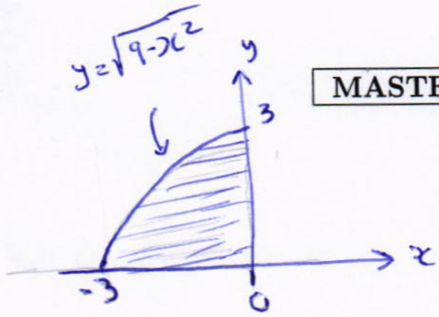
Answer KEY

Q	MASTER	CODE01	CODE02	CODE03	CODE04
1	A	C $_{3}$	D $_{5}$	A $_{6}$	B $_{5}$
2	A	E $_{6}$	A $_{7}$	C $_{1}$	A $_{2}$
3	A	C $_{5}$	C $_{4}$	C $_{2}$	B $_{1}$
4	A	E $_{1}$	B $_{6}$	E $_{4}$	A $_{4}$
5	A	C $_{4}$	E $_{3}$	B $_{5}$	A $_{3}$
6	A	E $_{2}$	B $_{1}$	C $_{7}$	E $_{6}$
7	A	D $_{7}$	D $_{2}$	B $_{3}$	D $_{7}$
8	A	D $_{9}$	A $_{10}$	A $_{11}$	E $_{8}$
9	A	C $_{14}$	B $_{12}$	D $_{9}$	C $_{10}$
10	A	C $_{11}$	E $_{9}$	A $_{12}$	B $_{12}$
11	A	E $_{13}$	D $_{11}$	B $_{8}$	C $_{11}$
12	A	E $_{12}$	B $_{14}$	E $_{14}$	C $_{13}$
13	A	C $_{8}$	C $_{13}$	D $_{13}$	D $_{14}$
14	A	C $_{10}$	E $_{8}$	A $_{10}$	D $_{9}$
15	A	B $_{20}$	B $_{17}$	D $_{18}$	C $_{20}$
16	A	A $_{16}$	B $_{15}$	B $_{16}$	C $_{15}$
17	A	D $_{21}$	C $_{16}$	B $_{21}$	E $_{21}$
18	A	E $_{17}$	C $_{18}$	E $_{19}$	C $_{17}$
19	A	E $_{15}$	E $_{19}$	B $_{20}$	C $_{16}$
20	A	C $_{18}$	B $_{21}$	A $_{15}$	C $_{19}$
21	A	A $_{19}$	B $_{20}$	B $_{17}$	E $_{18}$
22	A	E $_{25}$	C $_{26}$	D $_{24}$	A $_{22}$
23	A	D $_{27}$	C $_{22}$	D $_{22}$	A $_{28}$
24	A	A $_{26}$	E $_{23}$	A $_{26}$	E $_{25}$
25	A	A $_{28}$	A $_{25}$	D $_{23}$	A $_{23}$
26	A	D $_{23}$	E $_{27}$	A $_{25}$	C $_{26}$
27	A	D $_{24}$	B $_{28}$	C $_{27}$	E $_{27}$
28	A	C $_{22}$	D $_{24}$	B $_{28}$	D $_{24}$

Detailed Solution



1. $\int_{-3}^{0} \sqrt{9-x^2}\,dx =$ area of a quarter of a circle of radius 3

$= \frac{1}{4} \cdot \pi (3)^2$

$= \frac{9\pi}{4}$

(a) $\frac{9\pi}{4}$ ______ (correct)

(b) $-\frac{9\pi}{4}$

(c) $\frac{9\pi}{2}$

(d) $\frac{3\pi}{4}$

(e) $\frac{3\pi}{2}$

~ Example 3(c) § 5.3

2. $\int \frac{2x}{(x+1)^2}\,dx =$ Let $u = x+1$. Then $du = dx$ and $x = u-1$.

$= \int \frac{2(u-1)}{u^2}\,du$

(a) $2\ln|x+1| + \frac{2}{x+1} + C$ ______ (correct)

(b) $\frac{1}{2}\ln|x+1| + 2(x+1) + C$

(c) $\frac{1}{(x+1)^2} + \frac{2}{x+1} + C$

(d) $-2x + \ln|x+1| + C$

(e) $2x - \arctan x + C$

$= 2\int \left(\frac{1}{u} - \frac{1}{u^2}\right) du$

$= 2\left[\ln|u| + \frac{1}{u}\right] + C$

$= 2\ln|x+1| + \frac{2}{x+1} + C$

Example 6 § 5.7

3. If $F(x) = \int_{\sqrt{x}}^{\sqrt{2}} 4\cos(t^4)\,dt$, then $F(2) + F'(2) =$

$\cdot\ F(2) = \int_{\sqrt{2}}^{\sqrt{2}} 4\cos(t^4)\,dt = 0$

(a) $-\sqrt{2}\cos 4$ (correct)

(b) $1 - \sqrt{2}\cos 4$

(c) $\frac{\cos 4}{\sqrt{2}}$

(d) $1 - \frac{\cos\sqrt{2}}{\sqrt{2}}$

(e) $\sqrt{2}\cos(2\sqrt{2})$

$\cdot\ F(x) = -\int_{\sqrt{2}}^{\sqrt{x}} 4\cos(t^4)\,dt$

$F'(x) = -4\cos((\sqrt{x})^4) \cdot \frac{d}{dx}(\sqrt{x})$

$= -4\cos(x^2) \cdot \frac{1}{2\sqrt{x}}$

$= \frac{-2\cos(x^2)}{\sqrt{x}}$

$\cdot\ F'(2) = \frac{-2\cos(4)}{\sqrt{2}} = -\sqrt{2}\cos 4$

$\cdot\ F(2) + F'(2) = 0 - \sqrt{2}\cos 4 = -\sqrt{2}\cos 4$

~ #86
§5.4

4. The improper integral $\int_{-\infty}^{0} e^{3x}\,dx$ $= \lim_{t\to-\infty} \int_t^0 e^{3x}\,dx$

$= \lim_{t\to-\infty} \frac{1}{3} e^{3x}\Big]_t^0$

(a) converges to $\frac{1}{3}$ (correct)

(b) converges to $-\frac{2}{3}$

(c) converges to e^3

(d) converges to $3e$

(e) diverges

$= \lim_{t\to-\infty} \frac{1}{3}(1 - e^{3t})$

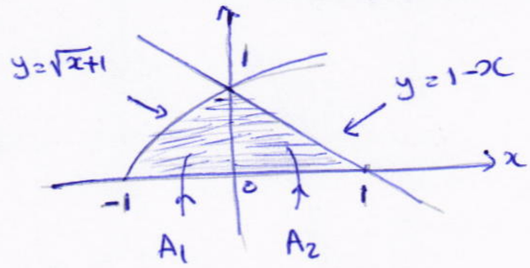
$= \frac{1}{3}(1 - 0) = \frac{1}{3}$

#16
§8.8

5. The **area** of the region enclosed by the curves

$$y = \sqrt{x+1},\ y = 1 - x, \text{ and } y = 0$$

is



(a) $\frac{7}{6}$ ______ (correct)

(b) $\frac{1}{6}$

(c) $\frac{5}{6}$

(d) $\frac{11}{6}$

(e) $\frac{13}{6}$

$$A_1 = \int_{-1}^{0} \sqrt{x+1}\, dx = \frac{2}{3}(x+1)^{3/2}\Big]_{-1}^{0} = \frac{2}{3}(1-0) = \frac{2}{3}$$

$$A_2 = \int_{0}^{1} (1-x)\, dx = x - \frac{x^2}{2}\Big]_{0}^{1} = \left(1-\frac{1}{2}\right) - 0 = \frac{1}{2}$$

$$A = A_1 + A_2 = \frac{2}{3} + \frac{1}{2} = \frac{7}{6}$$

~ #22
§7.1

6. $\displaystyle\int_{\ln 2}^{\ln\left(\frac{2}{\sqrt{3}}\right)} \frac{e^{-x}}{\sqrt{1-e^{-2x}}}\, dx =$

Let $u = e^{-x}$. Then

$du = -e^{-x}\, dx$

$x = \ln 2 \Rightarrow u = e^{-\ln 2} = \frac{1}{2}$

$x = \ln\left(\frac{2}{\sqrt{3}}\right) \Rightarrow u = e^{-\ln\left(\frac{2}{\sqrt{3}}\right)} = \frac{\sqrt{3}}{2}$

(a) $-\frac{\pi}{6}$ ______ (correct)

(b) $\frac{\pi}{3}$

(c) $-\frac{\pi}{3}$

(d) $\frac{\pi}{6}$

(e) π

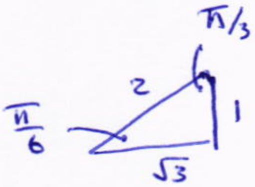
$$= \int_{1/2}^{\sqrt{3}/2} \frac{-1}{\sqrt{1-u^2}}\, du$$

$$= -\sin^{-1} u\Big]_{1/2}^{\sqrt{3}/2}$$

$$= -\left(\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right)\right)$$

$$= -\left(\frac{\pi}{3} - \frac{\pi}{6}\right)$$

$$= -\frac{\pi}{6}$$



~ #30
§5.8

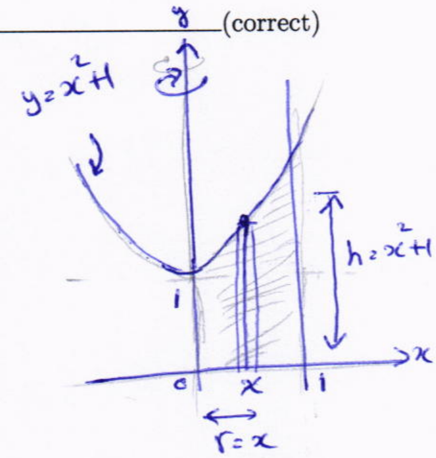
7. The **volume** of the solid generated by rotating the region bounded by the curves $y = x^2 + 1$, $y = 0$, $x = 0$ and $x = 1$ about the y-axis is

(a) $\frac{3\pi}{2}$ ______________________________ (correct)
(b) $\frac{\pi}{2}$
(c) $\frac{3\pi}{4}$
(d) $\frac{\pi}{4}$
(e) 2π

Example 3 §7.3

By the Shell Method

$$V = \int_0^1 2\pi \cdot x \cdot (x^2+1)\, dx$$
$$= 2\pi \int_0^1 (x^3 + x)\, dx$$
$$= 2\pi \cdot \left[\frac{x^4}{4} + \frac{x^2}{2} \right]_0^1$$
$$= 2\pi \cdot \left(\frac{1}{4} + \frac{1}{2} - 0\right)$$
$$= 2\pi \cdot \frac{3}{4}$$
$$= \frac{3\pi}{2}$$



8. A particle moves along a line so that its velocity at time t is

$$v(t) = \cos t \quad (\text{in} \quad \text{m/s})$$

The **total distance** traveled by the particle during the time interval $0 \leq t \leq \pi$ is

(a) 2 ______________________________ (correct)
(b) 0
(c) 1
(d) $\frac{1}{2}$
(e) 4

~ #98 §5.4

$$d = \int_0^{\pi} |\cos t|\, dt$$
$$= \int_0^{\pi/2} \cos t \, dt + \int_{\pi/2}^{\pi} -\cos t \, dt$$
$$= \sin t \Big]_0^{\pi/2} - \sin t \Big]_{\pi/2}^{\pi}$$
$$= (1 - 0) - (0 - 1)$$
$$= 1 + 1$$
$$= 2$$

9. $\displaystyle\int \frac{x+\ln(e^{2x-1})}{2x}\,dx =$ $\displaystyle\int \frac{x+(2x-1)}{2x}\,dx = \int \frac{3x-1}{2x}\,dx$

$= \frac{1}{2}\int\left(3-\frac{1}{x}\right)dx$

(a) $\frac{3}{2}x - \frac{1}{2}\ln|x| + C$ ____________________ (correct)

(b) $\frac{1}{2}x - \frac{1}{2x-1} + C$

(c) $\frac{1}{2}x + e^{2x-1} + C$

(d) $\frac{3}{2}x + \ln|x| + C$

(e) $\frac{1}{2}x - \frac{1}{2}\ln|2x-1| + C$

$= \frac{1}{2}\left[3x - \ln|x|\right] + C$

$= \frac{3}{2}x - \frac{1}{2}\ln|x| + C$

~ #54 §5.5

10. Let a be a real number such that $0 < a < 8$, and let

$$f(x) = \begin{cases} 4, & x < a \\ x, & x \geq a \end{cases}$$

If $\displaystyle\int_0^8 f(x)dx = 40$, then $2a - 5 =$

(a) 3 ____________________ (correct)

(b) −4

(c) 6

(d) −5

(e) 0

$\int_0^8 f(x)\,dx = \int_0^a f(x)\,dx + \int_a^8 f(x)\,dx$

$40 = \int_0^a 4\,dx + \int_a^8 x\,dx$

$40 = 4x\Big]_0^a + \frac{1}{2}x^2\Big]_a^8$

$40 = 4a + \frac{1}{2}(64 - a^2)$

$\Rightarrow 80 = 8a + 64 - a^2$

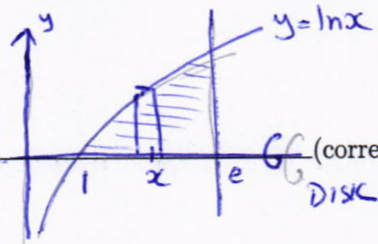
$\Rightarrow a^2 - 8a + 16 = 0$

$\Rightarrow (a-4)^2 = 0 \Rightarrow a = 4$

$\Rightarrow 2a - 5 = 8 - 5 = 3$

~ #55 §5.3

11. The **volume** of the solid generated by revolving the region bounded by the graphs of $y = \ln x$, $y = 0$ and $x = e$ about the x-axis is



(a) $\pi(e-2)$ _____ (correct) Disk

(b) πe^2

(c) $2\pi(2e-1)$

(d) $\pi(1+\ln 2)$

(e) $\pi(2+\ln 2)$

#87(b)
§ 8.2

Disk Method

$V = \pi \int_1^e (\ln x)^2 \, dx$

$= \pi \left[x(\ln x)^2 - \int 2 \ln x \, dx \right]$

$u = (\ln x)^2 \quad dv = dx$

$du = \frac{2\ln x}{x} dx, \quad v = x$

$= \pi \left[x(\ln x)^2 - 2(x\ln x - x) \right]_1^e$

$= \pi \left[(e - 2e + 2e) - (0 - 0 + 2) \right]$

$= \pi (e - 2)$

$u = \ln x \quad dv = dx$

$du = \frac{1}{x} dx, \quad v = x$

$\int \ln x \, dx = x\ln x - \int dx$

$= x\ln x - x + C$

$x^3 + x^2 - x - 1 = x^2(x+1) - (x+1) = (x+1)(x^2-1) = (x-1)(x+1)^2$

12. $\displaystyle\int_2^3 \frac{-4x}{x^3+x^2-x-1}\,dx = \int_2^3 \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2}$ decomp.

$\Rightarrow -4x = A(x+1)^2 + B(x-1)(x+1) + C(x-1)$

$x = 1 \Rightarrow -4 = 4A + 0 + 0 \Rightarrow \boxed{A = -1}$

$x = -1 \Rightarrow 4 = 0 + 0 - 2C \Rightarrow \boxed{C = -2}$

$x = 0 \Rightarrow 0 = A - B - C = -1 - B + 2 = 1 - B \Rightarrow \boxed{B = 1}$

(a) $\ln\left(\frac{2}{3}\right) - \frac{1}{6}$ _____ (correct)

(b) $\ln\left(\frac{3}{2}\right) + \frac{1}{6}$

(c) $\ln\left(\frac{1}{6}\right) - \frac{2}{3}$

(d) $\ln\left(\frac{2}{3}\right) + \frac{3}{2}$

(e) $\ln\left(\frac{1}{6}\right) + \frac{1}{6}$

$= \int_2^3 \frac{-1}{x-1} + \frac{1}{x+1} - \frac{2}{(x+1)^2} \, dx$

$= -\ln|x-1| + \ln|x+1| + \frac{2}{x+1} \Big]_2^3$

$= \ln\left|\frac{x+1}{x-1}\right| + \frac{2}{x+1} \Big]_2^3$

$= \ln\left(\frac{4}{2}\right) + \frac{2}{4} - \left(\ln\left(\frac{3}{1}\right) + \frac{2}{3}\right)$

$= \ln 2 - \ln 3 + \frac{1}{2} - \frac{2}{3}$

$= \ln\left(\frac{2}{3}\right) - \frac{1}{6}$

~ #11
§ 8.5

13. $\displaystyle\int_{3/2}^{9/4} \frac{1}{\sqrt{3x-x^2}}\,dx =$

(a) $\frac{\pi}{6}$ (correct)

(b) $\frac{2\pi}{3}$

(c) $\frac{\pi}{4}$

(d) $\frac{\pi}{3}$

(e) $\frac{5\pi}{6}$

$3x - x^2 = -(x^2-3x) = -(x^2 - 3x + \frac{9}{4} - \frac{9}{4})$

$= \frac{9}{4} - (x-\frac{3}{2})^2$

Let $u = x - \frac{3}{2}$. Then

- $du = dx$
- $x = \frac{3}{2} \Rightarrow u = 0$
- $x = \frac{9}{4} \Rightarrow u = \frac{9}{4} - \frac{3}{2} = \frac{3}{4}$

$\displaystyle\int_0^{3/4} \frac{1}{\sqrt{\frac{9}{4}-u^2}}\,du = \sin^{-1}\left(\frac{2}{3}u\right)\Big]_0^{3/4}$

$= \sin^{-1}\left(\frac{1}{2}\right) - \sin^{-1}(0)$

$= \frac{\pi}{6} - 0$

$= \frac{\pi}{6}$

Example 5
§ 5.8

14. The improper integral $\displaystyle\int_3^{\infty} \frac{1}{x\sqrt{x^2-9}}\,dx = \overset{(i)}{\int_3^4 \frac{1}{x\sqrt{x^2-9}}\,dx} + \overset{(ii)}{\int_4^{\infty} \frac{1}{x\sqrt{x^2-9}}\,dx}$

(a) converges to $\frac{\pi}{6}$ (correct)

(b) converges to $\frac{\pi}{2}$

(c) converges to $\ln 3$

(d) converges to $2\ln 3$

(e) diverges

(i). $\displaystyle\int_3^4 = \lim_{t\to 3^+} \int_t^4 \frac{1}{x\sqrt{x^2-9}}\,dx$

$= \lim_{t\to 3^+} \frac{1}{3}\sec^{-1}\frac{|x|}{3}\Big]_t^4$

$= \lim_{t\to 3^+} \frac{1}{3}\sec^{-1}\frac{|4|}{3} - \frac{1}{3}\sec^{-1}\frac{|t|}{3} = \frac{1}{3}\sec^{-1}\frac{4}{3} - 0$

$= \frac{1}{3}\sec^{-1}\frac{4}{3}$

(ii) $\displaystyle\int_4^{\infty} = \lim_{t\to\infty} \int_4^t \frac{1}{x\sqrt{x^2-9}}\,dx$

$= \lim_{t\to\infty} \frac{1}{3}\sec^{-1}\frac{|x|}{3}\Big]_4^t$

$= \lim_{t\to\infty} \frac{1}{3}\sec^{-1}\frac{|t|}{3} - \frac{1}{3}\sec^{-1}\left(\frac{4}{3}\right)$

$= \frac{1}{3}\cdot\frac{\pi}{2} - \frac{1}{3}\sec^{-1}\left(\frac{4}{3}\right)$

#45
§ 8.8

Sum $= \frac{\pi}{6}$

15. The sequence $\left\{ \dfrac{\cos(2n)}{3^n} \right\}$

$-1 \le \cos(2n) \le 1$ for all n

$-\dfrac{1}{3^n} \le \dfrac{\cos(2n)}{3^n} \le \dfrac{1}{3^n}$

$\downarrow$ 0 $\quad\quad$ $\downarrow$ 0

(a) converges to 0 ______________________________ (correct)

(b) converges to $\dfrac{1}{3}$

(c) converges to $\dfrac{2}{3}$

(d) converges to 2

(e) diverges

By the squeeze Theorem $\lim_{n\to\infty} \dfrac{\cos(2n)}{3^n} = 0$

#44
§ 9.1

16. Which one of the following series is **convergent**?

p-Series

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^3}}$ $\quad p = \frac{3}{2} > 1 \Rightarrow$ Conv. ______________________________ (correct)

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n^3}}$ $\quad p = 1 \Rightarrow$ div.

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[4]{n^3}}$ $\quad p = \frac{3}{4} \le 1 \Rightarrow$ div

(d) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[5]{n^3}}$ $\quad p = \frac{3}{5} \le 1 \Rightarrow$ div.

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt[6]{n^3}}$ $\quad p = \frac{3}{6} = \frac{1}{2} \le 1 \Rightarrow$ div.

§ 9.3
p-Series

17. The series $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3}$

$$\int_2^{\infty} \frac{1}{x(\ln x)^3}\,dx = \lim_{t\to\infty} \int_2^{t} \frac{1}{x(\ln x)^3}\,dx$$

$$= \lim_{t\to\infty} \left.\frac{-1}{2(\ln x)^2}\right]_2^t$$

(a) converges by the integral test ______________ (correct)

(b) diverges by the integral test

(c) is a series for which the integral test is not applicable

(d) diverges by the n^{th}-term test for divergence

(e) is a convergent geometric series

$$= \lim_{t\to\infty} -\frac{1}{2(\ln t)^2} + \frac{1}{2(\ln 2)^2}$$

$$= 0 + \frac{1}{2(\ln 2)^2} = \frac{1}{2(\ln 2)^2}$$

Conv.

~ #16

§ 9.3

18. The second Taylor polynomial for $f(x) = \sqrt{x}$ centered at $c = 4$ is

(a) $P_2(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$ ______________ (correct)

(b) $P_2(x) = 2 + \frac{1}{4}(x-4) - \frac{1}{32}(x-4)^2$

(c) $P_2(x) = 2 - \frac{1}{4}(x-4) + \frac{1}{32}(x-4)^2$

(d) $P_2(x) = 2 - \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$

(e) $P_2(x) = 2 + (x-4) + (x-4)^2$

$$f(4) = \sqrt{4} = 2$$

$$f'(x) = \frac{1}{2\sqrt{x}} \Rightarrow f'(4) = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

$$= \frac{1}{2}x^{-1/2}$$

$$f''(x) = -\frac{1}{4}x^{-3/2}$$

$$f''(4) = -\frac{1}{4}\cdot 4^{-3/2}$$

$$= -\frac{1}{4}\cdot 2^{-3} = -\frac{1}{4}\cdot\frac{1}{8}$$

$$= -\frac{1}{32}$$

$$P_2(x) = f(4) + f'(4)(x-4) + \frac{f''(4)}{2!}(x-4)^2$$

$$= 2 + \frac{1}{4}(x-4) - \frac{1}{64}(x-4)^2$$

$$\frac{f''(4)}{2} = -\frac{1}{64}$$

#29

§ 9.7

19. The series $\sum_{n=0}^{\infty} \left(\frac{2}{x}\right)^n$

a geometric series with $a = 1$ and $r = \frac{2}{x}$

$\therefore$ It conv. iff $|r| < 1 \iff \left|\frac{2}{x}\right| < 1 \iff |x| > 2$

$\iff x > 2$ or $x < -2$

$\text{Sum} = \frac{a}{1-r} = \frac{1}{1-\frac{2}{x}} = \frac{x}{x-2}$

(a) converges for $x < -2$ or $x > 2$ and its sum is $\dfrac{x}{x-2}$ ________ (correct)

(b) converges for $-2 < x < 2$ and its sum is $\dfrac{x}{x-2}$

(c) converges for $x \geq -2$ and its sum is $\dfrac{1}{x-2}$

(d) converges for $x < -1$ or $x > 1$ and its sum is $\dfrac{x}{x-1}$

(e) diverges for all values of x

#62
§ 9.2

20. Which one of the following series is **convergent**?

(a) $\sum_{n=1}^{\infty} (-1)^n \sin\left(\frac{1}{n}\right)$ Conv. by AST. ________ (correct)

(b) $\sum_{n=1}^{\infty} (-1)^n \cos\left(\frac{1}{n}\right)$

(c) $\sum_{n=1}^{\infty} (-1)^n$

(d) $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$

(e) $\sum_{n=1}^{\infty} (-1)^n \sqrt{n}$

all div. as $a_n \not\to 0$

§ 9.5
alternating series

21. The series $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2+\sqrt{n}}$

(i) $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{2+\sqrt{n}}$ Conv. by AST

(ii) $\sum_{n=1}^{\infty} \left|\frac{(-1)^{n+1}}{2+\sqrt{n}}\right| = \sum_{n=1}^{\infty} \frac{1}{2+\sqrt{n}}$ Div. by LCT with $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

(a) converges conditionally ______ (correct)

(b) converges absolutely

(c) diverges

(d) converges by the ratio test

(e) diverges by the n^{th}-term test for divergence

So the Series is Conditionally Convergent

~ #9
§ 9.5

22. The n^{th} partial sum of the series

$$\sum_{n=1}^{\infty} \frac{4}{n(n+2)}$$

is

(a) $S_n = \frac{n(3n+5)}{(n+1)(n+2)}$ ______ (correct)

(b) $S_n = \frac{(n+2)(n+3)}{n(n+1)}$

(c) $S_n = \frac{3n+5}{n+2}$

(d) $S_n = \frac{n}{(n+1)(n+3)}$

(e) $S_n = \frac{2n+1}{(n+1)(n+2)}$

$$S_n = \sum_{k=1}^{n} \frac{4}{k(k+2)} = \sum_{k=1}^{n} \left(\frac{2}{k} - \frac{2}{k+2}\right)$$

$$= 2\sum_{k=1}^{n} \left(\frac{1}{k} - \frac{1}{k+2}\right)$$

$$= 2\left[\left(1-\frac{1}{3}\right) + \left(\frac{1}{2}-\frac{1}{4}\right) + \left(\frac{1}{3}-\frac{1}{5}\right) + \left(\frac{1}{4}-\frac{1}{6}\right) + \left(\frac{1}{5}-\frac{1}{7}\right) + \cdots + \left(\frac{1}{n-2}-\frac{1}{n}\right) + \left(\frac{1}{n-1}-\frac{1}{n+1}\right) + \left(\frac{1}{n}-\frac{1}{n+2}\right)\right]$$

$$= 2\left[1+\frac{1}{2} - \frac{1}{n+1} - \frac{1}{n+2}\right]$$

$$= 2\left[\frac{3}{2} - \frac{2n+3}{(n+1)(n+2)}\right]$$

$$= 3 - \frac{4n+6}{(n+1)(n+2)}$$

$$= \frac{3(n^2+3n+2) - 4n - 6}{(n+1)(n+2)}$$

$$= \frac{3n^2+9n+6-4n-6}{(n+1)(n+2)}$$

$$= \frac{3n^2+5n}{(n+1)(n+2)} = \frac{n(3n+5)}{(n+1)(n+2)}$$

#24
§ 9.2

23. Which one of the following statement is **FALSE**?

F: Take $a_n = \frac{1}{n}$. Then $\sum a_n$ div. & $\sum(a_n + \frac{1}{n}) = \sum \frac{2}{n} = 2\sum \frac{1}{n}$ div.

(a) If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\sum_{n=1}^{\infty}\left(a_n + \frac{1}{n}\right)$ is convergent ______(correct)

(b) If $a_n \geq 2^n$, $n \geq 1$, then $\sum_{n=1}^{\infty} a_n$ is divergent T, by Comparison test

(c) If $0 \leq a_n \leq \frac{1}{n^2}$, $n \geq 1$, then $\sum_{n=1}^{\infty} a_n$ is convergent T, by Comparison test

(d) If $\sum_{n=1}^{\infty} a_n$ is convergent, then $\sum_{n=1}^{\infty}\left(a_n + \frac{1}{n}\right)$ is divergent, T since $\sum \frac{1}{n}$ div.

(e) If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\sum_{n=1}^{\infty} 2\,a_n$ is divergent T since $\sum 2a_n = 2\sum a_n$

§ 9.2 & § 9.4 properties

24. A power series representation for $f(x) = \frac{4}{2+x^2}$, centered at 0, is given by

Use $\frac{1}{1+x} = \sum_{n=0}^{\infty} (-1)^n x^n$, $|x|<1$

(a) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{2^{n-1}}$ ______(correct)

(b) $\sum_{n=0}^{\infty} \frac{x^{2n}}{2^{n-1}}$

(c) $\sum_{n=0}^{\infty}(-1)^n \frac{x^n}{2^n}$

(d) $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n-1}}{2^{n-1}}$

(e) $\sum_{n=0}^{\infty} 2^{n+1} \cdot x^{2n}$

$$\frac{4}{2+x^2} = \frac{4}{2(1+\frac{x^2}{2})}$$

$$= 2 \cdot \frac{1}{1+\frac{x^2}{2}} = 2 \cdot \sum_{n=0}^{\infty} (-1)^n \left(\frac{x^2}{2}\right)^n, \quad \left|\frac{x^2}{2}\right| < 1$$

$$= 2\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{2^n}, \quad |x^2| < 2$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{2^{n-1}}, \quad |x| < \sqrt{2}$$

~ Example 1 & #27

§ 9.9

25. The interval of convergence I and the radius of convergence R of the power series

$$\sum_{n=1}^{\infty} \frac{(-1)^n \cdot 2^n \cdot x^n}{(n+1)(n+2)}$$

are

a_n

$$\left|\frac{a_{n+1}}{a_n}\right| = \left| \frac{(-1)^{n+1} \cdot 2^{n+1} \cdot x^{n+1}}{(n+2)(n+3)} \cdot \frac{(n+1)(n+2)}{(-1)^n \cdot 2^n \cdot x^n} \right|$$

$$= \frac{2(n+1)\cancel{(n+2)}}{n+3} |x| \longrightarrow 2|x|$$

Conv. if $2|x| < 1 \iff |x| < \frac{1}{2} \iff -\frac{1}{2} < x < \frac{1}{2}$

(a) $I = \left[-\frac{1}{2}, \frac{1}{2}\right]$, $R = \frac{1}{2}$ ______ (correct)

(b) $I = \left[-\frac{1}{2}, \frac{1}{2}\right]$, $R = 1$

(c) $I = \left(-\frac{1}{2}, \frac{1}{2}\right)$, $R = \frac{1}{2}$

(d) $I = \{0\}$, $R = 0$

(e) $I = (-\infty, \infty)$, $R = \infty$

endpts

$x = -\frac{1}{2}$: $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$ Conv. by LCT, $\sum \frac{1}{n^2}$

$x = \frac{1}{2}$: $\sum_{n=1}^{\infty} \frac{(-1)^n}{(n+1)(n+2)}$ Conv. by AST.

$I = \left[-\frac{1}{2}, \frac{1}{2}\right]$, $R = \frac{1}{2}$

~ #22
§ 9.8

a_n

26. The series $\sum_{n=1}^{\infty} \frac{3 \cdot 5 \cdot 7 \ldots (2n+1)}{18^n \cdot (2n-1) \cdot n!}$;

$$\frac{a_{n+1}}{a_n} = \frac{\cancel{3 \cdot 5 \cdot 7 \cdots (2n+1)} \cdot (2n+3)}{\cancel{18^{n+1}}_{18} \cdot (2n+1) \cdot \cancel{(n+1)!}_{n+1}} \cdot \frac{\cancel{18^n} \cdot (2n-1) \cdot \cancel{n!}}{\cancel{3 \cdot 5 \cdot 7 \cdots (2n+1)}}$$

$$= \frac{(2n+3)(2n-1)}{18 \cdot (2n+1) \cdot (n+1)} \longrightarrow \frac{4}{18 \cdot 2} = \frac{1}{9} < 1$$

$\Rightarrow$ Conv. by the Ratio Test.

(a) converges by the ratio test ______ (correct)

(b) diverges by the ratio test

(c) is a series with which the ratio test is inconclusive

(d) converges by comparing it with $\sum_{n=1}^{\infty} \frac{1}{2n-1}$

(e) diverges by comparing it with $\sum_{n=1}^{\infty} \frac{1}{18^n}$

#70
§ 9.6

27. $\int_0^2 xe^{-2x^3}\,dx =$

(a) $\sum_{n=0}^{\infty}(-1)^n \frac{2^{4n+2}}{(3n+2)\cdot n!}$ ______ (correct)

(b) $\sum_{n=0}^{\infty}(-1)^n \frac{2^{3n+2}}{(3n+2)\cdot n!}$

(c) $\sum_{n=0}^{\infty}(-1)^n \frac{2^{3n+1}}{(3n+1)\cdot n!}$

(d) $\sum_{n=0}^{\infty}(-1)^n \frac{2^{3n+3}}{(3n+3)\cdot n!}$

(e) $\sum_{n=0}^{\infty}(-1)^n \frac{2^{4n+1}}{(3n+1)\cdot n!}$

· Use $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, $|x|<\infty$

$$e^{-2x^3} = \sum_{n=0}^{\infty} \frac{(-2x^3)^n}{n!} = \sum_{n=0}^{\infty}(-1)^n \cdot \frac{2^n \cdot x^{3n}}{n!}$$

$$xe^{-2x^3} = \sum_{n=0}^{\infty}(-1)^n \frac{2^n \cdot x^{3n+1}}{n!}$$

$$\int_0^2 xe^{-2x^3}dx = \sum_{n=0}^{\infty}(-1)^n \frac{2^n}{n!}\int_0^2 x^{3n+1}\,dx$$

$$= \sum_{n=0}^{\infty}(-1)^n \frac{2^n}{n!} \cdot \left.\frac{x^{3n+2}}{3n+2}\right]_0^2$$

$$= \sum_{n=0}^{\infty}(-1)^n \frac{2^n}{n!} \cdot \left(\frac{2^{3n+2}}{3n+2} - 0\right)$$

$$= \sum_{n=0}^{\infty}(-1)^n \frac{2^{4n+2}}{(3n+2)\cdot n!}$$

~ # 63, 64
§ 9.10

28. $\sum_{n=2}^{\infty} \frac{(-1)^{n+1}\cdot 6}{(2n+1)!} =$

$$-6\sum_{n=2}^{\infty} \frac{(-1)^n}{(2n+1)!} = -6\left[\sum_{n=2}^{\infty}\frac{(-1)^n}{(2n+1)!} + \sum_{n=0}^{1}\frac{(-1)^n}{(2n+1)!} - \sum_{n=0}^{1}\frac{(-1)^n}{(2n+1)!}\right]$$

$$= -6\left[\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!} - \left(1-\frac{1}{6}\right)\right]$$

(a) $5-6\sin 1$ ______ (correct)

(b) $5+6\sin 1$

(c) $1-2\sin 1$

(d) $3+2\sin 1$

(e) $5-2\sin 1$

$$= -6\left[\sin 1 - \frac{5}{6}\right]$$

$$= -6\sin 1 + 5$$

$$= 5 - 6\sin 1.$$

~ # 56, 58
§ 9.10

$$\sin x = \sum_{n=0}^{\infty}(-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\sin 1 = \sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}$$