + \frac{i}{n}$; $A = \lim_{n\to\infty} \sum_{i=1}^{n} f(c_i)\,\Delta x$

(a) $\lim_{n\to\infty}\left(\frac{3}{n}\sum_{i=1}^{n} 1 - \frac{2}{n^2}\sum_{i=1}^{n} i - \frac{1}{n^3}\sum_{i=1}^{n} i^2\right)$

(b) $\lim_{n\to\infty}\left(\frac{4}{n}\sum_{i=1}^{n} 1 - \frac{1}{n^2}\sum_{i=1}^{n} i - \frac{2}{n^3}\sum_{i=1}^{n} i^2\right)$

(c) $\lim_{n\to\infty}\left(\frac{1}{n}\sum_{i=1}^{n} 3 - \frac{1}{n^2}\sum_{i=1}^{n} i\right)$

(d) $\lim_{n\to\infty}\left(\frac{1}{n}\sum_{i=1}^{n} 3 + \frac{1}{n^2}\sum_{i=1}^{n} i + \frac{1}{n^3}\sum_{i=1}^{n} i^2\right)$

(e) $\lim_{n\to\infty}\left(\frac{1}{n}\sum_{i=1}^{n} 2 - \frac{3}{n^2}\sum_{i=1}^{n} i + \frac{1}{n^3}\sum_{i=1}^{n} i^2\right)$

$$= \lim_{n\to\infty} \sum_{i=1}^{\infty} f\left(1+\frac{i}{n}\right)\cdot\frac{1}{n}$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[4 - \left(1+\frac{i}{n}\right)^2\right]\cdot\frac{1}{n}$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(3 - \frac{2i}{n} - \frac{i^2}{n^2}\right)\frac{1}{n}$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{3}{n} - \frac{2i}{n^2} - \frac{i^2}{n^3}\right)$$
$$= \lim_{n\to\infty} \left(\frac{3}{n}\sum_{i=1}^{n} 1 - \frac{2}{n^2}\sum_{i=1}^{n} i - \frac{1}{n^3}\sum_{i=1}^{n} i^2\right)$$

~ # 94
Sec 5.4

16. The velocity (in m/s) of a particle moving along a line is given by

$$v(t) = t^2 - t - 12$$

The **total distance** the particle travels over the interval $0 \le t \le 5$ is

$d = \int_0^5 |v(t)|\,dt$; $t^2 - t - 12 = 0 \Rightarrow (t-4)(t+3) = 0 \Rightarrow t = 4,\ t = -3$

$v(t)$: $-$ on $(0, 4)$, $+$ on $(4, 5)$

(a) $\frac{77}{2}$

(b) $\frac{55}{3}$

(c) $\frac{125}{6}$

(d) $\frac{53}{2}$

(e) $\frac{67}{3}$

$$= \int_0^5 |t^2 - t - 12|\,dt$$
$$= \int_0^4 -(t^2 - t - 12)\,dt + \int_4^5 (t^2 - t - 12)\,dt$$
$$= -\frac{t^3}{3} + \frac{t^2}{2} + 12t\Big]_0^4 + \frac{t^3}{3} - \frac{t^2}{2} - 12t\Big]_4^5$$
$$= -\frac{64}{3} + 8 + 48 - 0 + \frac{125}{3} - \frac{25}{2} - 60 - \left(\frac{64}{3} - 8 - 48\right)$$
$$= -\frac{64}{3} + \frac{125}{3} - \frac{64}{3} + 8 + 48 - 60 + 8 + 48 - \frac{25}{2}$$
$$= \frac{-3}{3} + 56 - 60 + 56 - \frac{25}{2}$$
$$= -1 - 4 + 56 - \frac{25}{2}$$
$$= 51 - \frac{25}{2} = \frac{102 - 25}{2} = \frac{77}{2}$$

~ # 94
Sec 5.4

~ #32
Sec 5.8

17. $\int_0^{\frac{\pi}{6}} \frac{4\cos x}{1+4\sin^2 x}\,dx =$

(a) $\frac{\pi}{2}$
(b) $\frac{\pi}{4}$
(c) π
(d) 4π
(e) 2π

Let $u = 2\sin x$.
Then $du = 2\cos x\,dx$
$x = 0 \Rightarrow u = 0$
$x = \frac{\pi}{6} \Rightarrow u = 2\cdot\frac{1}{2} = 1$

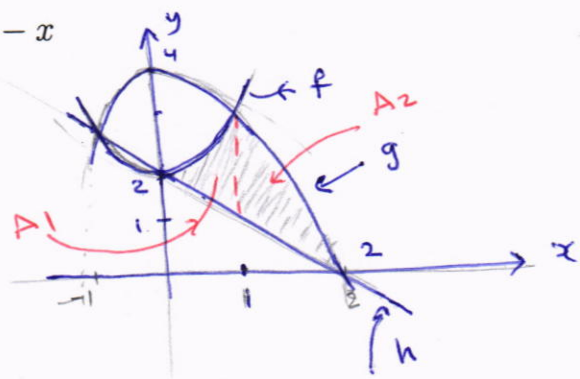
$$= \int_0^1 \frac{2}{1+u^2}\,du$$
$$= 2\tan^{-1} u \Big]_0^1 = 2\left(\frac{\pi}{4} - 0\right) = \frac{\pi}{2}$$

18. The **area** of the region in the first quadrant bounded by the graphs of

#51
§7.1

$$f(x) = x^2+2,\ g(x) = 4-x^2,\ h(x) = 2-x$$

is equal to



(a) 2
(b) 3
(c) $\frac{5}{2}$
(d) 4
(e) $\frac{7}{2}$

pts of intersection
$f(x) = g(x) \Rightarrow x = \pm 1$
$f(x) = h(x) \Rightarrow x = 0, -1$
$g(x) = h(x) \Rightarrow x = -1, 2$

$$A = A_1 + A_2$$
$$= \int_0^1 [(x^2+2) - (2-x)]\,dx + \int_1^2 [(4-x^2) - (2-x)]\,dx$$
$$= \int_0^1 (x^2+x)\,dx + \int_1^2 (2+x-x^2)\,dx$$
$$= \frac{x^3}{3} + \frac{x^2}{2}\Big]_0^1 + 2x + \frac{x^2}{2} - \frac{x^3}{3}\Big]_1^2$$
$$= \frac{1}{3} + \frac{1}{2} - 0 + \left(4 + 2 - \frac{8}{3}\right) - \left(2 + \frac{1}{2} - \frac{1}{3}\right)$$
$$= \frac{1}{3} + \cancel{\frac{1}{2}} + 4 - \frac{8}{3} \cancel{-\frac{1}{2}} + \frac{1}{3} = 4 - \frac{6}{3} = 4 - 2 = 2$$