~ #72
§ 8.3

1. $\int_0^1 \cos^2(\pi x)\,dx = \int_0^1 \frac{1}{2}(1+\cos(2\pi x))\,dx$

$= \frac{1}{2}\left[x + \frac{1}{2\pi}\sin(2\pi x)\right]_0^1$

(a) $\frac{1}{2}$ ______ (correct)

(b) 1

(c) 2

(d) $\frac{3}{2}$

(e) 3

$= \frac{1}{2}\left[(1+0)-(0+0)\right]$

$= \frac{1}{2}(1)$

$= \frac{1}{2}$

~ Example 2
§ 8.5

2. If $\frac{1}{x(x+1)^2} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$, then $A+B+C=$

$= \frac{A(x+1)^2 + Bx(x+1) + Cx}{x(x+1)^2}$

(a) -1 ______ (correct)

(b) 1

(c) 0

(d) 2

(e) -3

$\Rightarrow 1 = A(x+1)^2 + Bx(x+1) + Cx$

$x=0$: $1 = A + 0 + 0 \Rightarrow \boxed{A=1}$

$x=-1$: $1 = 0 + 0 - C \Rightarrow \boxed{C=-1}$

$x=1$: $1 = 4A + 2B + C$

$\Rightarrow 1 = 4 + 2B - 1 \Rightarrow 2B = -2 \Rightarrow \boxed{B=-1}$

So $A+B+C = 1-1-1 = -1$

~#11
§ 8.2

3. $\int_1^e 16x^3 \ln x \, dx =$ $16 \int_1^e x^3 \ln x \, dx$, By Parts $u = \ln x$ $dv = x^3 \, dx$

$du = \frac{1}{x} \, dx$ $v = \frac{x^4}{4}$

$16 \left[\frac{x^4}{4} \ln x - \int \frac{1}{4} x^3 \, dx \right]$

(a) $3e^4 + 1$ ______ (correct)

(b) $2e^3 + 4$

(c) $e^4 - 1$

(d) $2e^4 + 4$

(e) $e - 1$

$16 \left[\frac{1}{4} x^4 \ln x - \frac{1}{16} x^4 \right]$

$4x^4 \ln x - x^4 \Big]_1^e$ $\boxed{\ln e = 1}$

$= (4e^4 - e^4) - (0 - 1)$

$= 3e^4 + 1$

~ Example3
§ 7.2 ; an ordinary washer problem

4. The **volume** of the solid generated by revolving the region bounded by the curves $y^2 = 2x$, $y = 4$ and $x = 0$ about the x-axis is equal to

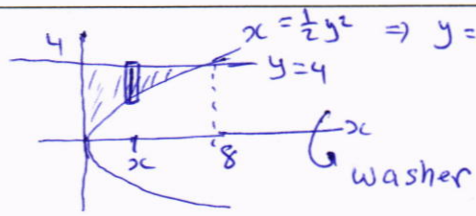
(a) $64\,\pi$ ______ (correct)

(b) $32\,\pi$

(c) $16\,\pi$

(d) $8\,\pi$

(e) $128\,\pi$



$V = \pi \int_0^8 \left[4^2 - (\sqrt{2x})^2 \right] dx$

$= \pi \int_0^8 (16 - 2x) \, dx$

$= \pi \cdot \; 16x - x^2 \Big]_0^8$

$= \pi \; (16 \cdot 8 - 8^2) \; = \pi \cdot 8 (16 - 8) = \pi \cdot 8 \cdot 8 = 64\pi$

#7 §7.4

5. The **arc length** of the curve $y = \frac{2}{3}(x^2+1)^{3/2}$ from $\left(0, \frac{2}{3}\right)$ to $\left(1, \frac{4\sqrt{2}}{3}\right)$ is equal to

$L = \int_0^1 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx$

(a) $\frac{5}{3}$ (correct)

(b) $\frac{2}{3}$

(c) $\frac{4}{3}$

(d) $\frac{1}{3}$

(e) $\frac{\sqrt{2}}{3}$

. $\frac{dy}{dx} = \frac{2}{3} \cdot \frac{3}{2} (x^2+1)^{1/2} \cdot 2x$

$\left(\frac{dy}{dx}\right)^2 = 4x^2(x^2+1) = 4x^4+4x^2$

$1+\left(\frac{dy}{dx}\right)^2 = 1+4x^4+4x^2 = (2x^2+1)^2$

$\sqrt{1+\left(\frac{dy}{dx}\right)^2} = 2x^2+1$

$= \int_0^1 (2x^2+1)\, dx = \frac{2}{3}x^3 + x\Big]_0^1 = \frac{2}{3}+1 = \frac{5}{3}$

Example 3, § 8.4

6. $\int_0^1 \frac{1}{(x^2+1)^{3/2}}\, dx =$

$x = \tan\theta \Rightarrow dx = \sec^2\theta\, d\theta$

. $(x^2+1)^{3/2} = (\tan^2\theta)^{3/2} = \tan^3\theta$

. $x=0 \Rightarrow \theta = 0$

$x = 1 \Rightarrow \theta = \frac{\pi}{4}$

$-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

(a) $\frac{1}{\sqrt{2}}$ (correct)

(b) $\sqrt{2}$

(c) $\frac{3}{\sqrt{2}}$

(d) $1 - \frac{1}{\sqrt{2}}$

(e) $\sqrt{2}-1$

$\int_0^{\pi/4} \frac{1}{\sec^3\theta} \cdot \sec^2\theta\, d\theta$

$= \int_0^{\pi/4} \frac{1}{\sec\theta}\, d\theta = \int_0^{\pi/4} \cos\theta\, d\theta$

$= \sin\theta\Big]_0^{\pi/4}$

$= \frac{1}{\sqrt{2}} - 0 = \frac{1}{\sqrt{2}}$

Example 6
§8.3

7. $\int \tan^4 x\, dx = \int \tan^2 x \cdot \tan^2 x \, dx = \int \tan^2 x \cdot (\sec^2 x - 1)\, dx$

$= \int (\tan^2 x \cdot \sec^2 x - \tan^2 x)\, dx$

(a) $\frac{1}{3}\tan^3 x - \tan x + x + C$ ______ (correct)

(b) $\tan^5 x + \tan^2 x + C$

(c) $\frac{1}{2}\tan x - \frac{1}{2}x + C$

(d) $\tan x + \sec^2 x + C$

(e) $2\tan^{-1} x - \tan x + x^2 + C$

$= \int (\tan^2 x \cdot \sec^2 x - \sec^2 x + 1)\, dx$

$= \frac{1}{3}\tan^3 x - \tan x + x + C$

~ #27 §7.3

8. Using the **shell method**, the **volume** of the solid obtained by revolving the region bounded by the curves $x = 4 - (y+1)^2$, $x = 0$ about the line $y = 2$ is given by

pt of int: $0 = 4 - (y+1)^2$

$(y+1)^2 = 4 \Rightarrow y+1 = \pm 2$

$\Rightarrow y = -1 \pm 2$

$= -3$ or

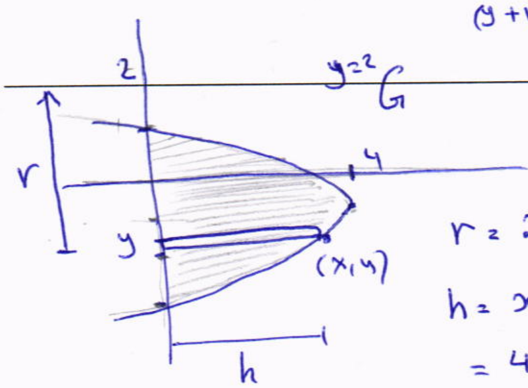
(a) $2\pi \int_{-3}^{1} (1-y)(2-y)(3+y)\, dy$ ______ (correct)

(b) $2\pi \int_{-3}^{1} (1+y)(2-y)(3+y)\, dy$

(c) $2\pi \int_{-3}^{1} (1-y)(2+y)(3-y)\, dy$

(d) $2\pi \int_{-3}^{1} (1+y)(2-y)(3-y)\, dy$

(e) $2\pi \int_{-3}^{1} (1+y)(2+y)(3+y)\, dy$



$r = 2 - y$

$h = x - 0 = x$

$= 4 - (y+1)^2$

$= (2 - (y+1))(2 + (y+1))$

$= (1-y)(3+y)$

$V = \int_{-3}^{1} 2\pi r h \, dy$

$= 2\pi \int_{-3}^{1} 2\pi (2-y)(1-y)(3+y)\, dy$

~#10
§8.3

9. $\int \frac{\cos^3 t}{\sqrt{\sin t}}\,dt =$ $\int \frac{\cos^2 t}{\sqrt{\sin t}} \cdot \cos t \, dt = \int \frac{1-\sin^2 t}{\sqrt{\sin t}} \cdot \cos t \, dt$

$u = \sin t \Rightarrow du = \cos t \, dt$

(a) $2\sqrt{\sin t} - \frac{2}{5}\sqrt{\sin^5 t} + C$ ________ (correct)

(b) $\sqrt{\sin t} + 2\sqrt{\sin^5 t} + C$

(c) $\frac{\cos t}{\sqrt{\sin t}} - \frac{1}{5}\sin^3 t + C$

(d) $\frac{2\cos t}{\sqrt{\sin t}} - \frac{2}{5}\sqrt{\sin^3 t} + C$

(e) $2\sqrt{\sin t} - \frac{2}{3}\sqrt{\sin^3 t} + C$

$= \int \frac{1-u^2}{\sqrt{u}}\,du$

$= \int u^{-1/2} - u^{3/2}\,du$

$= 2u^{1/2} - \frac{2}{5}u^{5/2} + C$

$= 2\sqrt{\sin t} - \frac{2}{5}\sqrt{\sin^5 t} + C$

#43 §7.4

10. The **area of the surface** generated by revolving the curve

$y = \sqrt{4-x^2}$, $-1 \le x \le 1$

about the x-axis is

$\therefore y' = \frac{-x}{\sqrt{4-x^2}}$

$1 + (y')^2 = 1 + \frac{x^2}{4-x^2} = \frac{4}{4-x^2}$

$\sqrt{1+(y')^2} = \frac{2}{\sqrt{4-x^2}}$

$S = \int_{-1}^{1} 2\pi y \sqrt{1+(y')^2}\,dx$

(a) 8π ________ (correct)

(b) 6π

(c) 4π

(d) 2π

(e) 5π

$= 2\pi \int_{-1}^{1} \sqrt{4-x^2} \cdot \frac{2}{\sqrt{4-x^2}}\,dx$

$= 2\pi \int_{-1}^{1} 2\,dx$

$= 2\pi \cdot 2x\Big]_{-1}^{1} = 2\pi(2+2) = 8\pi$

~#23
§ 8.5

11. $\int \frac{8-x}{x^3+4x}\,dx =$

$\frac{8-x}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$\Rightarrow 8-x = A(x^2+4) + x(Bx+C) = (A+B)x^2 + Cx + A4$

$\Rightarrow \cdot\ 4A = 8 \Rightarrow A = 2$

$\cdot\ C = -1$

$\cdot\ A+B=0 \Rightarrow B=-2$

(a) $2\ln|x| - \ln(x^2+4) - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$ ——— (correct)

(b) $2\ln|x| + \ln(x^2+4) + \tan^{-1}\left(\frac{x}{2}\right) + C$

(c) $\ln|x| - \frac{1}{2}\ln(x^2+4) + C$

(d) $3\ln|x| + \ln(x^2+4) - \tan^{-1}x + C$

(e) $\ln|x| - \frac{1}{2}\ln(x^2+4) - \tan^{-1}\left(\frac{x}{2}\right) + C$

$= \int \frac{2}{x} + \frac{-2x-1}{x^2+4}\,dx$

$= \int \frac{2}{x} - \frac{2x}{x^2+4} - \frac{1}{x^2+4}\,dx$

$= 2\ln|x| - \ln(x^2+4) - \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right) + C$

#44 § 8.3

12. $\int \cos(5\theta)\cos(3\theta)\,d\theta =$

(a) $\frac{1}{4}\sin(2\theta) + \frac{1}{16}\sin(8\theta) + C$ ——— (correct)

(b) $\frac{1}{4}\sin(2\theta) - \frac{1}{16}\sin(8\theta) + C$

(c) $-\frac{1}{4}\cos(2\theta) - \frac{1}{16}\cos(8\theta) + C$

(d) $\frac{1}{4}\cos(2\theta) + \frac{1}{16}\sin(8\theta) + C$

(e) $-\frac{1}{4}\sin(2\theta) + \frac{1}{16}\cos(8\theta) + C$

$= \frac{1}{2}\int [\cos(5\theta-3\theta) + \cos(5\theta+3\theta)]\,d\theta$

$= \frac{1}{2}\int [\cos(2\theta) + \cos(8\theta)]\,d\theta$

$= \frac{1}{2}\left(\frac{1}{2}\sin(2\theta) + \frac{1}{8}\sin(8\theta)\right) + C$

$= \frac{1}{4}\sin(2\theta) + \frac{1}{16}\sin(8\theta) + C$

~ # 73 (a)

§ 7.2 13. The base of a solid is the region bounded by the curves $y = x^3$, $y = 1$, $x = 0$. If the cross sections of the solid perpendicular to the x-axis are **squares**, then the **volume** of the solid is equal to

(a) $\frac{9}{14}$ (correct)

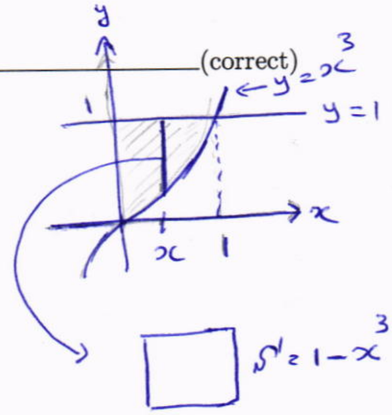
(b) $\frac{3}{7}$

(c) $\frac{5}{14}$

(d) $\frac{8}{7}$

(e) $\frac{2}{7}$

$$V = \int_0^1 A(x)\,dx = \int_0^1 (1-x^3)^2\,dx = \int_0^1 (1-2x^3+x^6)\,dx = x - \frac{1}{2}x^4 + \frac{x^7}{7}\Big]_0^1 = 1 - \frac{1}{2} + \frac{1}{7} = \frac{1}{2} + \frac{1}{7} = \frac{9}{14}$$



$A(x) = s^2 = (1-x^3)^2$

66 § 8.2

14. $\int_0^8 e^{\sqrt{2x}}\,dx =$

(a) $3e^4 + 1$ (correct)

(b) $4e^4 - 1$

(c) $2e^4 + 2$

(d) $e^4 - 1$

(e) $2e^4 + 1$

$y = \sqrt{2x} \Rightarrow 2x = y^2$

$\Rightarrow 2\,dx = 2y\,dy$

$\Rightarrow dx = y\,dy$

. $x = 0 \Rightarrow y = 0$ & $x = 8 \Rightarrow y = 4$

$$\int_0^4 e^y \cdot y\,dy = \int_0^4 y e^y\,dy = ye^y - e^y\Big]_0^4 = 4e^4 - e^4 - (0-1) = 3e^4 + 1$$

$u = y$ $dv = e^y\,dy$

$du = dy$ $v = e^y$

$\int y e^y\,dy = ye^y - \int e^y = ye^y - e^y + C$

~ Example 1
§ 8.4

15. $\displaystyle\int \frac{x^2}{\sqrt{3-x^2}}\,dx =$

$x = \sqrt{3}\sin\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$

. $dx = \sqrt{3}\cos\theta\, d\theta$

. $\sqrt{3-x^2} = \sqrt{3-3\sin^2\theta} = \sqrt{3}\sqrt{\cos^2\theta} = \sqrt{3}\cos\theta$

$\sqrt{3}$, θ, $\sqrt{3-x^2}$, x

(a) $\dfrac{3}{2}\sin^{-1}\left(\dfrac{x}{\sqrt{3}}\right) - \dfrac{1}{2}x\sqrt{3-x^2} + C$ (correct)

(b) $\dfrac{1}{\sqrt{3}}\sin^{-1}\left(\dfrac{x}{\sqrt{3}}\right) - \dfrac{3}{2}x\sqrt{3-x^2} + C$

(c) $\dfrac{1}{2}\sin^{-1}\left(\dfrac{x}{\sqrt{3}}\right) - x\sqrt{3-x^2} + C$

(d) $\dfrac{\sqrt{3}}{2}\sin^{-1}\left(\dfrac{x}{3}\right) - \dfrac{1}{2}\sqrt{3-x^2} + C$

(e) $\dfrac{3}{2}\sin^{-1}\left(\dfrac{x}{3}\right) + \dfrac{\sqrt{3-x^2}}{x} + C$

$$= \int \frac{3\sin^2\theta}{\cancel{\sqrt{3}\cos\theta}} \cdot \cancel{\sqrt{3}\cos\theta}\, d\theta$$

$$= \frac{3}{2}\int 1-\cos(2\theta)\, d\theta$$

$$= \frac{3}{2}\left[\theta - \frac{1}{2}\sin(2\theta)\right] + C$$

$$= \frac{3}{2}\left[\theta - \sin\theta\cos\theta\right] + C$$

$$= \frac{3}{2}\left[\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) - \frac{x}{\sqrt{3}}\cdot\frac{\sqrt{3-x^2}}{\sqrt{3}}\right] + C$$

$$= \frac{3}{2}\sin^{-1}\left(\frac{x}{\sqrt{3}}\right) - \frac{1}{2}x\sqrt{3-x^2} + C$$

#25 § 8.5

16. $\displaystyle\int \frac{\sin x}{\cos x + \cos^2 x}\,dx =$

$u = \cos x \Rightarrow du = -\sin x\, dx$

$$= \int \frac{-1}{u+u^2}\,du = -\int \frac{1}{u(u+1)}\,du$$

(a) $\ln|1+\sec x| + C$ (correct)

(b) $\ln|1-\sec x| + C$

(c) $\ln|1+\cos x| + C$

(d) $\ln|1-\cos x| + C$

(e) $\ln|\sin x + \cos x| + C$

partial fractions

$$\frac{1}{u(u+1)} = \frac{A}{u} + \frac{B}{u+1}$$

$\Rightarrow 1 = A(u+1) + Bu$

$u=0 \Rightarrow 1 = A$

$u=-1 \Rightarrow 1 = -B \Rightarrow B = -1$

$$= -\int\left(\frac{1}{u} - \frac{1}{u+1}\right)du$$

$$= -\left[\ln|u| - \ln|u+1|\right] + C$$

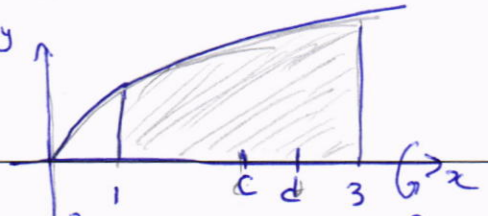
$$= \ln|u+1| - \ln|u| + C$$

$$= \ln\left|\frac{u+1}{u}\right| + C$$

$$= \ln\left|\frac{\cos x + 1}{\cos x}\right| + C = \ln|1+\sec x| + C$$

#58 §7.2

17. Let S be the solid formed by revolving the region bounded by the curves $y = \sqrt{x}$, $y = 0$, $x = 1$, and $x = 3$ about the x-axis. If c and d are two different numbers in the interval $[1, 3]$ that divide the solid S into three parts of equal volume, then $c^2 + d^2 =$



(a) 10 ________________ (correct)

(b) $\frac{8}{3}$

(c) $\frac{\sqrt{11}}{3}$

(d) $\frac{\sqrt{8}}{3}$

(e) $\frac{10}{3}$

Disk Method : $V = \int_1^3 \pi(\sqrt{x})^2\,dx = \pi\int_1^3 x\,dx = \frac{\pi}{2}x^2\Big]_1^3 = 4\pi$

- $V_1 = \int_1^c \pi(\sqrt{x})^2\,dx \Rightarrow \frac{4\pi}{3} = \int_1^c \pi x\,dx = \frac{\pi}{2}(x^2\Big]_1^c = \frac{\pi}{2}(c^2-1)$

$\Rightarrow c^2 - 1 = \frac{8}{3} \Rightarrow c^2 = \frac{11}{3}$

- $V_3 = \int_d^3 \pi(\sqrt{x})^2\,dx \Rightarrow \frac{4\pi}{3} = \frac{\pi}{2}x^2\Big]_d^3 = \frac{\pi}{2}(9-d^2)$

$\Rightarrow 9 - d^2 = \frac{8}{3} \Rightarrow d^2 = 9 - \frac{8}{3} = \frac{19}{3}$

- $c^2 + d^2 = \frac{11}{3} + \frac{19}{3} = \frac{30}{3} = 10$

18. If $\alpha = 2\tan^{-1}(-3)$, then $\Rightarrow \frac{\alpha}{2} = \tan^{-1}(-3) \Rightarrow \tan(\frac{\alpha}{2}) = -3$

~#57 §8.7

$$\int_\alpha^0 \frac{1}{\cos x - 3\sin x + 3}\,dx =$$

$x = 0 \Rightarrow u = 0$ & $x = \alpha \Rightarrow u = \tan(\frac{\alpha}{2}) = -3$

[Hint: Use the substitution $u = \tan\left(\frac{x}{2}\right)$, $-\pi < x < \pi$]

$= \int_{-3}^0 \frac{1}{\frac{1-u^2}{1+u^2} - 3\frac{2u}{1+u^2} + 3} \cdot \frac{2}{1+u^2}\,du$

(a) $\ln\left(\frac{8}{5}\right)$ ________________ (correct)

(b) $\ln\left(\frac{7}{5}\right)$

(c) $\ln\left(\frac{6}{5}\right)$

(d) $\ln\left(\frac{9}{5}\right)$

(e) $\ln(2)$

$= \int_{-3}^0 \frac{1}{1-u^2-6u+3+3u^2}\,du = \int_{-3}^0 \frac{2}{2u^2-6u+4}\,du$

$= \frac{2}{2}\int_{-3}^0 \frac{1}{u^2-3u+2}\,du$: $\frac{1}{(u-1)(u-2)} = \frac{A}{u-1} + \frac{B}{u-2}$

$1 = A(u-2) + B(u-1)$

$u = 1 \Rightarrow 1 = -A \Rightarrow A = -1$

$u = 2 \Rightarrow 1 = B$

$= \int_{-3}^0 \frac{-1}{u-1} + \frac{1}{u-2}\,du$

$= -\ln|u-1| + \ln|u-2|\Big]_{-3}^0$

$= \ln\left|\frac{u-2}{u-1}\right|\Big]_{-3}^0$

$= \ln 2 - \ln\left|\frac{-5}{-4}\right| = \ln 2 - \ln\frac{5}{4} = \ln 2 + \ln\frac{4}{5}$

$= \ln\left(\frac{8}{5}\right)$